Teacher Notes

Activity 9

Back and Forth—Analysis of Spring Motion

Objectives

- Discover the relationships between position, velocity, and acceleration
- Connect mathematical relationships to real-world phenomena

Materials

- TI-84 Plus / TI-83 Plus
- A spring with a mass attached

Teaching Time

- 90 minutes

Abstract

In this activity, students will graph (in parametric mode) the motion of a mass moving back and forth on a spring. They will also graph the position, velocity, and acceleration versus time graphs which will allow them to observe the relationship between position and its first two derivatives. Speed is discussed and analyzed in terms of madnitude of velocity on various time intervals.

Management Tips and Hints

Prerequisites

Students should know:

- how to take derivatives of basic trigonometric functions.
- that the period is related to the B value in $y = A \cos(Bx)$ and that A represents amplitude in the same equation.

Student Engagement

This activity is best done with students working in cooperative groups so that they can compare answers and look for connections. A class discussion of basic concepts after the activity would enhance understanding and ensure that students made correct observations.

Evidence of Learning

Students are asked to make predictions prior to graphing equations. This allows them to correct their own thinking. A class discussion or analysis of student answers to the questions will demonstrate student knowledge.

Common Student Errors/Misconceptions

- In the study of motion in one dimension, many students confuse the graph with the picture of the motion.
- Students may also have difficulties in interpreting velocity and acceleration graphs for a physical situation.

Teaching Hints

Show the motion of a mass on a spring both vertically and horizontally to introduce the activity. It may be easier to demonstrate the motion vertically. When you hang a mass on the spring, it stretches the spring. The position where the mass hangs while at rest is called the equilibrium position. The mass will oscillate about that position.

Extensions

1. Students could use a Calculator-Based Ranger™ (CBR™) data collection device and collect data for the mass oscillating at the end of a spring. They will observe the same relationships shown in this activity.
2. Use a force probe and motion detector with a Calculator-Based Laboratory™2 (CBL2™) system to collect force and position data. A graph of the force versus position data produces a linear graph as predicted by Hooke's Law ($F = {}^{-}kx$).

Activity Solutions

1. Check students' graphs for correct amplitude, period, and starting point.

2. $y = 4\cos(x)$

3. *A* represents amplitude, or the distance that the spring was pulled from its unstretched length.

4. *B* is related to the period. It represents the number of times the spring repeats its motion in the natural period of the cosine function.

$$B = \frac{2\pi}{\text{Period}}$$

5. n/a

6. n/a

7. The following are possible viewing window settings. Answers will vary.

Xmin = ⁻5 Xmax = 15

Ymin = ⁻5 Ymax = 6

8. When the mass moves toward the left, the position versus time graph decreases; when the mass moves toward the right, the position versus time graph increases. The position versus time graph has a maximum or minimum point where the mass changes direction.

9. The graph decreases when the mass moves left.

10. The graph increases when the mass moves right.

11. The mass moves fastest when it passes through the $x = 0$ position and moves slowest when it changes direction.

12. Answers will vary. Look for students to make some connections between the answers to Questions **8–11** when they make the sketch.

13. Answers will vary depending on students' predictions.

14. The velocity is positive or above the time axis when the mass moves towards the right and negative or below the axis when the mass moves left.

15. On the position graph, negative velocity is represented by a decreasing graph. On the velocity graph, it is represented by the graph when it is below the time axis.

16.

- **a.** The velocity is zero when the mass is at its maximum or minimum displacement.
- **b.** The position versus time graph has a relative maximum or minimum at these times.
- **c.** The mass is turning around or changing directions at these times.
- **d.** Answers will vary.

17. Answers will vary.

18. Answers will vary depending on what the predictions were in Question **17**.

19. The position and acceleration graphs are reflections of each other about the time axis.

20. When the mass is to the right of its equilibrium position, the force exerted on the mass by the spring is back toward its equilibrium position. So the acceleration for this system always acts in the opposite direction from the displacement.

21. When the acceleration is a maximum or minimum, the velocity is zero and the mass is turning around or changing direction.

22. The mass moves the fastest when it passes through its equilibrium position where $x = 0$.

23.

- **a.** Speed is a maximum at the center where $x = 0$.
- **b.** Speed is zero when $x = {}^-4$ or $x = 4$.
- **c.** Acceleration is a maximum when $x = {}^-4$.
- **d.** Acceleration is a minimum when $x = 4$.
- **e.** Acceleration is positive when the mass is between ${}^-4$ and 0.
- **f.** Acceleration is negative when the mass is between 0 and 4.

24.

- **a.** Speeds up.
- **b.** Speeds up.
- **c.** Slows down.
- **d.** Slows down.

25. When velocity and acceleration act in the same direction, the object speeds up. When velocity and acceleration act in opposite directions, the object slows down.

26. The position versus time plot would repeat twice as often because the period is now π.

27. The equation for the position would now be $y = 4 \cos(2t)$, and the maximum velocity would now be 8 instead of 4. The maximum acceleration would now be 16 instead of 4. The chain rule would still be applied when taking the derivatives.

28. Answers will vary.