Getting "A-round" Area

Math Objectives

- Students will give an informal derivation of the relationship between the circumference and area of a circle (CCSS).
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

- circumference
- area
- pi
- radius of a circle
- parallelogram

About the Lesson

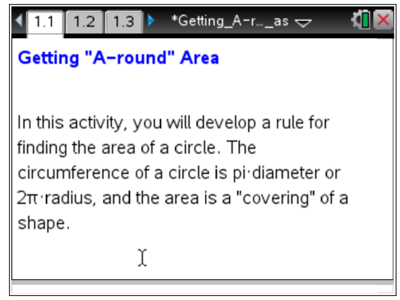
- The lesson assumes students have knowledge of the area formula for a parallelogram.
- The lesson involves using sectors of a circle to form a parallelogram and, from this shape, investigating the area formula for a circle.
- As a result, students will:
  - Use approximations for the circumference and radius of a circle.
  - Connect circumference and radius of a circle to the dimensions of the constructed parallelogram to develop an area formula for a circle.

TI-Nspire™ Navigator™ System

- Send out the *Getting_A-round.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
- Getting_A-round_Area_Student.pdf
- Getting_A-round_Area_Student.doc

TI-Nspire document
- Getting_A-round_Area.tns

Discussion Points and Possible Answers

You have probably already found the area of different kinds of polygons (closed shapes with straight sides) by using square tiles. Area is essentially a covering of the "space" inside the polygons measured in square units.

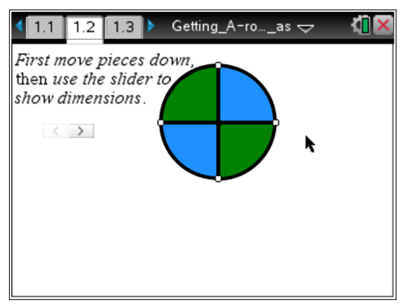
Finding areas of round shapes is a bit more of a challenge, especially when trying to cover the round shape with square tiles. The square tiles just won't fit "nicely". In this activity, you'll try another method to find the area of a circle. You will make the circle into another shape—one that's easier for calculating area.

You will need to know how to find the circumference (the distance around the outside, the perimeter) of a circle by using the formula $2\pi r$, where r is the radius of the circle.

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor (arrow) until it becomes a hand (☝) getting ready to grab the point. Also, be sure that the word point appears. Then select [ctrl] [click] to grab the point and close the hand (✊). When finished moving the point, select [esc] to release the point.

> **Tech Tip:** To drag a point, students should place their fingers over the point and drag it along the screen. The shapes in this simulation will move only along particular paths. If students experience difficulty dragging the shapes, they should try dragging the points in various directions.

Move to page 1.2.



1. If you were to move the pie-shaped sections of the circle to form another 2-dimensional shape where we could find the area more easily, what might this shape look like? Draw a sketch to show this new shape.

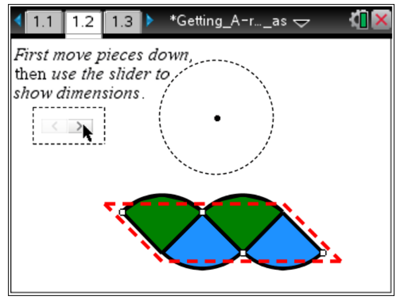
   Sample Answers: Student responses will vary. Some might draw a rectangle or parallelogram. Some might not have any idea.

> **Teacher Tip:** Students are asked about what shape they might be able to construct for you to see how they are initially thinking about rearranging the sectors into other shapes. You might want to suggest to some students that they can rotate or flip the pieces to fit together. You might also want to have students describe where they see the radii in the original circle, to

ensure that students see that all radii are segments from the center of the circle to the circle’s edge and that they can find more than 4!

2. On Page 1.2, select any “open” point on a section of the circle, and drag it below the circle to form another shape.

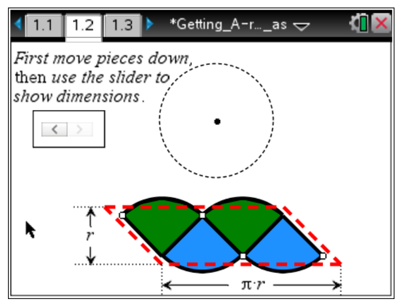
   a. After you move all 4 sections, what does the new shape appear to look like? Does it look similar to the shape you suggested in question 1?



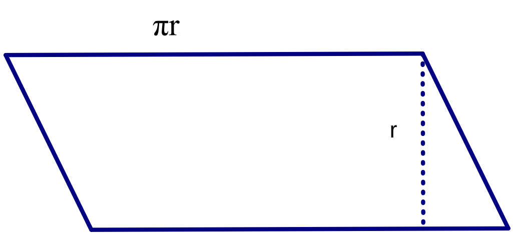
Sample Answers: It sort of looks like a parallelogram. Yes, I thought I could make a parallelogram from the circle pieces.

Teacher Tip: Some students might still not see a strong resemblance to any polygon due to the few number of sectors. The next page allows students to manipulate more pieces to help with the visualization.

   b. Where does the circumference of the circle show up in the new shape? Draw a sketch below and show where the circumference appears in the new shape.



Sample Answers:



The circumference is $2\pi r$, and it is split between the top and bottom of the parallelogram. So each base of the parallelogram is approximately $0.5(2\pi r) = \pi r$ units.

Teacher Tip: You might also want to ask where they see the radius in the shape. The radius could also represent an approximation for a side length of the parallelogram at this stage, but a representation for the height is critical to finding the area of the parallelogram whereas the side length is not.

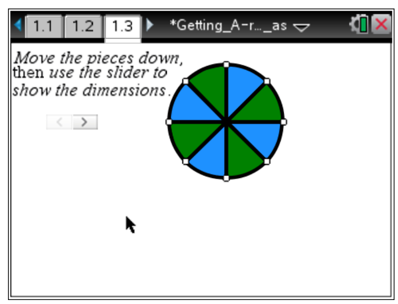
   c. Using this shape as a reference and its area rule, what might be an area formula for a circle? Explain your thinking.

Sample Answers: Since the area formula for a parallelogram is A=bh, the base of the new parallelogram is about πr, and the height is about r, then the area for the circle would be $A = (\pi r)r = \pi r^2$.

> **Teacher Tip:** Moving the pieces permits the students to begin to see the rough shape of a parallelogram. Given the curved sides, two of the side lengths are approximately πr. The height (the perpendicular distance between the two parallel sides) is approximated by the radius. Finding the area of the parallelogram by multiplying the base by the height might help students understand why the area rule for a circle has a value of "r^2".

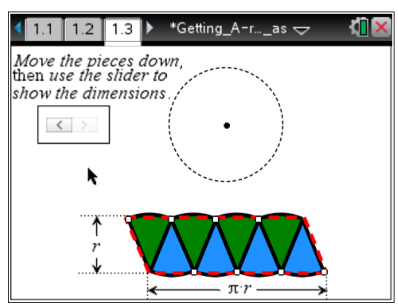
TI-Nspire Navigator Opportunity: ***Live Presenter***
See Note 1 at the end of this lesson.

Move to page 1.3.



3. Perhaps only having 4 sections in the circle didn't allow you to make a very convincing shape. Let's try using 8 sections and see what happens. Move the 8 sections in this circle by selecting and dragging to form a shape.
   a. What does this shape look like? How does it differ from what was created using the 4 pieces?

Sample Answers: It looks more like a parallelogram than when only using 4 pieces. The smaller pieces fill the parallelogram shape better.



   b. Using this shape as a reference, what might be an area formula for a circle?

Sample Answers: The area formula will still be A=bh for the parallelogram so the area formula for the circle will be $A= \pi r^2$.

   c. How is circumference connected to the area rule?

Sample Answers: The circumference shows up in the bases of the parallelogram; we only need one base so we use πr, which is half of the circumference.

4. Imagine cutting each of the 8 pieces of the circle in half so there are 16 pie-shaped pieces.

> **Teacher Tip:** You might want to have circular pieces of paper or paper plates on hand for students who have a hard time visualizing what happens with more sections. Students could fold and cut the paper/paper plates to actually make the new parallelogram. If students create a shape using the physical pieces, they might construct alternate shapes. Their goal is to construct a familiar shape where they know an area rule so that they can apply that rule to the circle. For example, they might create a "fatter" parallelogram where the height is 2r and the base is $\frac{1}{4} * 2 * \pi * r = \frac{1}{2} * \pi * r$. With that, the area of the circle is still $2r * \frac{1}{2} * \pi r = \pi r^2$.

a. If you rearranged those pieces in a similar way as you did above, what shape would you make? How might it differ from the previous ones?

Sample Answers: I could still make a parallelogram. It would fill in more gaps than the one made with only 8 pieces.

b. How does putting more congruent pie-shaped pieces in the circle change the overall appearance of the shape you make with the re-arranged pieces?

Sample Answers: It still is a parallelogram, but it fills in more of the gaps of the shape so it looks more and more like a parallelogram.

c. Does the area rule you made for the circle change if you put more and more pieces in the circle?

Sample Answers: No, the area rule doesn't change because the shape is still a parallelogram.

> **Teacher Tip:** The point of these questions is to have students notice the overall area of the circle won't change if you increase the number of sectors; however, the area of the parallelogram better approximates the area of the circle. In fact, as the sectors become smaller and smaller in size, the parallelogram approaches the appearance of a rectangle.

5. How can you use this activity to help explain the area formula for a circle?

Sample Answers: Since the circle has no straight sides, it is not as easy to find its area as with a rectangle. But by rearranging pieces of the circle, I can make it look like a parallelogram. The base of the parallelogram is half of the circumference, πr, and the height of the parallelogram is the radius of the circle so the area formula for the circle is $A = (\pi r)\, r = \pi r^2$.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to find the area formula for a circle.
- The relationship of the area formula of a circle to its circumference and radius.

Assessment

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Which of the following is the area formula for a circle?

a. $A = 2\pi r$

b. $A = \pi r^2$

c. $A = \pi d$

d. $A = 2\pi r^2$

Sample Student Response:

b. $A = \pi r^2$

TI-Nspire Navigator

Note 1

Question 2c, Name of Feature: Live Presenter

A student could share how they are seeing the circumference and radius of the original circle showing up in the base and height of the parallelogram.

Note 2

Assessment, Name of Feature: Quick Poll

You can use Quick Poll to see if students can distinguish the area formula from other distractors.

Tech Tip: There is a Quick Poll option "Allow document access" under the Tools menu. You might decide to have that turned on or off, depending upon whether you want the students to be able to return to the .tns file or not while they answer the Quick Poll.