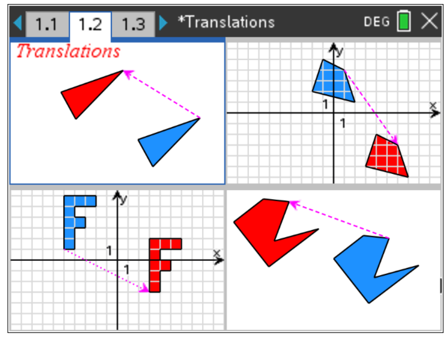
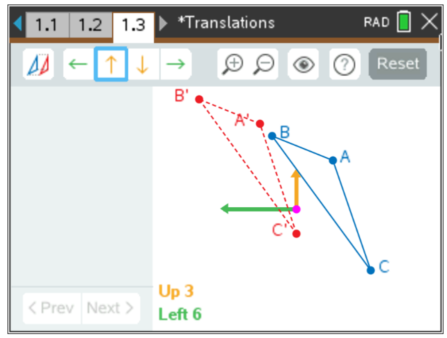


Translations Lesson

Transformational Geometry is a way to study geometry by focusing on geometric "movements" or "transformations" and observing/studying properties about these figures.

There are four geometric transformations:
< Reflections **< Translations** < Rotations < Dilations

Play - Investigate - Explore - Discover
P I E D



In the figure to the right, ΔABC is translated up 3, to the left 6.
ΔABC is called the pre-image while $\Delta A'B'C'$ is called the image (of translation).
$\Delta A'B'C'$ is read "triangle A prime, B prime, C prime."

Download and install the red TI-Nspire student software and the Translations TNS file from the website where you obtained this document.
Then you can interact with these figures, too. If you decide not to download the software, or if you cannot, you can still do this activity along with the video.
A **conjecture** is an opinion or conclusion based on what is observed.

1. What conjecture(s) can you make based upon what you observed about a triangle and its image after being translated?
< The pre-image triangle and image triangle are congruent.
< Corresponding sides and corresponding angles of the two triangles have the same measures (are congruent).

2. What is another word or phrase for what a translation does?
A slide

3. ΔPQR is typically called the **pre-image triangle** while $\Delta P'Q'R'$ is called the **image triangle** (of translation).
$\Delta P'Q'R'$ is read **Triangle P prime, Q prime, R prime**

4. a) If a triangle is translated, what appears to be true about the angles of the pre-image and image triangle? (please word your answer properly)
Corresponding angles of the pre-image and image triangles have the same measure.
Do not allow students to say that all the angles are equal. They are not.

b) If a triangle is translated, what appears to be true about the sides of the pre-image and image triangle? (please word your answer properly)

Corresponding sides of the pre-image and image triangles have the same measure.
Because the corresponding angles and the corresponding sides of the pre-image and image triangles are congruent (have equal measures), the triangles are congruent.
Therefore, a translation is called an **isometry**. An isometry is a transformation that does not change a figure's shape or size. A translation is also referred to as a **rigid motion** because it moves an object but preserves its shape and size (congruence).
We also say that a translation is a **distance-preserving** and an **angle-preserving** transformation.

5. Is a reflection an isometry? Explain.
Yes, a reflection is an isometry because when a figure is reflected about any line, it does not change the figure's shape or size.

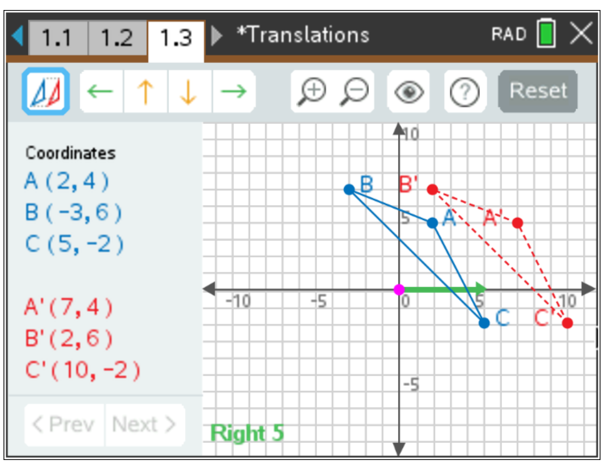
6. a) If a triangle is translated, what appears to be true about the perimeters of the pre-image and image triangle?
The perimeters of the pre-image triangle and image triangle are equal.

b) If a triangle is translated, what appears to be true about the areas of the pre-image and image triangle?
The areas of the pre-image triangle and image triangle are equal.

Grids and Coordinates

7. a) Translate ΔABC to the right 5 units.

Draw your answer on the grid below.



b) Write the ordered pair for each point:

A **(2, 4)** B **(– 3, 6)** C **(5, – 2)**

A' **(7, 4)** B' **(2, 6)** C' **(10, – 2)**

c) Grab and move the vertices of the pre-image triangle.

Write the *new* ordered pair for each point:
<u>(answers will vary)</u>

A **(4, 6)** B **(– 2, 4)** C **(8, 1)**

A' **(9, 2)** B' **(3, 0)** C' **(13, – 3)**

d) Using the pattern observed in the coordinates, if a point on the pre-image triangle has coordinates $(1,\ 2)$, what are the coordinates of its corresponding point on the image triangle?

That is, $(1,\ 2) \rightarrow \underline{\underline{(6,\ 2)}}$ '$\rightarrow$' means "maps to"

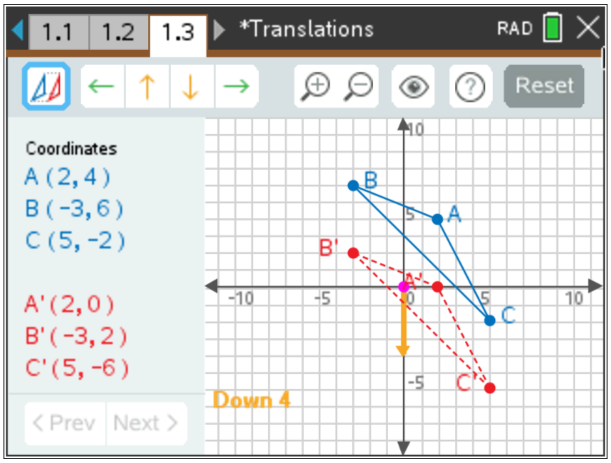
e) Similarly, the point $(-3,\ 7)$ would be translated to? That is, $(-3,\ 7) \rightarrow \underline{\underline{(2,\ 7)}}$

f) Generalize the pattern. If a point on the pre-image triangle has coordinates $(x,\ y)$, what are coordinates of its corresponding point on the image triangle?

That is $(x, y) \rightarrow \underline{\underline{(x+5, y)}}$

8. a) Translate ΔABC down 4 units.

Draw your answer on the grid below.



b) Write the ordered pair for each point:

A **(2, 4)** B **(– 3, 6)** C **(5, – 2)**

A' **(2, 0)** B' **(– 3, 2)** C' **(5, – 6)**

c) Grab and move the vertices of the pre-image triangle.

Write the *new* ordered pair for each point:
(answers will vary)

A **(0, 7)** B **(– 1, 2)** C **(5, 4)**

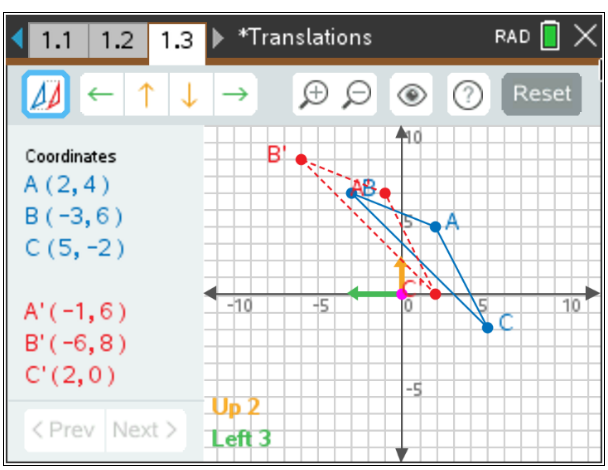
A' **(0, 3)** B' **(– 1, – 2)** C' **(5, 0)**

d) Using the pattern observed in the coordinates, if a point on the pre-image triangle has coordinates $(1, 2)$, what are the coordinates of its corresponding point on the image triangle?
That is, $(1, 2) \rightarrow \underline{\underline{(1, -2)}}$ $' \rightarrow '$ means "maps to"

e) Similarly, the point $(-3, 7)$ would be translated to? That is, $(-3, 7) \rightarrow \underline{\underline{(-3, 3)}}$

f) Generalize the pattern. If a point on the pre-image triangle has coordinates (x, y), what are coordinates of its corresponding point on the image triangle?That is $(x, y) \rightarrow \underline{\underline{(x, y-4)}}$

9. a) Translate ΔABC to the left 3 units
and up 2 units.
Draw your answer on the grid below.



b) Write the ordered pair for each point:

A **(2, 4)** B **(– 3, 6)** C **(5, – 2)**

A' **(– 1, 6)** B' **(– 6, 8)** C' **(2, 0)**

c) Grab and move the vertices of the pre-image triangle.

Write the *new* ordered pair for each point:

(answers will vary)

A **(4, 7)** B **(– 3, – 2)** C **(6, 2)**

A' **(1, 9)** B' **(– 6, 0)** C' **(3, 4)**

d) Using the pattern observed in the coordinates, if a point on the pre-image triangle has coordinates $(1, 2)$, what are the coordinates of its corresponding point on the image triangle?

That is, $(1, 2) \rightarrow$ $(-2, 4)$ $\quad$ '$\rightarrow$' means "maps to"

e) Similarly, the point $(-3, 7)$ would be translated to? That is, $(-3, 7) \rightarrow$ $(-6, 9)$

f) Generalize the pattern. If a point on the pre-image triangle has coordinates (x, y), what are coordinates of its corresponding point on the image triangle? That is $(x, y) \rightarrow$ $(x - 3, y + 2)$

10. Given: ΔDEF is translated to the right 4 units and down 2 units.

a) If D has coordinates (5, 7), what are the coordinates of D'? $(9, 5)$

b) If E has coordinates (– 3, – 7), what are the coordinates of E'? $(1, -9)$

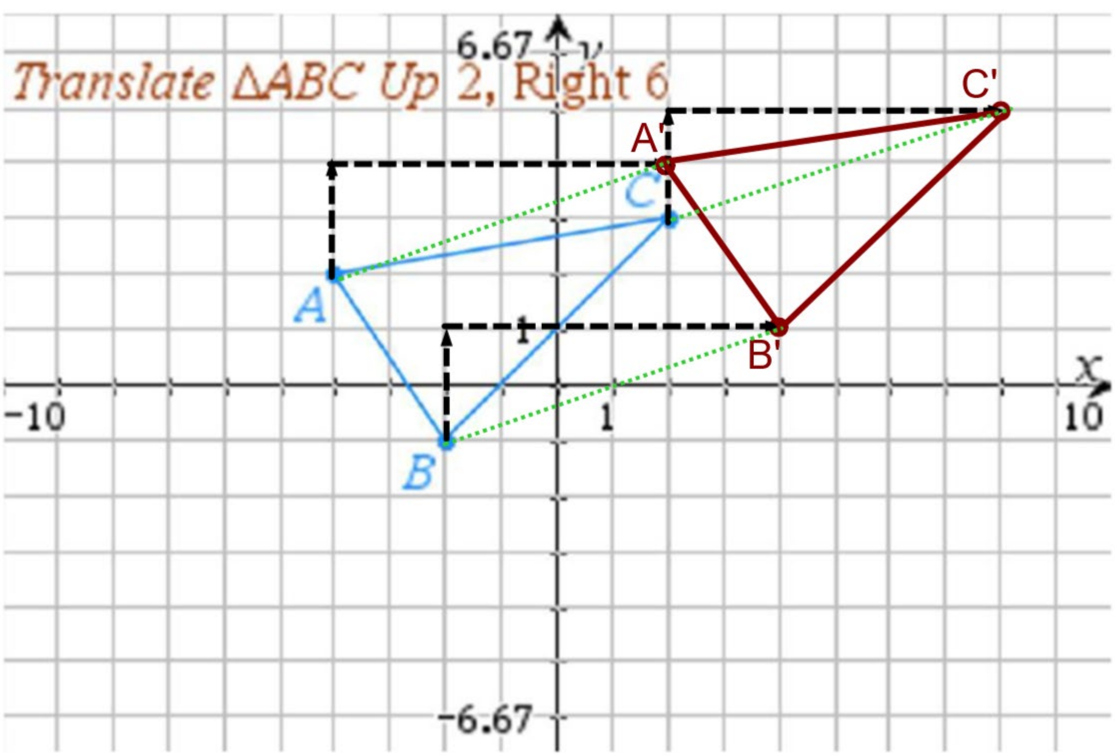
c) If F has coordinates (a, b), what are the coordinates of F'? $(a + 4, b - 2)$

d) If E' has coordinates (1, 6), what are the coordinates of E? $(-3, 8)$

e) If D' has coordinates (p, q), what are the coordinates of D? $(p - 4, q + 2)$

Translate by Hand

11. Translate ΔABC up 2 units, right 6 units, using a straightedge.
Label the vertices appropriately and show the 3 dashed segments that connect corresponding vertices.
a)



b) List the coordinates of each of the 6 vertices:

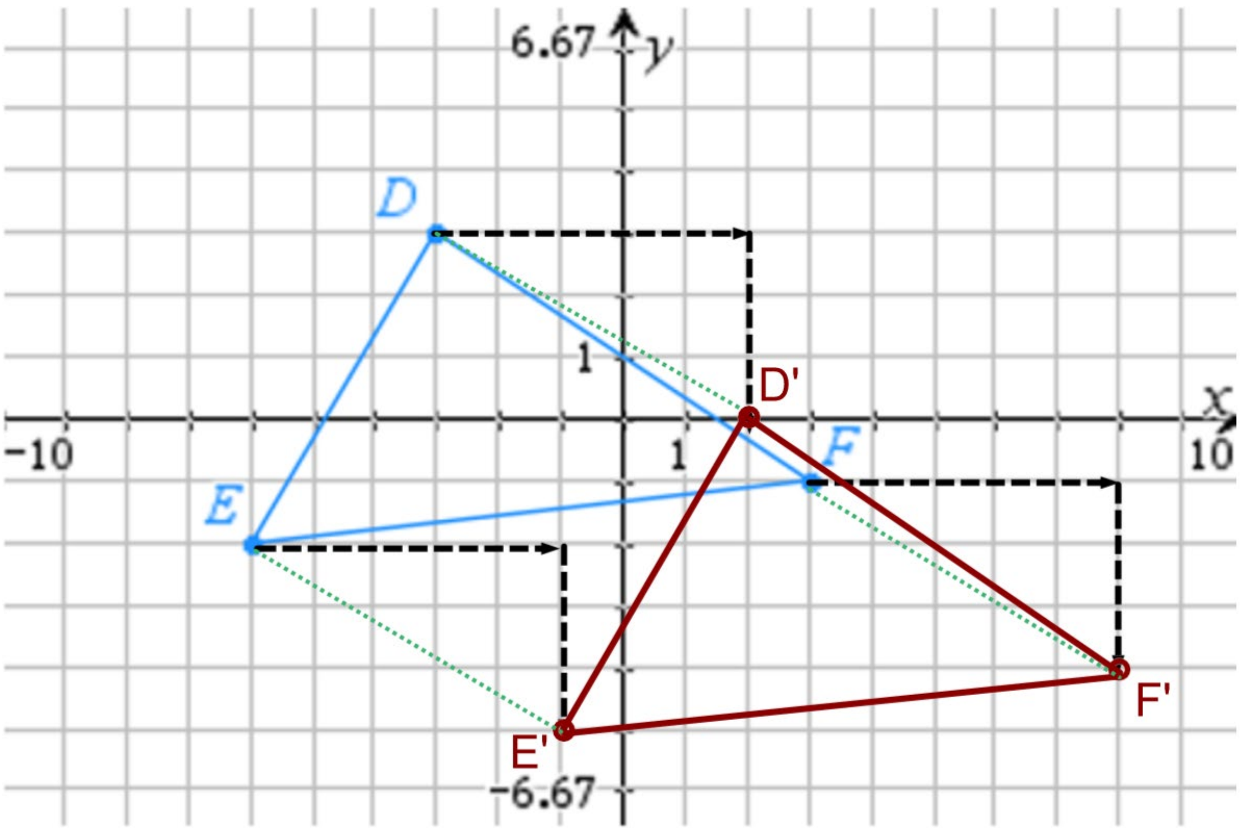
A: **(– 4, 2)** B: **(– 2, – 1)** C: **(2, 3)**

A': **(2, 4)** B': **(4, 1)** C': **(8, 5)**

c) If (x, y) is a point on ΔABC, what are the coordinates of its image on $\Delta A'B'C'$? **(x + 6, y + 2)**

d) If (g, h) is a point on $\Delta A'B'C'$, what are the coordinates of its pre-image on ΔABC? **(g – 6, h – 2)**

12. Translate ΔDEF down 3 units, right 5 units, using a straightedge.
Label the vertices appropriately and show the 3 dashed segments that connect corresponding vertices.
a)



b) List the coordinates of each of the 6 vertices:

D: **(– 3, 3)** E: **(– 6, – 2)** F: **(3, – 1)**

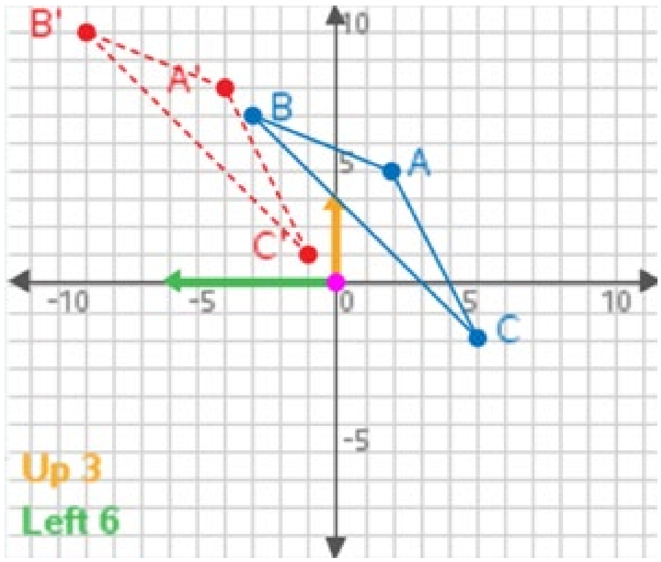
D': **(2, 0)** E': **(– 1, – 5)** F': **(8, – 4)**

c) If (x, y) is a point on ΔDEF, what are the coordinates of its image on $\Delta D'E'F'$? **(x + 5, y – 3)**

d) If (g, h) is a point on $\Delta D'E'F'$, what are the coordinates of its pre-image on ΔDEF? **(g – 5, h + 3)**

Properties of Corresponding Sides of Translated Triangles

13. Translate ΔABC up 3 units and to the left 6 units, using a straightedge.



a) Look at corresponding sides $\overline{AB}$ *and* $\overline{A'B'}$.
We have already established that these two segments have the same length.
What else appears to be true about these two segments?

They appear to be parallel to each other.

What about $\overline{BC}$ *and* $\overline{B'C'}$?

They appear to be parallel to each other.

What about $\overline{CA}$ *and* $\overline{C'A'}$?

They appear to be parallel to each other.

b) It appears that each pair of corresponding sides is parallel.
If segments (lines) are to be parallel, what must be true about their slopes?

Then their slopes must be the same (must be equal).

c) Calculate the slope of each pair of corresponding sides. Record your answers as fractions.

Slope of $\overline{AB}$ = $-\frac{2}{5}$ Slope of $\overline{A'B'}$ = $-\frac{2}{5}$

Slope of $\overline{BC}$ = -1 Slope of $\overline{B'C'}$ = -1

Slope of $\overline{CA}$ = -2 Slope of $\overline{C'A'}$ = -2

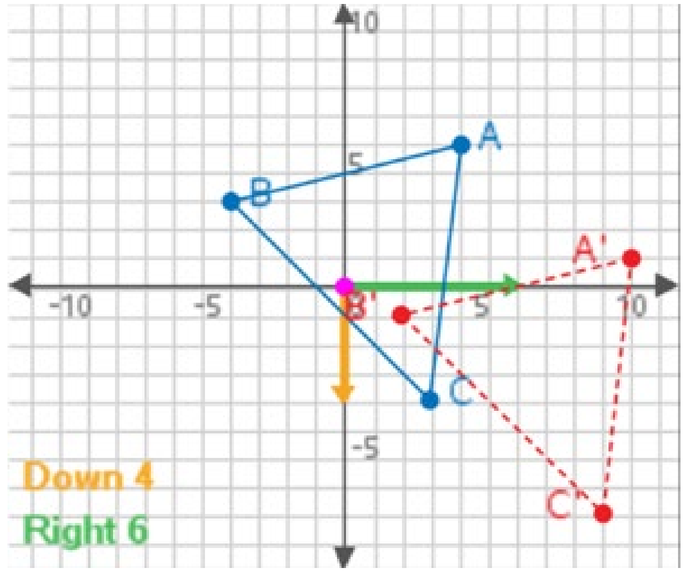
d) Based upon the results in part c above, is each pair of corresponding sides parallel?

Yes, each pair of corresponding sides is parallel because their slopes are equal.

e) This is not enough evidence to prove this conjecture for all triangles. We need to investigate more examples. Let's use the technology to do this.

Good idea.

14. Translate ΔABC down 4 units and to the right 5 units, using a straightedge.



a) Look at corresponding sides $\overline{AB}$ *and* $\overline{A'B'}$.
We have already established that these two segments
have the same length.
What else appears to be true about these two segments?

They appear to be parallel to each other.

What about $\overline{BC}$ *and* $\overline{B'C'}$?

They appear to be parallel to each other.

What about $\overline{CA}$ *and* $\overline{C'A'}$?

They appear to be parallel to each other.

b) Calculate the slope of each pair of corresponding sides. Record your answers as fractions.

Slope of $\overline{AB}$ = $\frac{1}{4}$ Slope of $\overline{A'B'}$ = $\frac{1}{4}$

Slope of $\overline{BC}$ = -1 Slope of $\overline{B'C'}$ = -1

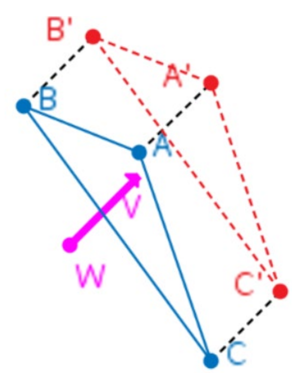
Slope of $\overline{CA}$ = -9 Slope of $\overline{C'A'}$ = -9

c) Based upon the results in part c above, is each pair of corresponding sides parallel?

Yes, each pair of corresponding sides is parallel because their slopes are equal.

Translate by Vector

A **vector** is a directed line segment which has both length and direction.



15. In the figure at the right, ΔABC is translated by **vector** $\overrightarrow{WV}$.

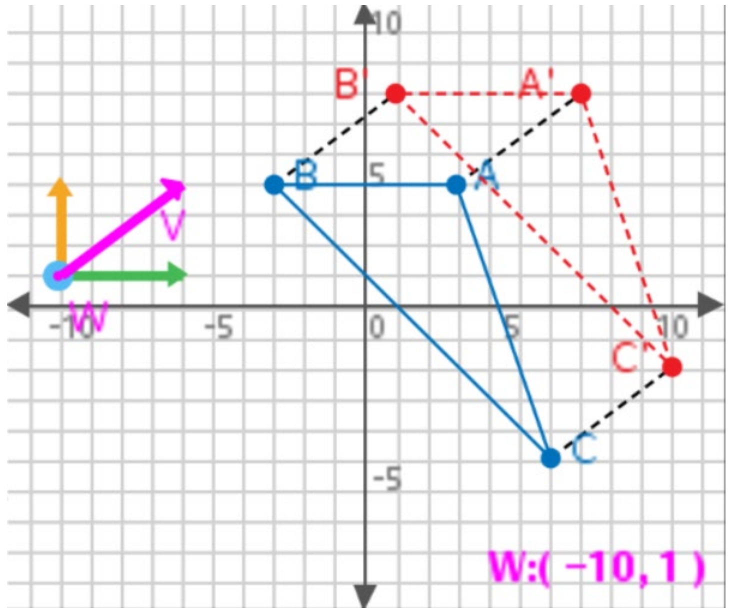
Look at the dashed segments , $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and the **vector** $\overrightarrow{WV}$.

Two things seem to be true about vector $\overrightarrow{WV}$ and these three dashed segments. Write two conjectures below.

The length of vector $\overrightarrow{WV}$ appears to be the same as the lengths of $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and the slope of vector $\overrightarrow{WV}$ appears to be the same as the slopes of $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$.

16. Using the figure to the right, calculate the slopes of the following segments. Write your answers as fractions.

Note: $m(\overline{CD})$ means the slope of segment CD



$m(\overline{BA}) = \underline{\underline{0}}$ $\qquad m(\overline{B'A'}) = \underline{\underline{0}}$

a) $m(\overline{BC}) = \underline{\underline{-1}}$ $\qquad m(\overline{B'C'}) = \underline{\underline{-1}}$

$m(\overline{AC}) = \underline{\underline{-3}}$ $\qquad m(\overline{A'C'}) = \underline{\underline{-3}}$

b)

$m(\overline{AA'}) = \underline{\underline{\frac{3}{4}}}$ $\qquad m(\overline{BB'}) = \underline{\underline{\frac{3}{4}}}$

$m(\overline{CC'}) = \underline{\underline{\frac{3}{4}}}$ $\qquad m(\overline{WV}) = \underline{\underline{\frac{3}{4}}}$

c) Name the segments that are parallel.

$\overline{BA} \parallel \overline{B'A'}, \quad \overline{AC} \parallel \overline{A'C'}, \quad \overline{BC} \parallel \overline{B'C'}; \qquad \overline{AA'} \parallel \overline{BB'} \parallel \overline{CC'} \parallel \overline{WV}$

17. Translate ΔDEF by vector $\overrightarrow{WV}$ using a ruler. Show these as dashed segments: $\overline{DD'}$, $\overline{EE'}$, $\overline{FF'}$.