Math Objectives

- Students will recognize the function $g(x) = \log_b(x)$ as the inverse of $f(x) = b^x$ where $b > 0$ and $b \neq 1$.
- Students will apply this inverse relationship and solve simple logarithmic equations.

Vocabulary

- exponential function
- one-to-one function
- domain and range
- logarithmic function
- inverse function

About the Lesson

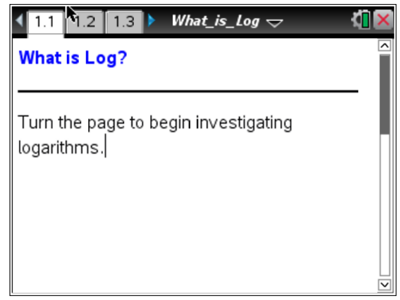
- This lesson involves the one-to-one function $f(x) = b^x$. In acknowledging the existence of its inverse, students will:
- Use the domain and range of $f(x)$ to determine the domain and range of $f^{-1}(x)$.
- Interpret the graph of $f^{-1}(x)$ as the reflection of $f(x)$ across the line $y = x$.
- Use this inverse relationship to write an equation for the graph of the inverse.
- Recognize the logarithmic notation needed to define the inverse function.
- Use the inputs and outputs of two inverse functions to complete a table.
- As a result, students will:
- Solve simple logarithmic equations and verify solutions using the corresponding exponential equations.

TI-Nspire™ Navigator™ System

- Send out the *What_is_Log.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- What_is_Log_Student.pdf
- What_is_Log_Student.doc

TI-Nspire document

- What_is_Log.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, make sure they have not selected more than one point. Press esc to release points. Check to make sure that they have moved the cursor (arrow) until it becomes a hand ( ) getting ready to grab the point. Also, be sure that the word point appears. Then select ctrl to grab the point and close the hand ( ). When finished moving the point, select esc to release the point.

Tech Tip: The pages that have drag-able values have been designed to easily allow students to move the point. Instruct students to move the cursor to the open point until they get the open hand ( ) and select the Touchpad or press enter. The point should slowly blink. Then the point can be moved by pressing the directional arrows of the Touchpad.

Tech Tip: To move the point on a slider, tap on the point to highlight it. Then begin sliding it.

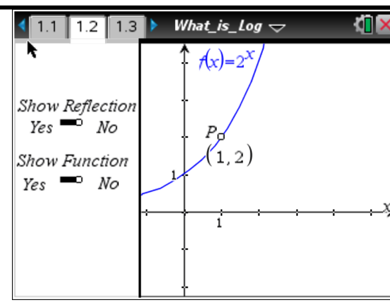
TI-Nspire Navigator Opportunity: *Quick Polls*

See Note 1 at the end of this lesson.

TI-Nspire Navigator Opportunity: *Live Presenter*

See Note 2 at the end of this lesson.

Move to page 1.2.



1. The graph of the function $f(x) = 2^x$ is shown.

   a. What are the domain and range of $f(x)$?

   Answer: The domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.

   b. Recall that $f(x) = 2^x$ is a one-to-one function, so it has an inverse reflected over the line $y = x$. What are the domain and range of $f^{-1}(x)$?

   Answer: The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.

c. Point P is a point on $f(x)$. Move the Show Reflection slider to *Yes* and then move point P. As you do so, point P' invisibly traces out the graph of $f^{-1}(x)$. Since $f(x)$ can be written as $y = 2^x$, write a corresponding equation for the inverse.

Answer: $x = 2^y$

> **Teacher Tip:** Point P and P' may not necessarily show the same number of digits, but will round to be the same.

d. The equation $x = 2^y$ cannot be written as a function of y in terms of x without new notation. Move the Show Function slider to *Yes*. The inverse of $f(x)$ is actually $f^{-1}(x) = \log_2(x)$. In general, $\log_b x = y$. is equivalent to $b^y = x$ for $x > 0$, $b > 0$ and $b \neq 1$. Why do you think x and b must be greater than 0? Why can b not be equal to 1?

Answer: x must be greater than 0 because the range of $f(x) = b^x$ is $(0, \infty)$ and thus the domain of $f^{-1}(x) = \log_b(x)$ must be $(0, \infty)$. b must be greater than 0 because negative values for b will result in negative values for x, and x has to be greater than 0. b cannot be equal to 1 because when $b = 1$, the function is linear, not exponential.

e. Move point P so that its coordinates are (1, 2). The point (1, 2) on $f(x) = 2^x$ indicates that $2^1 = 2$. P' has the coordinates (2, 1). The point (2, 1) on $f^{-1}(x) = \log_2(x)$ indicates that $\log_2 2 = 1$. Use this relationship between exponential expressions and logarithmic expressions to complete the following table. (Move point P as necessary.)

Answer: See table that follows.

> **Teacher Tip:** Students will not be able to drag point P to all possibilities in the table. Encourage them to use the relationships.

P	***P′***	**Exponential Expression**	**Logarithmic Expression**
(1, 2)	(2, 1)	$2^1 = 2$	$\log_2 2 = 1$
(2, 4)	(4, 2)	$2^2 = 4$	$\log_2 4 = 2$
(3, 8)	(8, 3)	$2^3 = 8$	$\log_2 8 = 3$
(0, 1)	(1, 0)	$2^0 = 1$	$\log_2 1 = 0$
$\left(-1, \frac{1}{2}\right)$	$\left(\frac{1}{2}, -1\right)$	$2^{-1} = \frac{1}{2}$	$\log_2 \frac{1}{2} = -1$
$\left(-2, \frac{1}{4}\right)$	$\left(\frac{1}{4}, -2\right)$	$2^{-2} = \frac{1}{4}$	$\log_2 \frac{1}{4} = -2$
$\left(-3, \frac{1}{8}\right)$	$\left(\frac{1}{8}, -3\right)$	$2^{-3} = \frac{1}{8}$	$\log_2 \frac{1}{8} = -3$

Teacher Tip: Students may need to be reminded that $2^{-n} = \dfrac{1}{2^n}$ and thus $\log_2 \dfrac{1}{2^n} = -n$.

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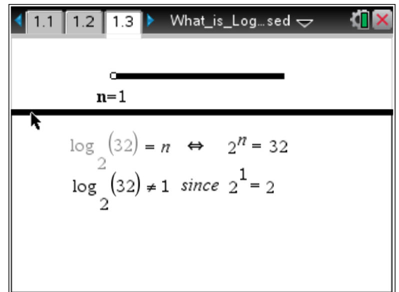

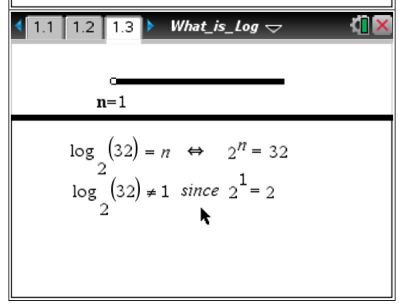

2. Solve the logarithmic equation $\log_2 32 = y$ using the patterns from question 1. Then, use the slider to change the n-value to solve the logarithmic equation. How does the exponential equation verify your result?

   <u>Answer:</u> $n = 5$ since $2^5 = 32$.

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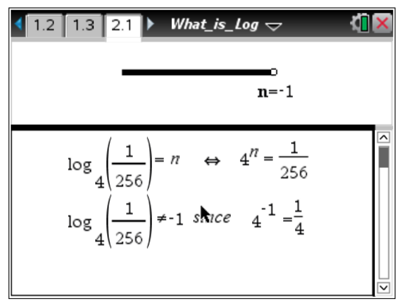

3. Solve the equation $\log_4 \dfrac{1}{256} = y$. Then, use the slider to change the n-value to solve the logarithmic equation. How does the exponential equation verify your result?

   <u>Answer:</u> $n = -4$ since $4^{-4} = \dfrac{1}{256}$.

4. Maya solved the logarithmic equation $\log_4 16 = y$. She says the answer is 4 since 4 times 4 is 16. Is her answer correct? Why or why not?

   <u>Answer:</u> Maya is not correct. The logarithmic equation $\log_4 16 = y$ is equivalent to the exponential equation $4^y = 16$. Although $4 \cdot 4 = 16$, the solution to the equation is an exponent and $4^4 \neq 16$. The correct solution is $y = 2$. Therefore, $\log_4 16 = 2$.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 3 at the end of this lesson.

5. Alex says that when solving a logarithmic equation in the form $\log_b a = y$, he can rewrite it as $b^a = y$. Is this a good strategy? Why or why not?

   <u>Answer:</u> Alex is not correct. There is an inverse relationship between logarithms and exponentials, but the correct exponential equation is $b^y = a$.

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 4 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- For all positive real b, where $b \neq 1$, $\log_b x = y$ if and only if $b^y = x$.

Assessment

Determine the value of the following logarithmic expressions and then justify each answer using an exponential expression.

1. $\log_3 27$
2. $\log_5 1$
3. $\log_7 7$
4. $\log_6 \frac{1}{6}$
5. $\log_4 \frac{1}{64}$

TI-Nspire Navigator

Note 1

Question 1b and 1c, *Quick Poll:* Send an Open Response Quick Poll, asking students to submit their answer to questions 1b and 1c. If students' answers are incorrect, consider taking a Class Capture. Identify incorrect responses, briefly discussing common misconceptions. Then identify and discuss correct responses.

Note 2

Question 1c, *Live Presenter:* Consider demonstrating or have a student demonstrate how to drag and move point *P* along the graph of the function or to drag the yes/no sliders.

Note 3

Question 4, *Quick Poll:*

Send an Open Response Quick Poll, asking students to submit their answer to question 4.

Note 4

Question 5, *Quick Poll:*

Send an Open Response Quick Poll, asking students to submit their answer to question 5.