EXPLORATIONS ™

Activity 4

Graphical Consequences of Continuity

Introduction

It is important to be able to describe the differences in behavior among various kinds of functions. Some functions, such as polynomial functions, have graphs that are unbroken. Their graphs can be drawn without lifting pencil from paper. Many functions do not share this property. For example, the graph of $y = \frac{1}{x}$ has unusual behavior near $x = 0$: It cannot be graphed from $x = {}^-1$ to $x = 1$ without lifting pencil from paper. Functions without any breaks or holes in their graphs are continuous. In this activity, you will explore the graphical behavior of continuous functions.

Exploration

Look at a polynomial function:

$$f(x) = \frac{x^3}{3} + \frac{x^2}{2} - 2x - 1$$

With your graphing handheld in **Func** mode, input **Y1** as **X^3/3+X^2/2–2X–1**.

From the **WINDOW Menu**, change the value of **Xres** to 1. This will ensure that the graphing handheld evaluates the function in every column of pixels as it graphs. Graph **Y1** in the **ZDecimal** viewing window, in which the distance between columns of pixels is 0.1.

1. Copy the graph onto the axes at the right. Now, look more closely at the graph at $x = 0$.
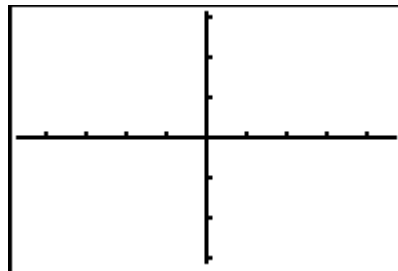
2. What is **Y1**(0)?

   To prepare for a more detailed close-up look, set your zoom factors.

   From the **ZOOM MEMORY Menu**, select **4:SetFactors**, and set **XFact** to 10 and **YFact** to 1. When the graphing handheld zooms in with these factors, the vertical scale of the graph is unchanged, but horizontally, the distance between columns of pixels is divided by 10.

   Now, zoom in at the origin. Select **2:Zoom In** from the **ZOOM Menu**, position the cursor at (0, 0), and press ENTER.
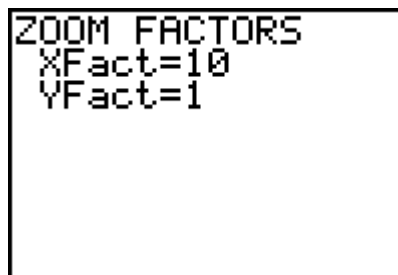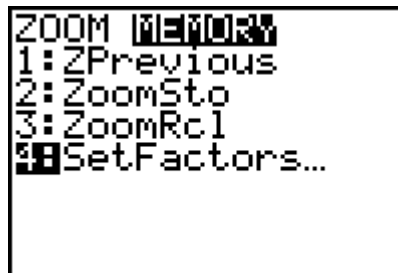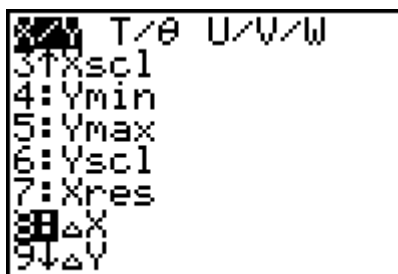
3. After you zoom in horizontally with a factor of 10, what is the distance between columns of pixels?
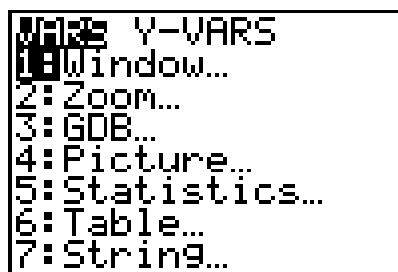
4. The graphing handheld stores the distances between columns and rows of pixels in variables ΔX and ΔY respectively. From the home screen press VARS, select **1: Window**, then **8:ΔX** to get the value of ΔX.
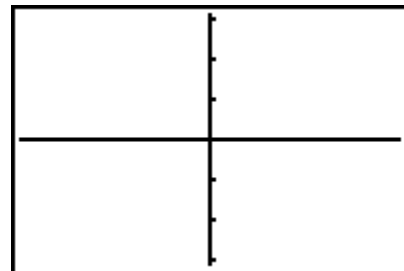
   What is the value of ΔX?

5. From the **1:Window…** secondary menu in the **VARS Menu**, select **9:ΔY** to get the value of ΔY. Remember, the vertical zoom factor is 1, which leaves the vertical scale unchanged.

   What is the value of ΔY?

6. Zoom in twice more at the point (0, 0).

   Copy the graph onto the axes in the illustration to the right and describe its shape.

7. What is the value of ΔX now?

   Trace along the graph, and observe the outputs from **Y1**. Do not trace so far off either the right-hand or left-hand edge of the graph that the viewing window changes.

8. What is the smallest value of *y* you can find?

9. What is the largest value of *y* you can find?

   You have seen that when the values of *x* are very close together, the values of *y* are also close together. That is, close inputs to the function produce close outputs from the function. This is a key feature of the graph of a continuous function. Eventually, the graph of a continuous function will resemble a horizontal line with no holes.
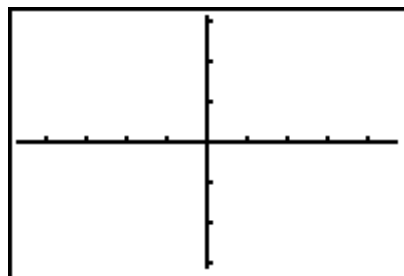
10. Explore a different function,

$$y = \frac{x^2 - \frac{1}{4}}{x - \frac{1}{2}}$$

   Input **Y1** as **(X^2−1/4)/(X−1/2).**

   Investigate this rational function at *x* = 0.5. Graph it in the **ZDecimal** viewing window.

   Copy the graph on the axes shown. Look closely at *x* = 0.5.

11. What is **Y1** (0.5)?

12. Now, zoom in at (0.5, 0). Select **2: Zoom In** from the **ZOOM Menu**, position the cursor at (0.5, 0), and press ENTER. Zoom in again at the same point.

Copy the graph onto the axes in the illustration to the right and describe its shape.

It appears that zooming in any number of times will not remove the hole in the graph at $x = 0.5$. However, the fact that the graph flattens out suggests that the hole could be filled in and made into a new function that is continuous.
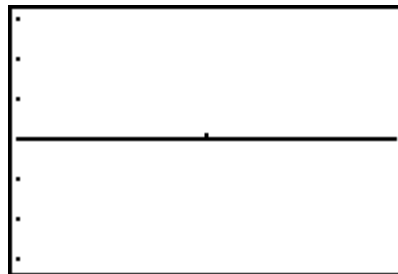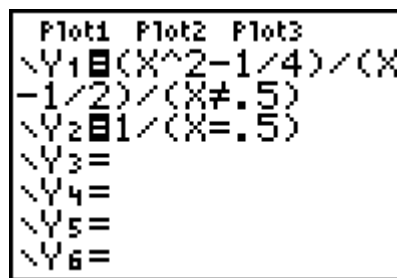
13. What do the values of y approach as x approaches 0.5?

14. Define **Y1** and **Y2** as shown in the screen to the right.

These definitions together define the split function,

$$g(x) = \begin{cases} \dfrac{x^2 - 0.25}{x - 0.5}, & x \ne 0.5 \\ 1, & x = 0.5 \end{cases}$$

Graph **Y1** and **Y2** in the same viewing window as Question **12**.

Describe the graph.

15. Look at a rather strange kind of function behavior (or misbehavior!) of

$y = \sin\left(\dfrac{1}{x}\right)$ near $x = 0$.

Make sure the graphing handheld is in **Radian** mode, and input **Y1** as **sin(1/X)**. Graph **Y1** in the **ZDecimal** viewing window.

Copy the graph onto the axes shown.

16. What is **Y1** (0)?

Zoom in several times at the origin.

17. Does the graph flatten out?

18. Describe what happens to the graph.

19. Do you think it would be possible to define **Y1** at $x = 0$ in such a way that it would be continuous there? Explain your answer.