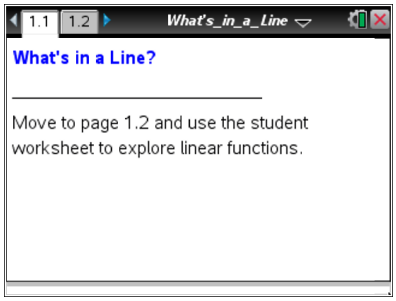
What's in a Line?

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line.
- Students will be able to recognize equations as defining linear functions from those that are not linear.
- Students will reason abstractly and quantitatively.
- Students will look closely to discern a pattern or structure.



Vocabulary

- equation
- coordinates
- function
- tables
- linear function
- graph
- slope *or* rate of change

About the Lesson

- This lesson involves identifying linear functions.
- As a result, students will:
  - Plot points on the coordinate graph as they begin to determine features of a linear function.
  - Plot new points for a visual check of the points "working" with the original plotted points.
  - Examine and compare the final equations that appear for each example.
  - See that equations in the form of y = mx + b represent linear functions and that other types of equations produce other functions or possibly non-functions.

TI-Nspire™ Navigator™ System

- Send out the *What's_in_a_Line.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- What's_in_a_Line_Student.pdf
- What's_in_a_Line_Student.doc

TI-Nspire document

- What's_in_a_Line.tns

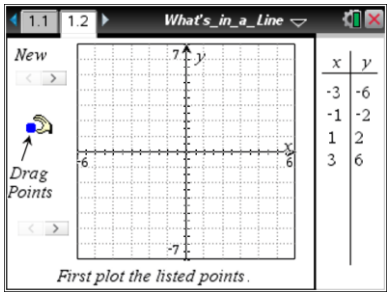
Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (☜) getting ready to grab the point. Press ctrl ☜ to grab the point and close the hand (☜). Press esc to release the point or deselect other objects.

Tech Tip: If students experience difficulty grabbing and dragging a point, have them tap and hold the desired point for a few seconds and then drag the point to the desired location.

Tech Tip: Use tab to select the arrows that are minimized sliders. Then use the arrows to select a "New" problem. The second slider will not do anything until students accomplish the instructions given at the bottom, "First plot the listed points."

Move to page 1.2.



1. Examine the features you see on Page 1.2 with your partner. Describe what you notice on the page itself as well as what actions you can perform. In this activity, you'll use these features to explore what makes a **linear function**.

   Sample Answers: I see a graph page, a table of *x*- and *y*-values, a "New" slider, and a slider near "First plot the listed points." I can drag points onto the graph, and when I match the points in the table, the values in the table turn blue.

2. Plot the four points listed in the initial table by grabbing and dragging one point at a time to its appropriate coordinates.
   a. Do the points appear to lie on a line (a linear function)? Test this idea by plotting three more points that you believe would be on the same line.

   Sample Answers: Yes, the points look like they lie on a line.

   b. Use the slider to check whether the points lie on a linear function. What did you find?

   Sample Answers: I found three more points by visually fitting three more points in between those that were plotted. I got (-2, -4), (0, 0) and (2, 4).

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

Teacher Tip: At this point in the activity, most students are visually finding extra points to plot. Have them share their plotted extra points BEFORE they use the test button to allow for class discussion on strategies for finding extra points. These visual strategies might begin to utilize important ideas of slope: "I saw that I could go four up and two over to get from one blue point to another so I tried to go two up and 1 over and the point fit." The next question moves the students' attention to the table, to examine patterns leading to slope/rate of change from a numerical perspective as well.

3. After you check to see whether your points are part of a linear function, an equation that represents the function will appear with the graph. Write the *x*- and *y*-coordinates, including your three from the first example, in the table below organizing the *x*-values in increasing order (and then matching the *y*-value). Write the equation that appeared next to the line:

Sample Answers: Answers are in red. Blue values are the original points.
y* = 2*x

x	*y*
-3	-6
-2	-4
-1	-2
0	0
1	2
2	4
3	6

a. If you were only given the equation, how could you determine the (x, y) coordinates for points that lie on the linear function?

Sample Answers: I could choose a number for x and then multiply that by 2 to find the *y*-value or I could choose a number for y and divide by 2 to find an *x*-value.

b. In your own words, describe the relationship between the *x*- and *y*-coordinates.

Sample Answers: The equation means that all of the *y*-values are twice the *x*-values.

Teacher Tip: The big idea for students to understand here is that the rule or equation is true for all the points that fit on the line AND, conversely, if the point is on the line, then its x and y coordinates have to make the equation a true statement.

c. Examine the ordered values in the table, but instead of looking for patterns between the *x*- and *y*-values, look for patterns as the *x*- and *y*- values increase. What do you notice?

Sample Answers: As the *x*-values increase by one each time, the *y*-values increase by 2.

Teacher Tip: Have students also search for the pattern they provided in the graph by describing moves in going from one point to another. Provide the vocabulary of "rate of change" and "slope" to describe this constant pattern. You can suggest to your students that rate of change or slope is a measure of the "steepness" to help them determine that the vertical change or "rise" (the change in *y*-values) will be compared to the horizontal change, or "run" (the change in *x*-values), or $\frac{rise}{run}$.

4. Use on the "New" button on the top left of the page to provide a new table of values. **Before** you plot the points, examine the pairs of coordinates, and look for a pattern within the table.
   a. Do you believe these points will lie on a line (a linear function)? Explain your thinking with others in your group. Provide support for your thinking by describing any pattern you notice in the table of points.

Sample Answers: I think the points lie on a line because I noticed that as the *x*-values get bigger by one, the *y*-values add 2, and that is true for all four points in the table. So they all have the same slope.

   b. Use this pattern to help you describe a rule or equation to find even more points. Share your thinking with others and write your "test" rule below:

Sample Answers: y = 2x + 3.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

Tech Tip: In TI-Nspire Navigator, on the Document Tab, click the Tools menu. You can change the Quick Poll Options and UNCHECK Allow Document Access. Students will then not be able to accidentally leave the poll by pressing Home/On, Scratchpad, etc.

Teacher Tip: Students might originally try y = 2x since the rate of change pattern in the table is the same as the first example. But one equation could not produce two different lines. Have students compare the *y*-values from both tables with the same *x*-values to notice the difference of 3. Allow students to determine how to incorporate that connection in the equation and then to test their ideas.

c. Now plot the points to test your *conjecture* (another word for "guess"). Do the points appear to lie on a line? Plot more points to test your thinking.

Sample Answers: Yes, the points appear to lie on a line.

d. What was the final equation for the second example?
How does this compare with your conjecture above? Does this equation describe a linear function? Explain your reasoning with others in your group.

Sample Answers: y = 2x + 3. Since the equation provides points that visually fit on the same line and I can get from one point to another by using the same slope, the equation describes a line. Since each *x*-value is assigned to only one *y*-value, it would be a linear function.

Teacher Tip: Understanding how to test for a function is assumed for this lesson. Students draw upon that understanding to determine if an equation represents a linear function.

5. In your group, examine the next three examples to determine whether they represent linear functions or not.
   a. **BEFORE** you plot the points for any example, examine the table of (x, y) values, and make a prediction about whether the points will lie on a line or not. If they will lie on a line, provide support for your response by referring to a "linear" pattern you notice in the table of (x, y) pairs and the points on the graph.
   b. After your group agrees on whether the points will lie on a line, plot the points to test your conjecture. After plotting the original four points, continue to plot more points, and finally use the slider to test your plotted points.
   In each example, develop a rule or equation that would describe how to find any possible

(x, y) pair. Write your equations below prior to testing them out.
Test equation for example 3: **y =**
Test equation for example 4: **y =**
Test equation for example 5: **y =**

c. For each example, write the final equation you found below.

Final equation for example 3: **y =**

Final equation for example 4: **y =**

Final equation for example 5: **y =**

Sample Answers: Student answers will vary.

6. Write all five equations you have worked with so far in the spaces below:
Sample Answers: (1) **y = 2x** (2) **y = 2x + 3** (3) **y = -0.5x + 2** (4) **y = -x -1** (5) **y = 1.5x -1**

a. Which of these equations represent linear equations?

Sample Answers: All of them do. Every graph produced a line, with every *x*-value mapping to only one *y*-value.

b. What common features are there among the linear equations that can help you describe any equation of a linear function? What might a general form for a linear equation look like?

Sample Answers: All of the equations have an x, a y, and some numbers. A general form might look like 'y = something times x plus/minus a number'.

> **Teacher Tip:** You will want to encourage them to think of the "something times x" as the slope or rate of change for the function. In further examples in the activity, they will have examples of y = x, y = a, and x = a to have them consider their general form further and to help make the connection with the slope being the coefficient of the x. This activity does not emphasize relationships between families of linear functions, however. That will need to be addressed in later lessons.

7. What were the patterns you noticed in the tables that suggested the plotted points would be on lines?
Sample Answers: When I looked for the slope in each table, it was always the same value, either 2 or -1 or whatever value I found.

8. In your group, look at the next four examples (6 through 9), and decide whether they represent linear functions or not. Use a similar process as before:
   1) look for and describe patterns in the table of (x, y) pairs that have previously indicated a linear function;
   2) plot the four points from the table to see if they visually appear to lie on a line;
   3) try to develop a rule or equation that will help you find three more points to plot; and finally
   4) check to determine the final equation for the example and to compare with your conjecture.

   Describe your findings below.

   Sample Answers: Example 6: $y = -3$; Example 7: $y= x^2$; Example 8: $x = 4$; Example 9: $y = |x|$

   Teacher Tip: In this set of examples, students will see equations that represent other types of functions, a non-function in x = a, and an "atypical" linear function of y = a. This function provides the opportunity to have students think about describing zero slope—a line with no steepness. The equation x = a will allow them to see an equation that is linear but is not a function. You might need to provide hints for the quadratic example of $y = x^2$ to see if they notice the squaring of values in the table. There is also an example of the absolute value function that might require hints, depending upon the students' experience with that function.

9. a. Describe all equations that would represent linear functions.

   Sample Answers: y = ax + b where a and b can be positive or negative numbers or even zero.

   Teacher Tip: You might want to ask students for examples of their positive and negative numbers to determine what kinds of "numbers" they are considering, in particular, are they including the rational numbers? Perhaps irrational? This can be an opportunity to introduce the language of the "real numbers".

   b. Give an example of an equation that would **not** represent a linear function.

   Sample Answers: $y = x^2$

   Teacher Tip: Students should be able to use one of the equations from the previous four examples to provide an equation that does not represent a linear function, an example that is either not a line or not a function.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- The definition for a function.
- How to describe equations that represent linear functions in general.
- What equations represent linear functions and non-linear functions.

Assessment

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)*
See Note 3 at the end of this lesson.

There are four examples remaining in the file that can be used as an individual or paired assessment of the lesson.
The solutions are:
Example 10: $y = -x^2$
Example 11: $x = -3$
Example 12: $y = -0.5x + 2$
Example 13: $y = x$

TI-Nspire Navigator

Note 1

Question 2b: Live Presenter

Have several students share their extra plotted points and how they determined that a point might fit with the original plotted points.

Note 2

Question 3b: Quick Poll

Use Quick Poll to see the types of equations that students are finding and to promote discussion of their choices. How are they using the values in the table to support their rules? Encourage them to look for numeric patterns both between corresponding *x*- and *y*-values and between consecutive *x*- and consecutive *y*-values, promoting the idea of slope/rate of change.

Note 3

Assessment: Quick Poll

A Quick Poll can be given at the conclusion of the lesson to get a sense of student understanding.