The Geometric Mean

TIMATH.COM: GEOMETRY

Activity Overview

In this activity, students will establish that several triangles are similar and then determine that the altitude to the hypotenuse of a right triangle is the geometric mean between the segments into which it divides the hypotenuse.

Topic: Ratio, Proportion, & Similarity

- Prove and apply the Mean Proportional Theorem for triangles.

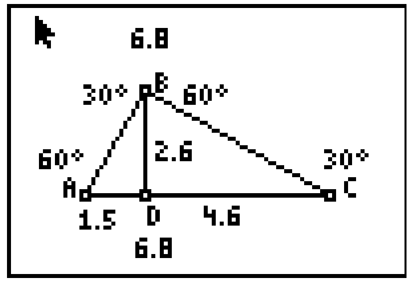
Teacher Preparation and Notes

- In a right triangle, the altitude from the right angle to the hypotenuse will be the geometric mean between the segments of the hypotenuse.
- X is the geometric mean between A and B if $\frac{A}{X} = \frac{X}{B}$; this can also be expressed as $X^2 = AB$, or $X = \sqrt{AB}$.
- **To download the student worksheet, go to education.ti.com/exchange and enter “8183” in the keyword search box.**

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- Geometry: Concurrent Lines, Medians, and Altitudes with Cabri™ Jr. (TI-84 Plus family) — 7288
- Geometric Mean Investigation (TI-Nspire™ technology) — 9944
- Geometric Mean with TI-Nspire (TI-Nspire™ technology) — 9655



This activity includes screen captures taken from the TI-84 Plus Silver Edition. It is also appropriate for use with the TI-83 Plus and TI-84 Plus but slight variances may be found within the directions.

Compatible Devices:

- TI-84 Plus Family

Software Application:

- Cabri™ Jr.

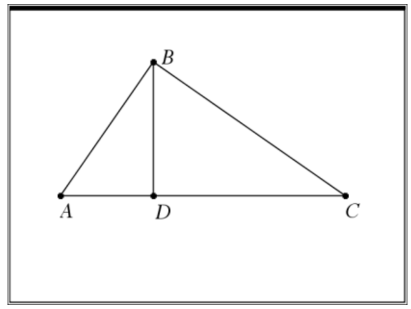
Lesson Files:

- GeometricMean_Student.pdf
- GeometricMean_Student.doc

Click [HERE]() for Graphing Calculator Tutorials.

Introduction

Triangle *ABC* is a right triangle with right angle $\angle ABC$ and an altitude from the right angle to the hypotenuse. Therefore, $m\angle ADB = 90°$ and $m\angle CDB = 90°$. Using knowledge of similar triangles, we could conclude that $\triangle ABC$ is similar to $\triangle ADB$ and is similar to $\triangle BDC$. When we create proportions from the two smaller triangles, we get: $\frac{AD}{BD} = \frac{BD}{DC}$.

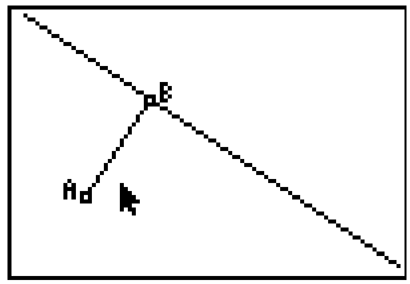


Simplifying this we have $BD^2 = AD \times DC$.

The line segment $\overline{BD}$ is called the **Geometric Mean** between $\overline{AD}$ and $\overline{DC}$.

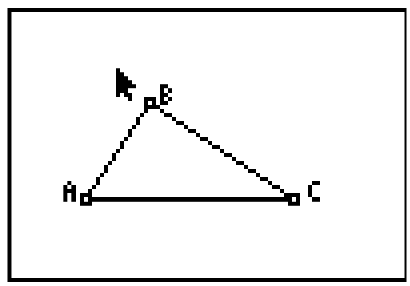
The Geometric Mean

Students are to construct $\overline{AB}$ and a perpendicular line to $\overline{AB}$ through *B*.



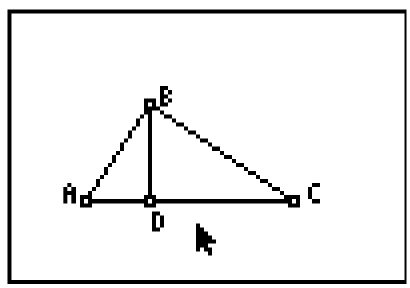
Students are to use the **Point On** tool to create a point *C* on the perpendicular line (**F2 > Point > Point on**).

Then, they can hide the perpendicular line with the **Hide/Show** tool and construct line segments connecting *A* to *C* and *B* to *C*.



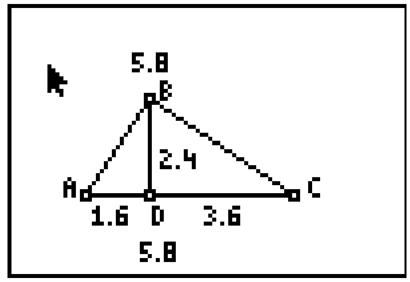
Students will construct a line that is perpendicular to $\overline{AC}$ through point *B*. They can then construct the point of intersection of this perpendicular line and $\overline{AC}$, labeling it *D*.

They should hide the perpendicular line and construct a line segment connecting *B* to *D*.

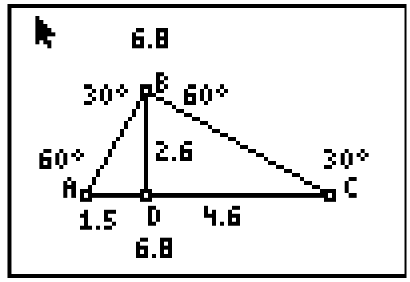


Direct students to construct line segments $\overline{AD}$ and $\overline{DC}$ and measure the lengths of $\overline{AD}$, $\overline{DC}$, and $\overline{BD}$.

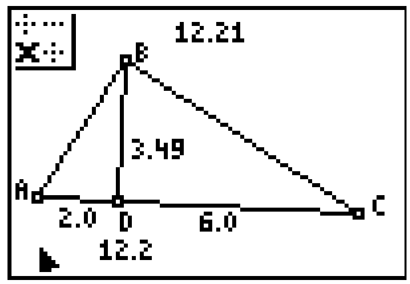
Students should use the **Calculate** feature to find the product of the lengths of $\overline{AD}$ and $\overline{DC}$. Then, they should use the **Calculate** feature again to find the square of *BD*. To do this, click on the measure of $\overline{BD}$, press ⊠ and click on the measure of $\overline{BD}$ again.



To verify that the triangles are similar, have students measure the angles in the figure. All of the angles in this figure have been measured.

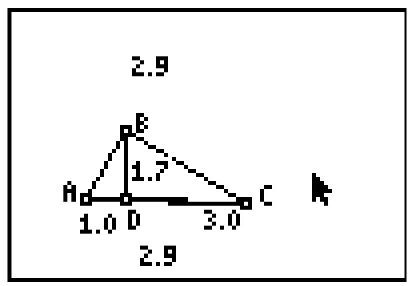


Students should drag point *A* and point *C* so that *AD* = 2 and *DC* = 6. It may be very difficult to get these values exactly due to the screen resolution. In this figure, the accuracy of the lengths of $\overline{AD}$ and $\overline{DC}$ are shown to one decimal place.



Students should manipulate their figure again so that *AD* = 1 and *DC* = 3. Note that the numbers in this figure are slightly off again.

They should see that the measurement of $\overline{BD}$ is the square root of ***AD*DC***. They are to consider how to use this figure to find other radical values.



Student Solutions

1. *BD* = $\sqrt{3}$ cm ≈ 1.732 cm

2. *Sample answers:*

a. *AD* = 2 cm, *DC* = 5 cm

b. *AD* = 2 cm, *DC* = 3 cm

Exercises

The exercises in this section reinforce the skills learned in this activity.

Student Solutions

1. $BD = 9$ in.
2. $DC = 16$ in.
3. $BD = 3\sqrt{6}$ in.
4. $BD = 5\sqrt{2}$ cm, $AB = 5\sqrt{3}$ cm, and $BC = 5\sqrt{6}$ cm
5. $AD = \frac{1}{2}$ cm, $AB = \frac{\sqrt{17}}{2}$ cm, and $BC = 2\sqrt{17}$ cm