Factor & Remainder Theorem

Student Activity

7 8 9 10 **11** 12

TI-Nspire™ Investigation Student 50 min

Introduction

Division of whole numbers is a great way to start thinking about division of polynomials. When a number such as 36 is divided by 12 the result is 3. We can claim that 12 is a factor of 36 because there is no remainder. We can also subsequently claim that 3 is a factor of 36. Our result can be re-written as: 3 × 12 = 36.

Now consider 36 divided by 15; the result is 2 with 6 'left over'. The 2 is referred to as the quotient and the 6 'left over' is referred to as the remainder. We conclude that 15 is not a factor of 36 since the remainder is not zero. Our result can be re-written as: 15 × 2 + 6 = 36.

A polynomial $p(x)$ can be divided by another polynomial $g(x)$; if the remainder is zero then $g(x)$ must be a factor of $p(x)$. This can be written symbolically as: $p(x) \div g(x) = f(x)$, and the converse: $p(x) = f(x) \times g(x)$.
The converse equation may be more familiar by consideration of a specific example:

$$f(x) = x + 2\ ,\ g(x) = x + 3 \text{ therefore } p(x) = (x+2)(x+3) \text{ or } p(x) = x^2 + 5x + 6$$

Now consider a polynomial $p(x)$ where $g(x)$ is not a factor. In this case the converse could be written as:

$$p(x) = f(x) \times g(x) + r(x) \text{ where } r(x) \text{ is the remainder function.}$$

This investigation explores polynomial division numerically, symbolically and graphically with a view to establishing a greater conceptual understanding.

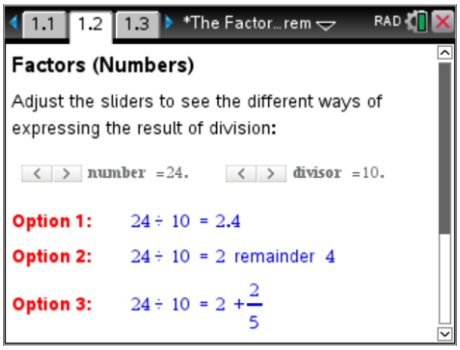
Instructions

Open the TI-nspire file "Factor and Remainder Theorem".

Navigate to page 1.2. Adjust the sliders to review the different ways that numerical division can be expressed, including the terminology:

- Quotient
- Remainder

You can scroll down to see the quotient and remainder expressed individually. Note the different ways of representing the numerical result.



Question: 1.

For each of the following write down the quotient and remainder:

a. 24 ÷ 10 =

b. 30 ÷ 7 =

c. 32 ÷ 6 =

d. 36 ÷ 9 =

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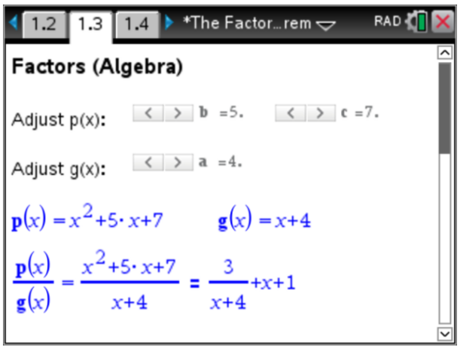
Question: 2.

For each of the results in Question 1, rewrite your answers as a product.

Navigate to page 1.3.

Use the sliders to adjust the coefficients and constants in the two functions: $p(x)$ and $g(x)$.



Question: 3.

Use the sliders to set up the equations listed below. In each case state the quotient and remainder.

a. $\dfrac{x^2+6x+10}{x+2}$

b. $\dfrac{x^2+5x+7}{x+3}$

c. $\dfrac{x^2+4x+9}{x+3}$

d. $\dfrac{x^2+6x+5}{x+5}$

Question: 4.

Given: $\dfrac{x^2+7x+12}{x+2}=x+5+\dfrac{2}{x+2}$, relate the fractional component of the result: $\dfrac{2}{x+2}$ to the numerical results in Question 1 and the different options on page 1.2 of the TI-Nspire document.

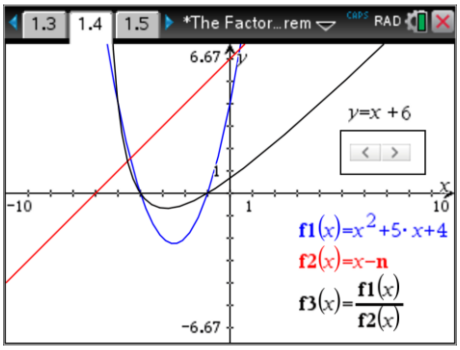
Question: 5.

Given: $\dfrac{x^2+8x+14}{x+2}=\dfrac{x^2+2x+6x+12+2}{x+2}$, express the numerator as three separate expressions and hence express $\dfrac{x^2+8x+14}{x+2}$ in quotient and remainder form.

Navigate to page 1.4

The graphical representation of polynomial division helps to understand what is meant by "express the polynomial as a product of its linear factors".

In this graph $f_1(x)$ is divided by $f_2(x)$. Adjust the slider to change the expression for $f_2(x)$



Question: 6.

Adjust the slider until it is apparent that $f_1(x)$ is represented as the product of two linear factors:

a. Write down the linear factors of $f_1(x)$

b. Where do the linear factors cross the x axis?

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Question: 7.

Given that $\dfrac{p(x)}{g(x)} = f(x)$ then it follows $p(x) = f(x) \cdot g(x)$. If $f(a) = 0$ what will $p(a)$ equal?

Question: 8.

Discuss the relationship between the answers to Question 6(b) and Question 7 and the factor theorem that states:

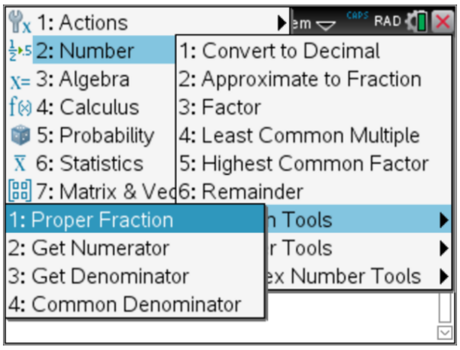
If polynomial $p(x)$ has a root $x = a$ then $p(a) = 0$ and $x - a$ is a factor of $p(x)$

Navigate to page 1.5

Page 1.5 is a calculator application. Use the menu to access the "Proper Fraction" command:

Number > Fraction Tools > Proper Fraction

A numerical fraction is 'improper' if the numerator is greater than the denominator. An algebraic fraction can be considered improper if the numerator is a higher degree polynomial than the denominator.



Question: 9.

Use the proper fraction command to re-write each of the following rational, algebraic fractions and identify the quotient and remainder for each.

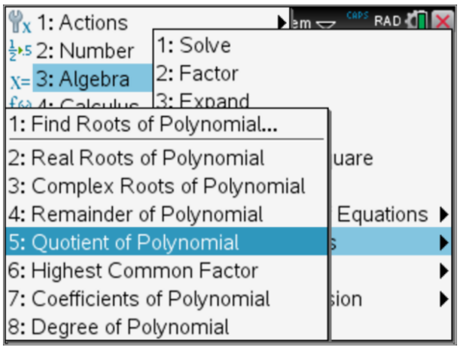
a. $\dfrac{x^2 + 6x + 18}{x + 5}$

b. $\dfrac{x^2 - 8x + 12}{x + 3}$

c. $\dfrac{x^3 + 4x^2 + 6x + 11}{x + 4}$

Additional polynomial tools allow you to use only the quotient or the remainder of polynomial division.

Algebra > Polynomial Tools > Quotient of Polynomial



Question: 10.

Use the **Remainder of Polynomial** and **Quotient of Polynomial** commands to check your answers to Question 9.

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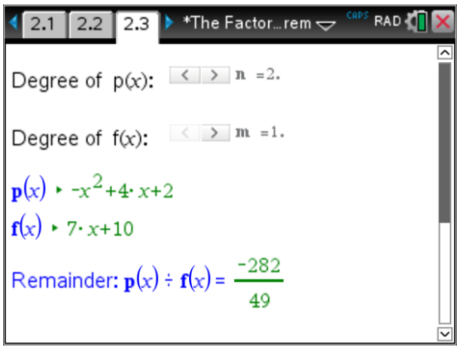
Extension

Navigate to page 2.1.

In this section we are interested in the degree of the remainder when $p(x)$ is divided by $f(x)$.

Change the degree of each polynomial and observe the remainder, paying particular attention to the degree of the remainder.



Question: 11.

Explore the how the degree of $p(x)$ and $f(x)$ relate to the degree of the remainder. If $f(x)$ is a linear function (degree = 1), what will be the degree of the remainder?

Question: 12.

If $p(x) = f(x)(x-a)+c$ and $c \neq 0$ then $(x-a)$ is not a factor of $p(x)$.

Determine an expression for $p(a)$ and discuss the significance of this result.

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