The Hinge Theorems

ID: 7897

Time required
40 minutes

Activity Overview

In this activity, students will explore the inequality relationships that arise when some of the triangle congruence conditions are in place but others are not. The SAS Inequality Theorem and the SSS Inequality Theorem are often referred to as the Hinge Theorem and its converse. These two theorems concern inequalities involving the sides and angles of two triangles.

Topic: Triangles and Congruence

Use necessary and sufficient conditions for congruence to conjecture theorems about congruent triangles.

Teacher Preparation

This activity is designed to be used in a high school or middle school geometry classroom. This activity uses the Cabri Jr application.

- *The SAS Inequality Theorem (Hinge Theorem) states:*
  If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first triangle is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.
- *The SSS Inequality Theorem (Converse of Hinge Theorem) states:*
  If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first triangle is larger than the included angle of the second triangle.
- ***To download the student worksheet, go to education.ti.com/exchange and enter "7897" in the keyword search box.***

Associated Materials

- *HingeTheorems_Student.doc*

Suggested Related Activities

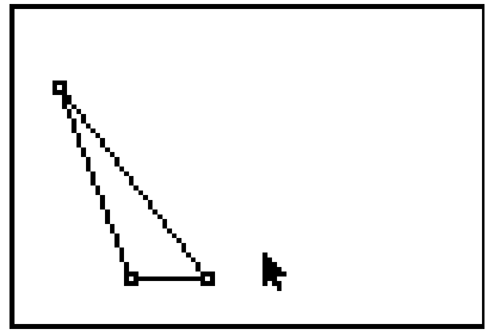
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Congruent or Not? (TI-84 Plus family) — 11136*
- *Similar or Congruent? (TI-84 Plus family) — 11063*

Problem 1 – SAS Inequality Theorem

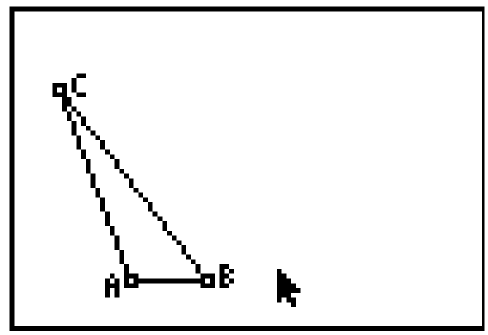
Students should open a new Cabri Jr. file.

They should then construct a scalene triangle using the **Triangle** tool.

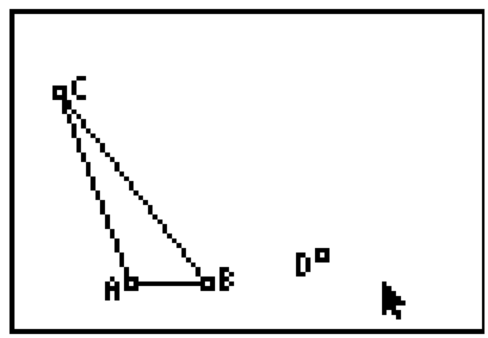


Select the **Alph-Num** tool to label the vertices *A*, *B*, and *C* as shown.

Note: Press [ENTER] to begin the label, type the label, then press [ENTER] again to end the label.



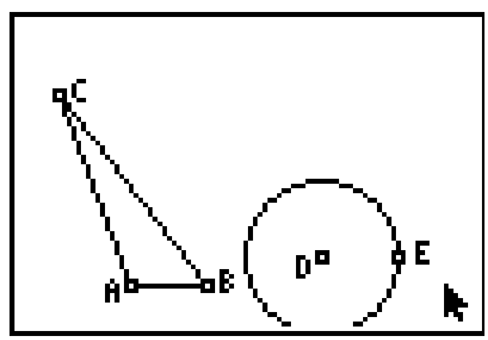
Have students construct a point on the screen using the **Point** tool. Again use the **Alph-Num** tool to label this point *D*.



Students should now select the **Compass** tool to copy $\overline{AB}$ to point *D*.

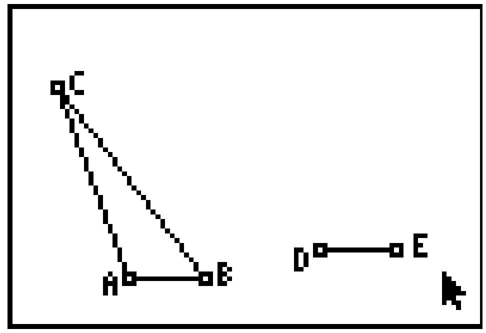
- Press [ENTER] on $\overline{AB}$. A dashed circle will appear and follow the pointer.
- Press [ENTER] on point *D*. The compass circle is anchored at center *D*.

Have students construct a point on the compass circle. Label this point *E*.



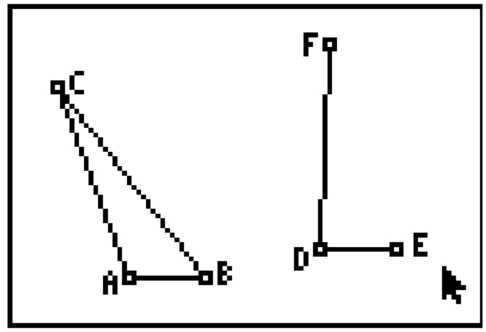
Direct students to create $\overline{DE}$ with the **Segment** tool. This segment is a copy of $\overline{AB}$.

Hide the compass circle with the **Hide/Show > Object** tool.



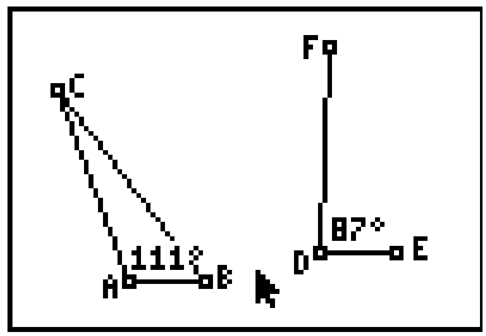
Repeat the use of the **Compass** tool to create a segment congruent to $\overline{AC}$ at point *D*. Construct $\overline{DF}$ so that point *F* is on the compass circle. Then hide the compass circle.

Save this file as *Hinge*. This setup will be used again for Problem 2.



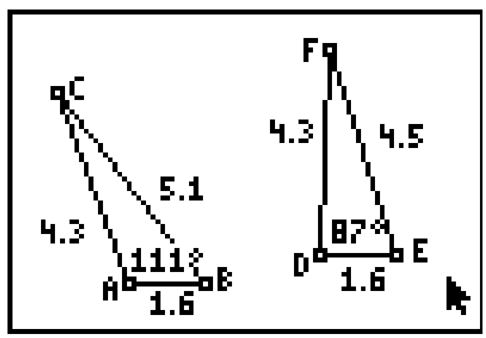
Students should measure $\angle BAC$ and $\angle EDF$ using the **Measure > Angle** tool. If needed, they can drag either point *E* or point *F* to ensure that $\angle BAC$ is larger than $\angle EDF$.

Discuss with students why this theorem is called the "Hinge" theorem.



Students now need to construct $\overline{EF}$, the third side of the new triangle, and measure the sides of both triangles using the **Measure > D. & Length** tool.

Confirm that $\overline{DE}$ and $\overline{DF}$ are congruent to $\overline{AB}$ and $\overline{AC}$, respectively.

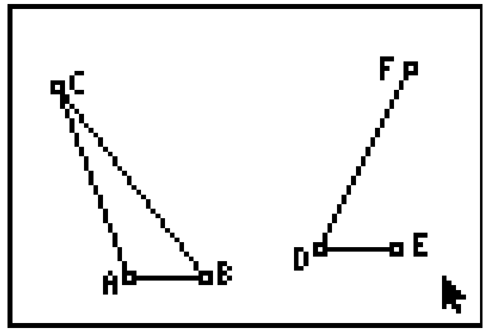


Students will verify that the property stated in the SAS Inequality Theorem holds true for this example. They should drag point *F* so that m$\angle EDF$ increases but remains less than m$\angle BAC$. Then they can drag point *F* so that m$\angle EDF$ is greater than m$\angle BAC$. This enables them to explore changes in the lengths of $\overline{BC}$ and $\overline{EF}$ as $\angle EDF$ changes in size.

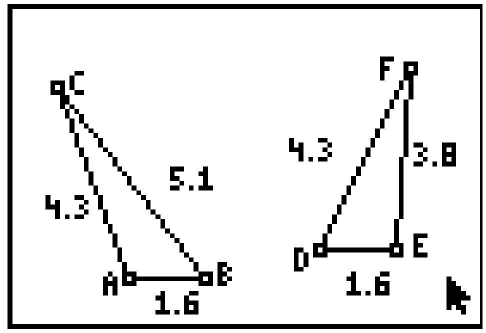
Problem 2 – SSS Inequality Theorem

Students should open the Cabri Jr. file *Hinge* that they created in Problem 1.

Recall that $\overline{DE}$ is a copy of $\overline{AB}$ and $\overline{DF}$ is a copy of $\overline{AC}$.

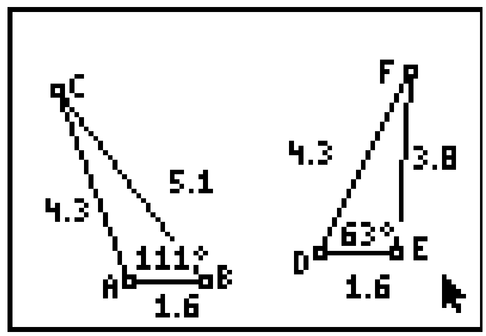


Students will construct $\overline{EF}$, the third side of the new triangle. This side should be smaller than side $\overline{BC}$ of the original triangle. They will drag either point *E* or point *F* if needed to make $\overline{EF}$ shorter and measure the sides of both triangles. Confirm that $\overline{DE}$ and $\overline{DF}$ are congruent to $\overline{AB}$ and $\overline{AC}$, respectively.



Students will measure $\angle BAC$ and $\angle EDF$.

Ask: *Do the angle measures in this example support the property stated in the SSS Inequality Theorem?*



Students should drag either point *E* or point *F* so that the length of $\overline{EF}$ approaches the length of $\overline{BC}$. Then they can make $\overline{EF}$ longer than $\overline{BC}$. This enables them to explore changes in the size of $\angle BAC$ and $\angle EDF$ as $\overline{EF}$ changes in length.