Math Objectives

- Students will identify the defining characteristics of a normal curve related to shape, center, spread, and area.
- Students will recognize that normal curves form a family whose members share these same characteristics.
- Students will use appropriate tools strategically (CCSS Mathematical Practices).
- Students will model with mathematics (CCSS Mathematical Practices).

Vocabulary

- normal curve
- mean
- point of inflection
- standard deviation

About the Lesson

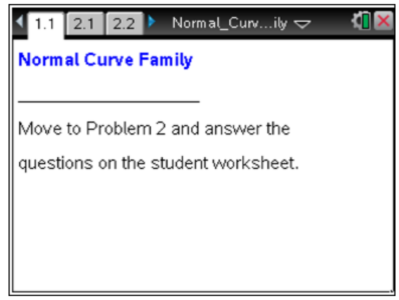
- This lesson involves investigating the relationship of the equation of a normal curve to its graph. As a result, students will:
  - Identify the axis of symmetry as the line $x = \mu$, where μ is the mean of the distribution represented by the normal curve, and the standard deviation as the horizontal distance from that line to the point of inflection.
  - Use a slider to change the values of two parameters, μ and σ, to investigate their effects on the normal curve, noting in particular that μ represents the location of the mean and that σ represents the horizontal distance from the mean to either point of inflection.
  - Estimate the area under a normal curve graphed on a coordinate grid, and investigate the area as the mean and standard deviation are changed.

TI-Nspire™ Navigator™ System

- Send out the *Normal_Curve_Family.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes class captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions might be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Normal_Curve_Family_Student.pdf
- Normal_Curve_Family_Student.doc

TI-Nspire document

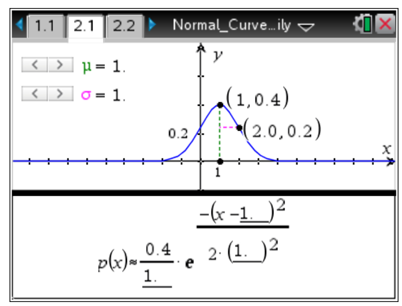
- Normal_Curve_Family.tns

Discussion Points and Possible Answers

Tech Tip: In this activity, students will **not** move the two points on the curve manually. To change the value of μ or σ, students should move the cursor to the left or right arrow displayed on the screen and press .

Teacher Tip: The initial discussion refers only to the parameters **μ** and **σ** and not to the characteristics of a distribution. That comes into the investigation at Question 7, where the curve is connected to the mean and standard deviation of a distribution.

Move to page 2.1.



1. The distributions of many real-world variables can be closely approximated by a normal distribution. The equation of a normal curve is approximately $p(x) \approx \dfrac{0.4}{\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$, where μ is the mean and σ the standard deviation.
   a. Describe the shape, center, and spread of the curve on page 2.1.

   <u>Sample Answers</u>: The curve is mound-shaped (unimodal) and seems to be symmetric to a line through the center—the axis of symmetry. The axis of symmetry is $x = 1$. The spread seems to go from negative infinity to positive infinity, but the curve is bunched up, mostly between –1.5 and 3.5 or –2 and 4. The maximum point, or height, of the curve seems to be about 0.4 at $x = 1$.

Teacher Tip: The actual equation for the normal curve is $p(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$.

If you show the equation to students, be sure they recognize that π and *e* are constants. Students might be asked to determine how the equation explains the behavior of the graph as *x* approaches infinity. They should observe that the number *e* raised to a negative power is really 1 over *e* to that power, so as *x* increases, the fraction becomes 1 divided by a very large number. That number will get closer and closer to 0, and the product of a number very close to 0 and 0.4 will also get increasingly closer to 0.

b. Find $p(1)$ when $\mu = 1$ and $\sigma = 1$. Explain how this point relates to the graph.

Answer: $p(1) = 0.4$, which represents the height of the curve at $x = 1$. The point (1, 0.4) is the maximum point on the curve.

c. Use the arrows to change μ and σ. Describe the changes in the graph of the normal curve.

Answer: The graph of the curve shifts horizontally when μ is changed. It moves to the right when μ increases, and moves to the left when µ decreases. When σ is changed, the height of the graph changes.

2. The point at which a graph changes from concave up to concave down is called the **point of inflection**. How far is a point of inflection from the center of the graph? Explain how you know.

Answer: The distance from a point to a line is the perpendicular distance from the point to the line, which is represented by the horizontal segment in the graph. The length of the segment from the axis of symmetry to either point of inflection is the same as the value of σ, the standard deviation. Using the displayed ordered pairs, you can subtract the *x*-coordinates.

> **Teacher Tip:** To help students understand the points of inflection, you might ask them how they would construct the curve using parts of curves they already know. They might suggest using part of a parabola that opens down and part of an exponential curve. The point at which the two would be glued together is the point of inflection. It can also be described as the point at which the curve, like a bowl with sloped edges, switches from "spilling water" to "holding water." Stress, however, that the curves in the graphs of the normal curves are not really parabolas.

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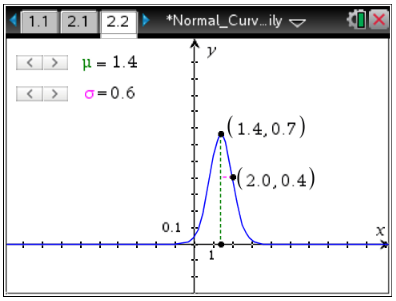

3. a. Two characteristics of this curve are the maximum point (center) and the distance from the center to the point of inflection (measure of spread). Use the arrows to change μ and σ. Describe how the parameters in the equation affect the maximum point and why.

Answer: The height of the maximum point is affected by changing σ. The curve gets "squeezed up" as σ gets smaller. A smaller σ means the points of inflection on either side of the axis of symmetry are closer to the mean. The horizontal location of the maximum point is affected by changing μ.

b. Predict the center, shape, and spread of the curve if $\mu = 3$ and $\sigma = 2$. Verify your prediction using the sliders.

Answer: The center of the curve will be at $x = 3$ on the horizontal axis, and the axis of symmetry will be the line $x = 3$. The curve will be flatter than when the standard deviation is 1, and the tails will still approach but not touch the x-axis. The distance from the axis of symmetry to either point of inflection is 2.

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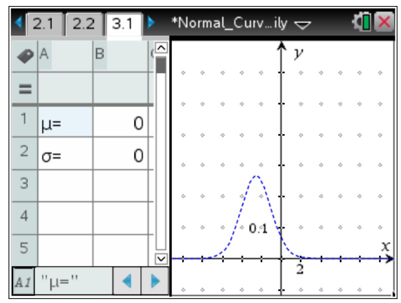


4. Consider the dashed curve.
   a. Predict the values for μ and σ that were used to create the graph. Explain why you think your prediction makes sense.

Sample Answers: Students might suggest the values are about $\mu = -2.5$ and $\sigma = 1.5$. The value for μ is the x-coordinate of the maximum point on the curve, and the distance from the value of μ to the point of inflection is the value of σ.

Tech Tip: To add or modify the data in a spreadsheet cell, double-tap the cell.

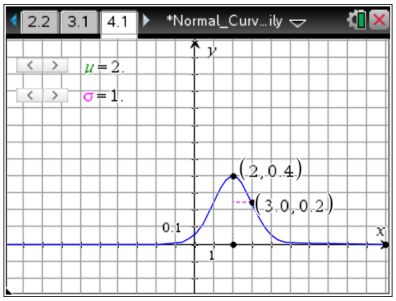
Tech Tip: To hide the keyboard after entering a value into the spreadsheet, press the keyboard down button in the lower right corner .

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

b. Verify the predictions by typing values into Column B of the spreadsheet. (The dotted line will become solid when you have the correct values.)

Answer: $\mu = -2.5$ and $\sigma = 1.5$

Move to page 4.1.



5. a. Describe the axis of symmetry for the curve.

Answer: The axis of symmetry is a line that divides the curve into two congruent parts. In this case, it is the vertical line that goes through $x = \mu$.

b. What happens to the axis of symmetry as μ and σ change?

Answer: The axis of symmetry is not affected at all by a change in σ. It is determined by μ; the line of symmetry is the vertical line $x = \mu$.

6. a. The length of the segment connecting the point of inflection and the axis of symmetry represents the standard deviation. Describe the changes in the graph as the standard deviation increases.

Answer: As the standard deviation increases, the graph becomes flatter and flatter, with the *y*-coordinate of the peak decreasing proportionately with increases in the value of σ.

b. Compare a normal curve with a mean of –2 and a standard deviation of 1 to a normal curve with a mean of 1 and a standard deviation of 1.

Answer: The curves are congruent. The curve with a mean of 1 could be translated 3 units to the left, and it would then match the curve with a mean of –2 exactly.

7. a. Calculate the area of one grid box, and then count boxes to approximate the area between the curve and the horizontal axis when $\mu = 0.6$ and $\sigma = 1.8$. (Note that the horizontal scale is marked in 1 unit intervals and the vertical scale is marked in 0.1 unit intervals.)

Answer: The area of one grid box is 0.1 by 1 or 0.1 square units. By counting and estimating, the number of rectangles between the curve and the horizontal axis is approximately 9.5, so the total area is approximately 0.95 plus the small parts under the tails as the *x*-values continue to increase or decrease, for a total area of about 1.

> **Teacher Tip:** Students might need to be reminded of the extra area that is not clearly visible and that accumulates as *x* goes to plus and minus infinity. A smaller grid might be a way to help them get a better estimate, which can be done by going to **Menu > Window/Zoom > Window Settings** and resetting the *x*-scale to a smaller value, e.g., 0.5 or 0.25.

Tech Tip: The window settings can be accessed by selecting > **Window/Zoom > Window Settings**.

b. Change the value of μ. Predict the total area between the curve and the horizontal axis. Verify by counting the boxes.

Answer: The value of μ will not change the area because the shifted curve will be congruent to the original; thus areas bounded by the curve and the horizontal axis for both curves will be equal.

c. Set μ to 0, and change the value of σ to 0.5. Use the grid boxes to approximate the area between the curve and the horizontal axis.

Answer: If the *x*-scale is not changed, each rectangle formed by the grid still has an area of 0.1. There are still about 9.5 rectangles that are 1 by 0.1 plus the extra area as the curve continues along the *x*-axis. The total area is approximately 1 square unit.

d. Change σ to a new value. Predict the area between the curve and the horizontal axis. Verify by counting the boxes.

Answer: The area continues to be about 1 square unit.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

8. A normal curve has defining characteristics related to shape, center, spread, and area. What are these characteristics and how can you recognize them in a graph?

Answer: (i) The curve is mound-shaped (unimodal) and is symmetric to a line through the axis of symmetry ($x = \mu$).

(ii) The *x*-coordinate of the maximum point is the mean of the distribution. The vertical line through the mean is the axis of symmetry for the curve.

(ii) The standard deviation of the distribution is the horizontal distance from the axis of symmetry to either point of inflection.

(iii) The total area bounded by any normal curve and the horizontal axis is always 1 square unit.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- A normal curve has certain defining characteristics. (These characteristics are given in the answer to Question 8.)
- Normal curves form a family of curves whose members share these characteristics.

Assessment

Identify the following as true or false. Be prepared to explain your reasoning in each case.

a. A normal curve can be short and flat or tall and skinny.

Answer: True; a large value of σ creates a short, flat curve, and a small value of σ creates a tall, skinny curve.

b. You can make two normal curves so that one would fit completely inside the other.

Answer: False; the area between every normal curve and the *x*-axis is 1 square unit, so it is impossible to have one normal curve completely under another.

c. It is possible to create a normal curve such that the area between the curve and the *x*-axis is more than 1.

Answer: False; the area between every normal curve and the *x*-axis is 1 square unit.

d. All normal curves are symmetric.

Answer: True; the curves are symmetric about the line $x = \mu$, the mean of the distribution.

e. There is some value of *x* such that the point (*x*, 0) is on the normal curve.

Answer: False; if this were true, the curve would touch the *x*-axis, but that is impossible given the equation of the normal curve. The *y*-coordinate of a point on the normal curve is never equal to zero.

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Note 1

Question 4 part a, *Quick Poll*

Use Quick Poll to gather students' predictions for the values of μ and σ. Tip: Poll the variables one at a time by having them enter just the number. This will minimize the variation in the responses.

Pick one student's value from those submitted and discuss with students why they think it will or will not match the curve graphed.

Note 2

Question 7, *Quick Poll*

To assess student understanding of the area under a normal curve, send one or several open response Quick Polls. For example, "What is the area under a normal curve with $\mu = 5$ and $\sigma = 2$?" Students should always response with "1" or "approximately 1".