Riemann Sums

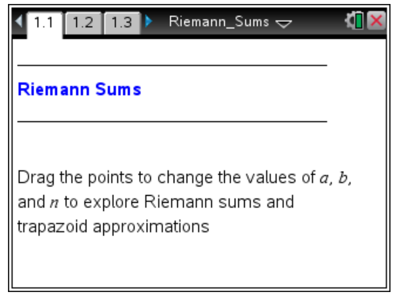
Student Activity

Name ______________

Class ______________

Open the TI-Nspire document *Riemann_Sums.tns*.

The purpose of this activity is to investigate three Riemann sums: left-hand endpoint, right-hand endpoint, and midpoint. One application of the definite integral is used to learn about the accuracy of each estimation and the relationship among these Riemann sums.



Suppose the function f is continuous on the interval $[a, b]$ and $f(x) \geq 0$ for all x in $[a, b]$. One interpretation of the definite integral $\int_a^b f(x)dx$ is the area of the region bounded above by the graph of $y = f(x)$, below by the x-axis, and by the vertical lines $x = a$ and $x = b$. One method to approximate this definite integral—equivalently, the area of the region—is by using a Riemann sum, the area of rectangles. You will consider three different Riemann sums in this activity: left, right, and midpoint.

Read the exercise on page 1.4. Then, move to page 1.3.

Consider the definite integral $\int_1^6 (x^2 + 2)dx$ (the area of the region bounded by the graph of $y = x^2 + 2$, the x-axis, and the lines $x = 1$ and $x = 6$).

1. In the bottom screen, select **L**, for left-hand endpoint Riemann sum. Move the slider for n to set the number of rectangles used to estimate the area of the region. Give answers to 3 decimal places. Note that a decimal approximation of the exact area of the region is also given.

   a. Complete the following table.

n	2	4	8	16	32
Left					

   b. How do your results change in relation to the 'exact' area as n increases? Explain your answer geometrically.

   c. As n increases, the left-hand endpoint Riemann sum in this case always returns an underestimate of the exact area of the region. Explain this result. What characteristic of the function f can be used to justify your answer?

2. Move the point to select **R**, for right-hand endpoint Riemann sum.

   a. Complete the following table.

n	2	4	8	16	32
Right					

   b. When the rectangles are formed from a right-hand endpoint Riemann sum, how does this change in relation to the 'exact' area as *n* increases? Explain your answer geometrically.

   c. As *n* increases, the right-hand endpoint Riemann sum in this case always returns an overestimate of the exact area of the region. Explain this result. What characteristic of the function *f* can be used to justify your answer?

3. Select **M**, for midpoint Riemann sum.

   a. Complete the following table.

n	2	4	8	16	32
Midpoint					

   b. How does the Riemann sum formed from rectangles made from the midpoint of the subintervals change in relation to the exact area as *n* increases? Explain your answer geometrically.

   c. As *n* increases, the midpoint Riemann sum in this case appears to return values that are close to the exact area of the region. Explain this result. Why do you think the midpoint Riemann sum in this case is a better estimate of the area of the region than the left-hand endpoint or right-hand endpoint Riemann sums? That is, why is the midpoint sum closer to the exact answer than the left or right sum for fixed values of *n*?

4. Change the values of n, the type of Riemann sum (**L**, **M**, or **R**), and the definition of the function $f1$ as necessary to answer the following questions.

   a. Consider the function $F(x) = \frac{x^3}{3} + 2x$. Compute $F(6) - F(1)$. How does this value compare to the exact answer in questions 1, 2, and 3? What is the relationship between F and f?

   b. Consider the function $f1(x) = -x^2 + 36$ on the interval [1, 6]. Compute left, right, and midpoint Riemann sums for various values of n. Explain what happens to these values as n increases. Why is *left* always an overestimate and why is *right* Riemann sums always an underestimate?

   c. Consider the function $f1(x) = 3x^2 - 18x + 32$ on the interval [1, 6]. Compute left, right, and midpoint Riemann sums for various values of n. Explain what happens to these values as n increases. Is it possible to predict whether left or right Riemann sums will always be an underestimate or overestimate? Why or why not?

   d. Can you suggest a better sum to estimate the area of the region based on different geometric figures (other than rectangles)? Explain why your method would produce a more accurate estimate of the area.