**Using the Document:** EulersMethod.tns

On page 1.2, the derivative  is defined in a Math Box. The default definition for  is  . This expression can be changed by the user to allow for more in-depth and conceptual questions concerning Euler’s Method. The initial condition, the endpoint

-value, and the number of Euler steps are also defined on page 1.2.

Page 1.3 is a Lists and Spreadsheet page that displays , , and . Page 1.4 shows a graph of the points obtained using Euler’s Method. The slider for  is used to change the number of steps and the slider for  is used to step through each Euler approximation

**Suggested Applications and Extensions**

Use the default initial value problem, , , to answer questions 1-3. The values for , , , and  can be set either in a Math Box or by using a slider. The default values are , , , and . The numerical approximations are given on page 1.3, a Lists and Spreadsheet page, and a visualization of the approximation is given on page 1.4.

1. Use Euler’s Method to approximate  for each of the following values for : (i) , (ii) , (iii) . Which value of  do you think produces the best estimate for ? Why?
2. Use Euler’s Method to approximate  for each of the following values for : (i) ,

(ii) , (iii) . Which value of  do you think produces the best estimate for ? Why?

1. Use Euler’s Method to approximate  for . Use separation of variables to find an expression for  in terms of . Add the graph of  on page 1.4 and compare it to approximation produced by Euler’s Method. Use the graph of  to explain why the Euler approximation for  is an underestimate of the true value for .

**Additional Problems**

1. Use Euler’s Method with  to estimate  where  is the solution to the initial-value problem , .
2. Use Euler’s Method with  to estimate  where  is the solution to the initial-value problem , . Consider each step in this Euler approximation. Explain why the estimate for  is so much larger than the estimate for .
3. Use Euler’s Method with  to estimate  where  is the solution to the initial-value problem , . Find  in terms of  and , and use this expression to explain why this approximation is an underestimate or an overestimate for the true value of .
4. Use Euler’s Method with  to estimate  where  is the solution to the initial-value problem , . Use separation of variables to find an expression for  in terms of . Graph  and the Euler approximation on the same coordinate axes. Explain why the first few Euler approximations are below the graph of  and the remaining approximations are above the graph of .
5. Use Euler’s Method with  to estimate  where  is the solution to the initial-value problem , . Use  to estimate . Which estimate do you think is better? Why?
6. Use Euler’s Method with  to estimate  where  is the solution to the initial-value problem , . Use separation of variables to find an expression for  in terms of . Graph  and the Euler approximation on the same coordinate axes. Find  and use this to explain why the Euler approximation for  is an underestimate of the true value for .
7. Let the function  be the solution to the differential equation  such that .
8. The function  has a critical point at . What is the -coordinate of this critical point?
9. Find  in terms of  and . Use  to determine whether the critical point found in part (a) is a relative minimum, relative maximum, or neither. Justify your answer.
10. The function  has an inflection point at . Use Euler’s Method with  to estimate  where . Is this approximation an overestimate or an underestimate. Justify your answer.