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| Problem 1 – Graphical Riemann Sums  Consider the function *f*(*x*) = –0.5*x*2 + 40.  Suppose we want to find the area bounded by this function and the *x*-axis from *x* =1 to *x* = 3. We can approximate this area with different rectangles: left, right, and midpoint Riemann sums. Using the **AREAPPRX** program, we can approximate this area using the three different Riemann sums mentioned above.  To begin, run the program by pressing ¼ and arrowing down until you reach the AREAPPRX program. Then press Í. And press Í again.  You will be prompted to provide four pieces of information. The first one asks you to enter the function after the **Y=**. After entering the function and pressing Í, you will be prompted to provide the lower bound (the *x*-value of the left endpoint), followed by the upper bound (the *x*-value of the right endpoint). Finally you will be prompted for the number of subintervals, **N**, which represents the number of rectangles (or trapezoids) to use. This time we will use 4 rectangles. The sums for the four different types of approximations are displayed. | |  |
| **Example 1:** Record the following three types of approximations below.  Using 4 rectangles:  Left Riemann sum = \_\_\_\_\_\_\_\_\_\_\_\_\_  Right Riemann sum = \_\_\_\_\_\_\_\_\_\_\_\_\_  Midpoint Riemann sum = \_\_\_\_\_\_\_\_\_\_\_\_\_  Restart the program and calculate the same three area approximations from *x* = 1 to *x* = 3 using 12 rectangles and record the results below.  Using 12 rectangles:  Left Riemann sum ≈ \_\_\_\_\_\_\_\_\_\_\_\_\_  Right Riemann sum ≈ \_\_\_\_\_\_\_\_\_\_\_\_\_  Midpoint Riemann sum ≈ \_\_\_\_\_\_\_\_\_\_\_\_ | | |
| Let’s compare our answers to the result we get using the definite integral command on the calculator. Press ‘ to obtain a fresh screen. Then type » **9:fnInt** followed by Í.  Enter the lower and upper boundaries as 1 and 3 respectively as well as the expression –0.5*x*2 + 40 as shown to the right. Be sure to enter **X** in the last field to denote that you are integrating the function with respect to *x*.  **Example 2:**  \_\_\_\_\_\_\_\_\_\_\_  Compare this answer with the approximations above. |  | |
| **1.** Letting *y* = – 0.5*x*2 + 40 again, run the **AREAPPRX** program from *x* = 0 to *x* = 4 and use 4 rectangles. How do the left, midpoint, and right Riemann sums compare? Explain why.  **2.** Describe what happens to the left, midpoint, and right Riemann sums as you increase the number of subintervals, *n*.  **3.** Is the midpoint Riemann sum an over or under approximation if the graph is:  **a.** Increasing and concave down? \_\_\_\_ over \_\_\_\_\_ under  **b.** Increasing and concave up? \_\_\_\_ over \_\_\_\_\_ under  **c.** Decreasing and concave down? \_\_\_\_ over \_\_\_\_\_ under  **d.** Decreasing and concave up? \_\_\_\_ over \_\_\_\_\_ under  After graphically exploring (especially with a small number of subintervals), explain why. | | |

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| **Problem 2 – Summation Notation** |
| Examine the function Y1(*x*) = –0.5*x*2 + 40.  **4.** The thickness of each rectangle is . If *a* = 1, *b* = 6, and *n* = 5. What is ∆*x*?  **5.** Expand  by writing the sum of the five terms and substituting *i* = 1, 2, 3, 4, and 5.  **6.** Explain why this is the summation notation for LEFT Riemann sums and not the RIGHT.  **7.** Let *y*(*x*) = –0.5*x*2 + 40, *a* = 1, and *b* = 6. Write the sigma notation and use the HOME screen (y  z[quit]) to evaluate the left Riemann sum for 10, 20, 50, and 100 subintervals.  **a.** *n* = 10  **b.** *n* = 20  **c.** *n* = 50  **d.** *n* = 100 |
| **Extension – Area Programs** |
| Use the Area Approximation program **AREAPPROX** to answer the following questions.  **8.** Let *y*(*x*) = *x*2, *a* = 1, and *b* = 6. Write the results for midpoint and trapezoid area approximations when:  **a.** *n* = 10  **b.** *n* = 50  **c.** *n* = 100  **9.** Compare the above midpoint and trapezoid values with the actual area. |