Background

The purpose of this file is to enable users to numerically investigate the limit of a function as $x \to a^+$ and as $x \to a^-$, or the limit as $x \to \infty$ and as $x \to -\infty$. Sliders are used to quickly examine the behavior of each function, and a table of values is also computed and displayed.

Course and Exam Description Unit

1.4: Estimating Limit Values from Tables
1.15: Connecting Limits at Infinity ad Horizontal Asymptotes

Calculator Files

LimitSliders.tns

Using the Document

LimitSliders.tns: This calculator file provides a technology tool for investigating the limit of an arbitrary function as x approaches a specific value from the left and from the right. In a separate calculator problem, the user can investigate the limit of a function as x increases or decreases without bound, that is, as $x \to \infty$ or as $x \to -\infty$. A short table of values for each type of limit is also automatically computed and displayed in a Lists and Spreadsheet page.

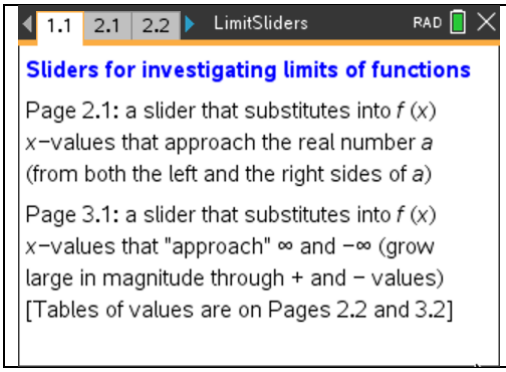
The default limits are

$\lim_{x \to 3} \frac{x^2 - 9}{3 - x}$ (calculator problem 2)

and

$\lim_{x \to \infty} \tan^{-1} x$ and $\lim_{x \to -\infty} \tan^{-1} x$ (calculator problem 3)

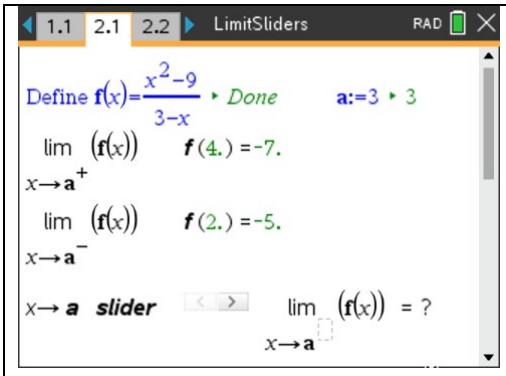
Page 1.1



This introductory screen provides information to help utilize this tns file. In examining either the limit as x approaches a specific number, or as $x$increases or decreases without bound, the function is defined as $f(x)$. A slider is used to change the value of x.

A table of values is given on pages 2.2 and 3.2.

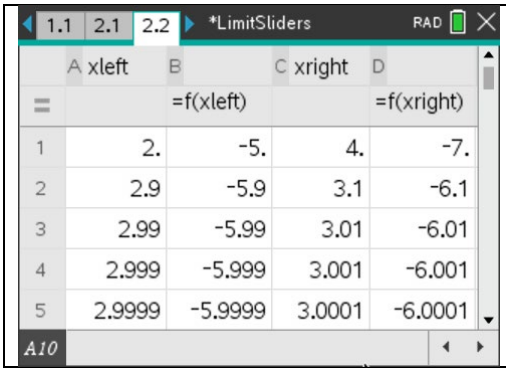
Page 2.1



Page 2.1 is a Notes page that contains two math boxes, values of the function f, and a slider to change the value of x. The function is defined in the first math box, and the value of a is set in the second math box. The function f is initially evaluated at $a \pm 1$. As you click the slider, the value of x move closer to a from the right and from the left in steps of 10^{-n}, for $n = 1,2,3,4,5$. This provides data that can be used to guess the overall limit, as indicated in the lower right side of the screen.

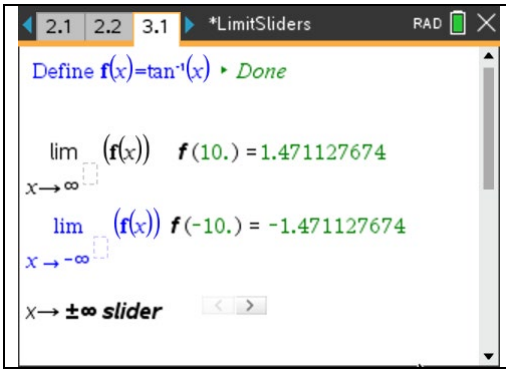
Note that there are calculations at the bottom of this screen, used to compute and display the values of x and the function values.

Page 2.2



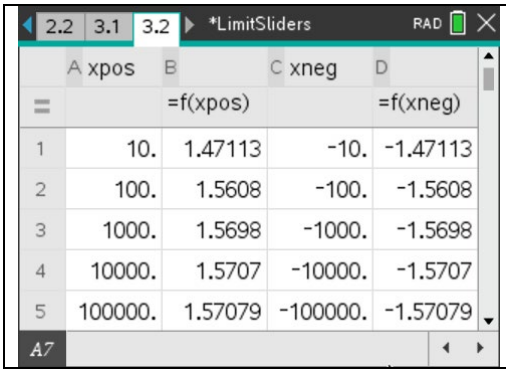
This Lists and Spreadsheet page contains the values of x close to a on the left and the right, and the corresponding values of the function f. This spreadsheet presentation might make it easier to conjecture if the values of f are converging or diverging.

Page 3.1



The function f is defined at the top of this Notes page in the Math Box. By default, the value of the function is given for $x = 10$ and $x = -10$. The slider at the bottom of the page is used to increase the magnitude of each of these x-values, up to $\pm 10^7$. The value of the function is computed and displayed automatically. These numerical values for f may provide insight into, or confirmation of, the limits $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

Page 3.2



This Lists and Spreadsheet page displays the values of x used on Page 3.1, and the corresponding values of the function f. This spreadsheet presentation might make it easier to conjecture if the values of f are converging or diverging as x increases or decreases without bound.

Suggested Applications and Extensions

Use Pages 2.1 and 2.2 to estimate the value of the limit, if it exists.

1. $\lim_{x\to 0}\frac{\sin x}{x}$
2. $\lim_{x\to 1}\frac{\ln x}{x-1}$
3. $\lim_{h\to 0}\frac{(2+h)^4-16}{h}$
4. $\lim_{x\to 0}\frac{x}{e^{2x}-1}$
5. $\lim_{x\to 0}\frac{\tan(2x)}{\sin(5x)}$
6. $\lim_{x\to 0}|x|^x$
7. $\lim_{x\to 4}[\sin x+\ln|x-4|]$
8. $\lim_{x\to 0}x^2-\frac{2^x}{10000}$
9. $\lim_{x\to 0}\sin\frac{1}{x}$
10. $\lim_{x\to 0}\frac{1}{1+e^{1/x}}$
11. $\lim_{x\to 0}f(x)$ where $f(x)=\begin{cases}-(x^2+1) & x<0\\ e^x & x\geq 0\end{cases}$

Use Pages 3.1 and 3.2 to estimate the value of the limit, if it exists.

1. $\lim_{x\to\infty} \frac{3x^2 + 7x - 11}{15 - 2x - 9x^2}$
2. $\lim_{x\to-\infty} \sqrt{\frac{7x^2 - x - 1}{11 - x + 5x^2}}$
3. $\lim_{x\to\infty} \frac{\sqrt{x} + x}{5x - 3}$
4. $\lim_{x\to\infty} \left(\sqrt{3x^2 + 5} - x\right)$ and $\lim_{x\to-\infty} \left(\sqrt{3x^2 + 5} - x\right)$
5. $\lim_{x\to\infty} \frac{x^3}{3^x}$ and $\lim_{x\to-\infty} \frac{x^3}{3^x}$
6. $\lim_{x\to\infty} \frac{\ln|x|}{x}$ and $\lim_{x\to-\infty} \frac{\ln|x|}{x}$
7. $\lim_{x\to\infty} \frac{\sqrt{3x^2 + 7}}{2x + 5}$ and $\lim_{x\to-\infty} \frac{\sqrt{3x^2 + 7}}{2x + 5}$
8. $\lim_{x\to\infty} \frac{\sin x}{e^x}$ and $\lim_{x\to-\infty} \frac{\sin x}{e^x}$
9. $\lim_{x\to\infty} \left(\sqrt{x^2 + 3x + 7} - \sqrt{3x^2 - 7x - 5}\right)$
10. $\lim_{x\to\infty} \frac{e^{-x}}{\tan^{-1} x}$ and $\lim_{x\to-\infty} \frac{e^{-x}}{\tan^{-1} x}$