This document provides the teacher with additional information about using pattern blocks to model addition and subtraction of fractions that can be used in the lesson Addition and Subtraction of Rational Numbers, Part 1.

1. *How to Use Pattern Blocks to Model Fractional Operations* – provides the teacher with the description of pattern blocks and explanations on how to use this manipulative in the classroom for modeling operations with fractions. This handout will be useful for the teachers who had limited experience with pattern blocks.

HOW TO USE PATTERN BLOCKS TO MODEL FRACTIONAL OPERATIONS

For many years addition, subtraction, multiplication, and division of fractions in elementary and middle schools have been taught algorithmically. Students do not *understand* the procedures they use to perform operations on fractions, so they memorize them.

All too often, children's disenchantment with mathematics begins late in elementary school or early in middle school when, even after years of practice, they cannot remember how to "do" fractions after summer vacation, or when they can perform steps, but are totally bored because they do not know what the steps mean or why they are doing them. Understanding fractions marks only the beginning of the journey toward rational number understanding, and by the end of the middle school years, as a result of maturation, experience, and fraction instruction, it is assumed that students are capable of a formal thought process called *proportional reasoning*. The form of reasoning opens the door to high school mathematics and science, and eventually, to careers in the mathematical sciences. The losses that occur because of the gaps in conceptual understanding about fraction, ratio, and related topics are incalculable. The consequence of *doing*, rather than understanding, directly or indirectly affect a person's attitudes toward mathematics, enjoyment and motivation in learning, course selection in mathematics and science, achievement, career flexibility, and even the ability to fully appreciate some of the simplest phenomena in everyday life. (Lamon, 2005)

If seventh grade students have never used models (fraction strips, paper folding, Cuisenaire rods, fraction circles, fraction squares, pattern blocks, etc.), they have difficulty visualizing what a fraction represents much less how to perform mathematical operations using them.

Students' understanding and ability to reason will grow as they represent fractions and decimals with physical materials and on number lines and as they learn to generate equivalent representations of fractions and decimals. (NCTM, 2000)

A possible pre-test to determine students' understanding of fractional operations consists of 4 problems:

Model the following operations as well as indicate the numerical solutions. Indicate what 1 unit represents in each problem:

1) $\dfrac{1}{2} + \dfrac{1}{3} =$ 2) $\dfrac{1}{2} - \dfrac{1}{3} =$ 3) $\dfrac{1}{2} \times \dfrac{1}{3} =$ 4) $\dfrac{1}{2} \div \dfrac{1}{3} =$

The examples below move students from concrete models to abstract thinking. Requiring students to model as well as provide a numerical answer to problems involving addition, subtraction, multiplication and division will increase student understanding.

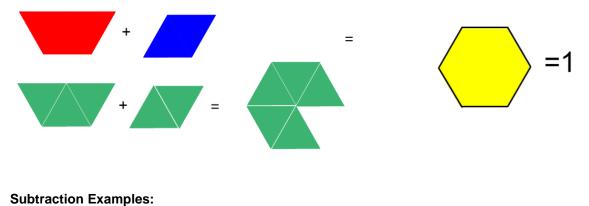**Addition Example:** Find $\dfrac{1}{2}+\dfrac{1}{3}$

$$\dfrac{1}{2}+\dfrac{1}{3}=$$

$$\dfrac{3}{6}+\dfrac{2}{6}=\dfrac{5}{6}$$

Students must find a common denominator to combine the addends. In the numerical answer the lowest common denominator is 6 and students can find it by calculating the least common multiple of denominators 2 and 3. (Please note all that is needed is a common denominator. Fractions are simplified to expedite computation. However, students need to realize that sometimes in real-life problems this process can lead to values that are harder to interpret. For example $\dfrac{3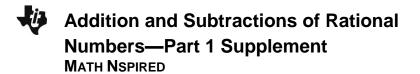}{5}$ is harder to interpret as percent than $\dfrac{60}{100}$. In other situations important information may be lost.) In the concrete model the common denominator is represented by the green triangle. The process of finding a common denominator in the concrete model is equivalent to finding a single shape that can be used to construct both shapes that represent the addends.
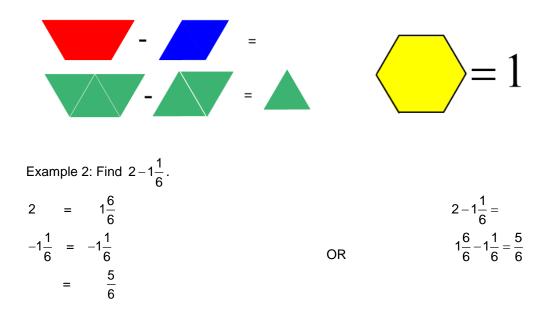

Subtraction Examples:

Example 1: Find $\dfrac{1}{2}-\dfrac{1}{3}$.

$$\dfrac{1}{2}-\dfrac{1}{3}=$$

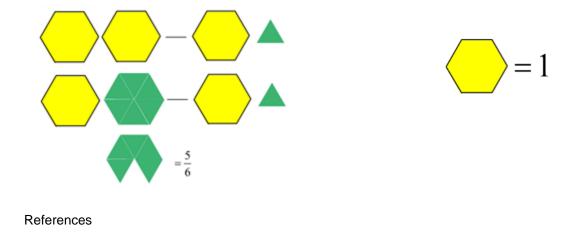$$\dfrac{3}{6}-\dfrac{2}{6}=\dfrac{1}{6}$$

Again, students must get a common denominator of two fractions to subtract. In the numerical answer the lowest common denominator is 6. In the concrete model the common denominator is represented by the green triangle.

Example 2: Find $2 - 1\frac{1}{6}$.

$$2 \quad = \quad 1\frac{6}{6}$$
$$-1\frac{1}{6} \quad = \quad -1\frac{1}{6}$$
$$= \quad \frac{5}{6}$$

OR

$$2 - 1\frac{1}{6} =$$
$$1\frac{6}{6} - 1\frac{1}{6} = \frac{5}{6}$$

In the numerical method above students rewrite 2 as a mixed number using the lowest common denominator 6 and the equivalent fraction, $1\frac{6}{6}$. When the problem is written vertically, students will often record the answer as $1\frac{1}{6}$ rather than $\frac{5}{6}$ because they have no understanding of the process of subtraction. They never consider more than one is being subtracted from 2 and they are recording the answer as a number larger than one. In the concrete model below students exchange one yellow hexagon for 6 green triangles so one yellow hexagon and one green triangle can be removed.

References

Lamon, Susan J. Teaching fractions and Ratios for Understanding, 2nd edition, Mahwah, New Jersey: Lawrence Erlbaum Associates, 2005.

Principles and Standards for School Mathematics. Reston, Virginia: National Council of Teachers of Mathematics, 2000.