



Math Objectives

- Students will use an interactive area model to multiply and divide positive decimals and mixed numbers.
- Students will use the distributive and commutative properties to multiply decimals and mixed numbers.
- Students will use the idea of division as an inverse operation of multiplication to divide decimals and mixed numbers.
- Students will apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers (CCSS).
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- factor
- product
- dividend
- divisor
- quotient
- commutative property of multiplication
- distributive property
- inverse

About the Lesson

- This lesson involves using an interactive area model to solve multiplication and division problems with positive decimals and mixed numbers. As a result, students will:
 - Make connections between the geometric model and mathematical operations.
 - Record their steps and observations on their worksheets.




TI-Nspire™ Navigator™

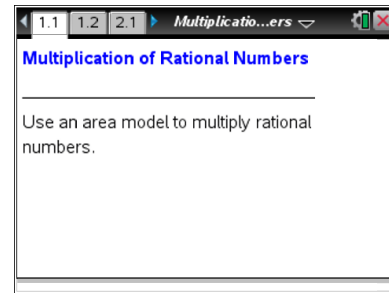
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to debrief solutions.
- Use Quick Poll to assess students' understanding.

Activity Materials

Compatible TI Technologies:  TI-Nspire™ CX Handhelds,



TI-Nspire™ Apps for iPad®,  TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

Lesson Files:

Student Activity

- Multiplication_and_Division_of_Rational_Numbers_Student.pdf
- Multiplication_and_Division_of_Rational_Numbers_Student.doc




TI-Nspire document

- Multiplication_and_Division_of_Rational_Numbers.tns



Discussion Points and Possible Answers



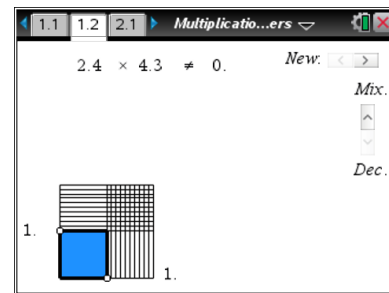
Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand  ready to grab the solid white point. Then press **ctrl**  to grab the point and close the hand . To release a shape, students should press either **esc** or the **enter** key. After using any of the arrows, click once anywhere, and the “grab hand” is operable.

Move to page 1.2.

For each multiplication problem, the factors represent the height and the width of the rectangle.

1. What is the geometric meaning of the product of two factors?

Answer: The product of two factors represents the area of the rectangle.



Teacher Tip: Discuss with the students whether it matters which factor is the height and which factor is the width. This should lead to the discussion of the commutative property of multiplication. In some problems in this activity, students will be able to model multiplication factors as height or width. Note: for all generated problems, the first factor has a value between 1 and 4, and the second factor has a value between 1 and 6. These restrictions are provided to keep the rectangle within the page.

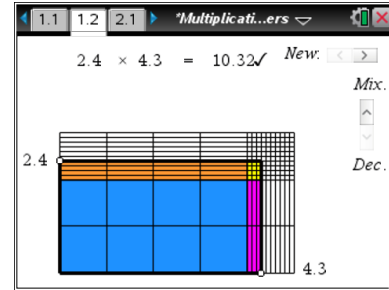
Teacher Tip: When the TI-Nspire document is first opened, all students will have the same first problem, 2.4×4.3 , which gives you the opportunity to have a whole class discussion about modeling the solution to the first problem with the students. Once students have clicked to generate new problems, they can always go back to the first problem by clicking on the left arrow until they cannot click anymore.



Grab and drag the white points to adjust the height and the width of the rectangle in order to model the multiplication problem. After you find the area, double-click the solution box on Page 1.2, and enter your answer. If your answer is correct, a check mark will appear.

Note: Do not delete the decimal point in the box.

- Find the value of 2.4×4.3 using the area model on Page 1.2. Show all the steps you used to calculate your answer and explain your reasoning.



Sample Answers: The area of a large blue square is 1, and there are 8 of them. The area of an orange horizontal rectangle is 0.1, there are 4 rows by 4 columns of these rectangles for a total of 16. The area of a vertical purple rectangle is 0.1; there are 2 rows by 3 columns of these rectangles for total of 6. Area of a small yellow square is 0.01; there are 4 rows by 3 columns of these squares for total of 12. Thus, $(8 \times 1) + (16 \times .1) + (6 \times .1) + 12 \times .01 = 10.32$

OR

Consider a whole rectangle of a given color. The area of the blue rectangle is $2 \times 4 = 8$, the area of the orange rectangle is $.4 \times 4 = 1.6$, the area of the purple rectangle is $2 \times .3 = .6$, and area of the yellow rectangle is $.4 \times .3 = 0.12$. Thus $2.4 \times 4.3 = 2 \times 4 + .4 \times 4 + 2 \times .3 + .4 \times .3 = 10.32$.

Teacher Tip: This is a good place to discuss with the students how the distributive property is used in the area model. Even if students do not clearly state that they are using the distributive property, as long as they find an area of each colored rectangle, they are using the distributive property implicitly. To help students to make connections between the distributive property and the area model, encourage them to consider each color to find the height and width of each rectangle and then to calculate the area as shown in a second answer above. Then show the students that the total area can be written as $(2 + .4) \times (4 + .3)$ and leads to the same result, such as $2 \times 4 + .4 \times 4 + 2 \times .3 + .4 \times .3$.

TI-Nspire Navigator Opportunity: Quick Poll, Class Capture, and Live Presenter
See Note 1 at the end of this lesson.



3. Click the right arrow in the top right corner of Page 1.2 to generate several new multiplication problems. For each problem:
 - a. Record the problem in the chart provided at the end of this worksheet.
 - b. Model the problem using the interactive area model on Page 1.2.
 - c. Find the product, and show all the steps of your calculations in the chart.
 - d. Check your calculations by entering your answer into the solution box on Page 1.2. Correct your work as needed.
 - e. Place a check mark in the “✓” box when your answer is correct.

Answer: Students’ problems and answers will vary. Sample problems with answers are given in the chart at the end of the document.

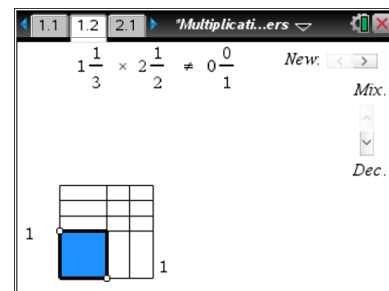
Teacher Tip: The new problems are generated randomly, so different students might have different problems. This gives you an opportunity to select a few students to share their solutions. Another possibility would be to put students in small groups and ask each group to share their solution. Make sure that students explain how they use the commutative and distributive properties.



TI-Nspire Navigator Opportunity: Quick Poll, Screen Capture, and Live Presenter

See Note 1 at the end of this lesson.

4. Click the up arrow on the right side of Page 1.2 to change the problem to multiplication of mixed numbers. Find the value of $1\frac{1}{3} \times 2\frac{1}{2}$ using the area model. Show all the steps you used to calculate your answer and explain your reasoning.



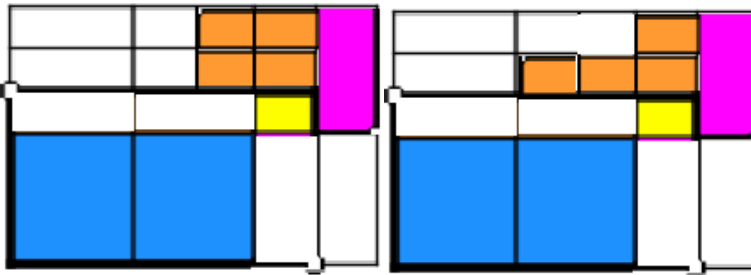
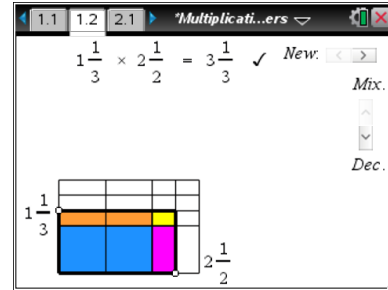
Teacher Tip: When the problem is first changed to multiplication of mixed numbers, all students will have the same first problem, $1\frac{1}{3} \times 2\frac{1}{2}$, which again gives you the opportunity to have a whole class discussion about modeling the solution to the problem using an area model. Once students have clicked to generate new problems, they can always go back to the first problem by clicking on the left arrow until they cannot click anymore.



Answer: The area of the blue rectangle is $1 \times 2 = 2$; the area of the orange rectangle is $\frac{1}{3} \times 2 = \frac{2}{3}$; the area of the purple rectangle is $1 \times \frac{1}{2} = \frac{1}{2}$; and the area of the yellow rectangle is $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. The total area can be found by re-arranging

orange and purple rectangles as shown below. Thus,

$$1\frac{1}{3} \times 2\frac{1}{2} = \left(1 + \frac{1}{3}\right) \times \left(2 + \frac{1}{2}\right) = 1 \times 2 + \frac{1}{3} \times 2 + 1 \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{2} = 3\frac{1}{3}$$



Teacher Tip: Ask students to explain the visual representation of the product and how they can justify that the total area is $3\frac{1}{3}$. Then,

encourage student discussion about the use of the distributive property in the area model for multiplication of mixed numbers. Again point their attention to the fact that the area of a whole rectangle is equal to the sum of the areas of 4 rectangles, each of a particular color. Let students write the height of a whole rectangle as the sum of heights of blue and orange rectangles. Similarly, they can write the width of the whole rectangle as the sum of widths of blue and purple rectangles. This will help them to make a connection between the area model and the distributive property. As students explain their problem solving process, ask them to identify properties of operations they used in their solutions.

Tech Tip: Students will need to enter their answers as mixed numbers. They will double-click on the whole number part of the product. When they see the cursor for text, they enter 3; then, they double-click on the fractional part of the product to enter $\frac{1}{3}$. They will need to move the cursor to the numerator and enter 1; then move the cursor to the denominator and enter 3.



TI-Nspire Navigator Opportunity: *Quick Poll, Class Capture, and Live Presenter*

See Note 1 at the end of this lesson.

5. Click the right arrow in the top right corner of Page 1.2 to generate several new multiplication problems. For each problem:
 - a. Record the problem in the chart provided at the end of this worksheet.
 - b. Model the problem using the interactive area model on Page 1.2.
 - c. Find the product, and show all the steps of your calculations in the chart.
 - d. Check your calculations by entering your answer into the solution box on Page 1.2. Correct your work as needed.
 - e. Place a check mark in the “✓” box when your answer is correct.

Answer: Students’ problems and answers will vary. Sample problems with answers are given in the chart at the end of the document.

Teacher Tip: The new problems are generated randomly, so different students might have different problems. This gives you an opportunity to select a few students to share their solutions. Another possibility would be to put students in small groups and ask each group to share their solution. Make sure that students explain how they use commutative and distributive properties.



TI-Nspire Navigator Opportunity: *Quick Poll, Class Capture, and Live Presenter*

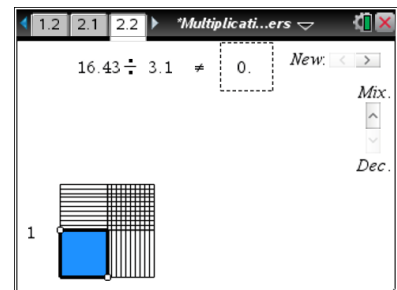
See Note 1 at the end of this lesson.

Move to page 2.1 and then page 2.2.

For each division problem, the dividend represents the area of the rectangle and the divisor represents the height of the rectangle.

6. What is the geometric meaning of the quotient?

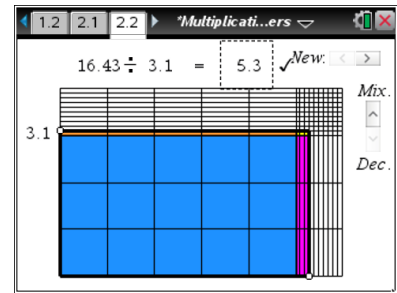
Answer: The quotient represents the width of the rectangle.



Teacher Tip: When Problem 2 is first opened, all students will have the same first problem, $16.43 \div 3.1$. Once students have clicked to generate new problems, they can always go back to the first problem by clicking on the left arrow until they cannot click anymore.



Grab and drag the white points to adjust the height and the width of the rectangle in order to model the division problem. Move the white point on the left side of the model to represent the divisor; move the white point on the bottom of the model until the rectangular area represented by the model equals the dividend. After you find the quotient, double-click the solution box on Page 1.2, and enter your answer. If your answer is correct, a check mark will appear. **Note:** Do not delete the decimal point in the box.



- Find the value of $16.43 \div 3.1$ using the area model on Page 2.2. Show all steps used to calculate your answer and explain your reasoning.

Sample Answers: The height of the rectangle is 3.1. As the width of the rectangle increases, so does the area of the rectangle. In order to find the width of the rectangle, we need to make sure that the area is equal to 16.43. Since $3 \times 5 = 15$ and $3 \times 6 = 18$, the width is a little larger than 5. When the width is 5, the area is $15 + 5 \times 0.1 = 15.5$ counting the whole squares and the one-tenths across the top. The dividend has a 3 in the hundredths place so a good number to try would be 5.3. Then additional areas comes from the purple rectangle that has area of $3 \times .3 = .9$ and the yellow rectangle that consists of 3 squares of 0.01 with area of $3 \times 0.01 = 0.03$. Adding all of them, we get $15.5 + .9 + .03 = 16.43$. Thus, the quotient is 5.3.



TI-Nspire Navigator Opportunity: Quick Poll, Class Capture, and Live Presenter

See Note 1 at the end of this lesson.

Teacher Tip: The area model for division uses the definition of division as the inverse operation to multiplication. Students should realize the quotient (width of the rectangle) multiplied by the height of the rectangle (divisor) is equal to the given area (dividend). Encourage student discussion to help them recognize this fact.



8. Click the right arrow in the top right corner of Page 2.2 to generate several new division problems. For each problem:
- Record the problem in the chart provided at the end of this worksheet.
 - Model the problem using the interactive area model on Page 2.2.
 - Find the quotient, and show all the steps of your calculations in the chart.
 - Check your calculations by entering your answer into the solution box on Page 2.2. Correct your work as needed.
 - Place a check mark in the “✓” box when your answer is correct.

Answer: Students’ problems and answers will vary. Sample problems with answers are given in the chart at the end of the document.

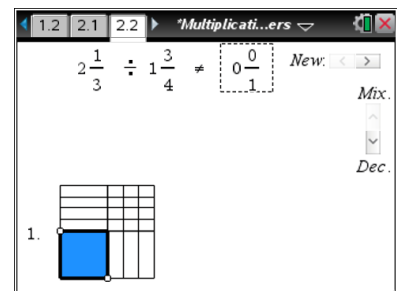
Teacher Tip: The new problems are generated randomly, so different students might have different problems. This gives you an opportunity to select a few students to share their solutions. Another possibility would be to put students in small groups and ask each group to share their solution. Make sure that students explain how they use the commutative and distributive properties.



TI-Nspire Navigator Opportunity: Quick Poll, Class Capture, and Live Presenter

See Note 1 at the end of this lesson.

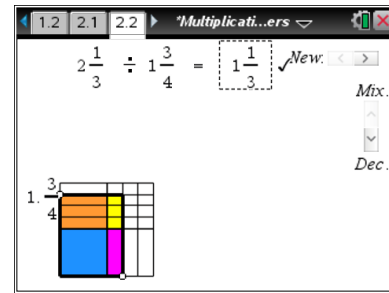
9. Click the up arrow on the right side of Page 2.2 to change the problem to division of mixed numbers. Find the value of $2\frac{1}{3} \div 1\frac{3}{4}$ using the area model. Show all the steps you used to calculate your answer and explain your reasoning.



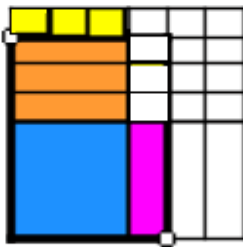
Teacher Tip: When the problem is first changed to division of mixed numbers, all students will have the same first problem, $2\frac{1}{3} \div 1\frac{3}{4}$, which again gives you the opportunity to have a whole class discussion about modeling the solution of the problem using an area model. Once students have clicked to generate new problems, they can always go back to the first problem by clicking on the left arrow until they cannot click anymore.



Answer: The height of the rectangle is $1\frac{3}{4}$. In order to find the width of the rectangle, we need to make sure that the area is equal to $2\frac{1}{3}$. Since one group of $1\frac{3}{4}$ is contained in $2\frac{1}{3}$, but $2 \times 1\frac{3}{4} = 3\frac{1}{2}$ which is larger than $2\frac{1}{3}$, the width is between 1 and 2.



When the width is 1, the area is $1 \times 1\frac{3}{4} = 1\frac{3}{4}$. Next, increase the width by $\frac{1}{3}$. Then additional areas come from the purple rectangle that has area of $\frac{1}{3}$ and the yellow rectangle that consists of 3 squares of $\frac{1}{12}$ with area of $\frac{3}{12} = \frac{1}{4}$. Adding all of them, we get $1\frac{3}{4} + \frac{1}{3} + \frac{1}{12} = 2\frac{1}{3}$. Visually, it can be shown by re-arranging the shapes to complete 2 unit squares as shown below. Thus, the quotient is $1\frac{1}{3}$.



Tech Tip: Students will need to enter their answers as mixed numbers. Students double-click on the integer part of the quotient. When they see the cursor for text, they enter 1; then, they double-click on the fractional part of the quotient to enter $\frac{1}{3}$. They will need to move the cursor to the numerator and enter 1; then move the cursor to the denominator and enter 3.



TI-Nspire Navigator Opportunity: Quick Poll, Class Capture, and Live Presenter

See Note 1 at the end of this lesson.



10. Click the right arrow in the top right corner of Page 2.2 to generate several new division problems.

For each problem:

- Record the problem in the chart provided at the end of this worksheet.
- Model the problem using the interactive area model on Page 2.2.
- Find the quotient, and show all the steps of your calculations in the chart.
- Check your calculations by entering your answer into the solution box on Page 2.2. Correct your work as needed.
- Place a check mark in the “✓” box when your answer is correct.

Answer: Students’ problems and answers will vary. Sample problems with answers are given in the chart at the end of the document.

Teacher Tip: The new problems are generated randomly, so different students might have different problems. This gives you an opportunity to select a few students to share their solutions. Another possibility would be to put students in small groups and ask each group to share their solution. Make sure that students explain how they use commutative and distributive properties.



TI-Nspire Navigator Opportunity: *Quick Poll, Class Capture, and Live Presenter*

See Note 1 at the end of this lesson.

Teacher Tip: These problems were presented without context to develop basic understanding of operations with rational numbers. In order to bring real-world context to these problems, ask students to develop stories for some problems they solved in this activity. As an extension, provide students with real-world context that results in solving multiplication and division problems with rational numbers. See assessment questions below for several examples of real-world problems.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- Using the distributive property to solve problems involving multiplication of rational numbers.
- Using the definition of division as the inverse operation of multiplication to solve problems involving division of rational numbers.
- Using an area model to solve problems involving multiplication and division of rational numbers.



Assessment

1. Kristen is making butterscotch brownies. The recipe calls for $1\frac{3}{4}$ cups of brown sugar. If she makes $3\frac{1}{2}$ batches of brownies, how much brown sugar will she need?

Answer: $6\frac{1}{8}$ cups

2. Ella is working on an order for bows. She uses $\frac{5}{8}$ yard of ribbon to make one bow. How many bows can Ella make from $4\frac{1}{3}$ yards? How much ribbon will she have left?

Answer: 6 bows; $\frac{14}{15}$ yards

3. You purchased $2\frac{1}{4}$ lbs of cheese at \$1.75 per pound. How much money did you spend?

Answer: $3\frac{15}{16} \approx \$3.94$

4. You have 3 dollar bills and 7 dimes in your pocket. How many water bottles can you buy from a vending machine that takes one dollar and one quarter per bottle? How much will you have left?

Answer: 2 bottles ; $2\frac{14}{15} \approx \$2.96$

5. In order to frame a large window, you need $14\frac{3}{8}$ feet of wooden plank. It is sold in pieces that measure $2\frac{1}{2}$ feet in length. You can also ask the store to cut the pieces so you only get what you need. How many pieces do you need?

Answer: 5 pieces

6. If Jannie walks 3.4 miles in .8 hours, how fast did she walk?

Answer: 4.25 miles per hour

7. If Mary walks 3.6 miles per hour for 1.9 hours, how far does she walk?

Answer: 6.84 miles



TI-Nspire Navigator

Note 1

Quick Poll, Class Capture, and Live Presenter

Use Quick Poll to collect answers to numerical questions. Use Class Capture to monitor how students model the problems. Use Live Presenter to have students explain the method they used to solve problems. Ask students to describe the process they used to model each problem with the area model and to describe how they determined a product or a quotient from the model.

Multiplication and Division of Rational Numbers – Sample Answers.

PROBLEM	COMPUTATION METHOD	ANSWER	✓
2.1×4.4	$(8 \times 1) + (4 \times .1) + (8 \times .1) + (4 \times .01) =$ $8 + (12 \times .1) + .04 = 8 + 1.2 + .04 = 9.24$	9.24	✓
3.8×2.9	$(6 \times 1) + (16 \times .1) + (27 \times .1) + (72 \times .01) =$ $6 + (43 \times .1) + .72 = 6 + (4 + .3) + .72 =$ 11.02	11.02	✓
3.2×5.5	$(15 \times 1) + (10 \times .1) + (15 \times .1) + (10 \times .01) =$ $15 + (1.0) + (1.5) + .1 = 17.6$	17.6	✓
$2\frac{4}{5} \times 4\frac{2}{3}$	$2\frac{4}{5} \times 4\frac{2}{3} = (2 + \frac{4}{5}) \times (4 + \frac{2}{3}) =$ $2 \times (4 + \frac{2}{3}) + \frac{4}{5} \times (4 + \frac{2}{3}) =$ $2(4) + 2(\frac{2}{3}) + \frac{4}{5}(4) + \frac{4}{5}(\frac{2}{3}) =$ $8 + (1 + \frac{1}{3}) + (3 + \frac{1}{5}) + \frac{8}{15} =$ $12 + \frac{1}{3} + \frac{1}{5} + \frac{8}{15} = 12 + \frac{5}{15} + \frac{3}{15} + \frac{8}{15} =$ $12 + \frac{1}{3} + \frac{1}{5} + \frac{8}{15} = 12 + \frac{5}{15} + \frac{3}{15} + \frac{8}{15} =$ $12 + \frac{16}{15} = 13\frac{1}{15}$	$13\frac{1}{5}$	✓
$3\frac{1}{2} \times 3\frac{1}{2}$	$(3 + \frac{1}{2}) \times (3 + \frac{1}{2}) = 3(3) + 3(\frac{1}{2}) + \frac{1}{2}(3) + \frac{1}{2}(\frac{1}{2}) =$ $9 + 3 + \frac{1}{4} = 12\frac{1}{4}$	$12\frac{1}{4}$	✓



$2\frac{1}{5} \times 2\frac{1}{4}$	$4 + \frac{2}{5} + \frac{1}{2} + \frac{1}{20} = 4 + \frac{8}{20} + \frac{10}{20} + \frac{1}{20}$	$4\frac{19}{20}$	✓
$16.24 \div 2.8$	$16 \div 3$ is about 5.3; If the width of the rectangle is 5.3, then the area would be $10 + 5(.8) + 2(.3) + 3(.08) = 14.84$ which is not enough. Every time the width increased by .1, .28 is added to the area. $5 \times .28 = 1.4$. $1.4 + 14.84 = 16.24$	5.8	✓
$12.6 \div 3.6$	The quotient will be around 3. If the width of the model is 3, then the area would be $9 + 18$ tenths or 10.8 which is not enough. Every time the width is increased by .1, then the area will increase by .36.	3.5	✓
$17.39 \div 3.7$	$17 \div 4 > 4$; If the width is pulled to 4.5, then the area will be $12 + 2.8 + 1.5 + .35 = 16.65$. Every time the wide is increased by .1, the area increased by .37. The width needs to be increased by an additional .2.	4.7	✓
$20\frac{4}{9} \div 3\frac{5}{6}$	$21 \div 4 > 5$ so start with a rectangular width of $5\frac{1}{3}$. This width gives $15 + \frac{25}{6} + 1 + \frac{5}{18} = 15 + 4\frac{1}{6} + 1 + \frac{5}{18} = 20 + \frac{3}{18} + \frac{5}{18} = 20\frac{8}{18} = 20\frac{4}{9}$	$5\frac{1}{3}$	✓
$5\frac{1}{18} \div 2\frac{1}{6}$	The quotient will be between 2 and 3. If the width is $2\frac{1}{3}$, then the area is $= 4 + \frac{2}{3} + \frac{2}{6} + \frac{1}{18} = 4 + \frac{2}{3} + \frac{1}{3} + \frac{1}{18} = 5\frac{1}{18}$	$2\frac{1}{3}$	✓
$12\frac{2}{3} \div 2\frac{2}{3}$	$13 \div 3 > 4$. A width of $4\frac{3}{4}$ will give the following area: $8 + 6(\frac{1}{4}) + 8(\frac{1}{3}) + \frac{6}{12} = 8 + 1\frac{1}{2} + 2\frac{2}{3} + \frac{6}{12} = 9\frac{1}{2} + 2\frac{2}{3} + \frac{1}{2} = 10 + 2\frac{2}{3} = 12\frac{2}{3}$	$4\frac{3}{4}$	✓