

Inside Out

Student Activity

7 8 9 10 11 12



TI-Nspire



Investigation



Student



50min

Aim

Where is the centre of a triangle? This investigation explores the circumcentre of a triangle and how it is identified as 'the centre' by consideration of congruency.

Equipment

For this activity you will need:

- TI-nspire
- TI-nspire document – Inside Out

Introduction

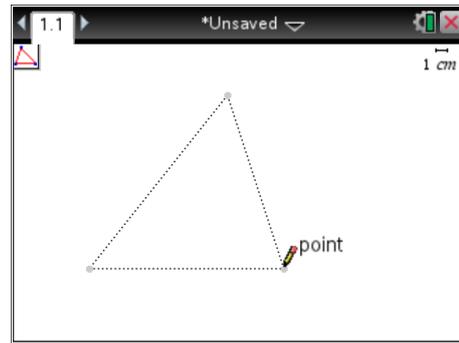
What shapes have a centre? A circle can be defined as a set of points equidistant from a fixed point, the centre of the circle. So the distance from the centre of the circle is the same to all points on its circumference. The diagonals of a square intersect at a single point, this point is equidistant from each of the square's vertices. An equilateral triangle, just like the circle and square, has a point that is equidistant from points on its perimeter. What about an isosceles or scalene triangle, do they have a similar point?

Introduction – Setting up the calculations

Press the **Home** key, select **New Document** and insert a **Geometry** Application.

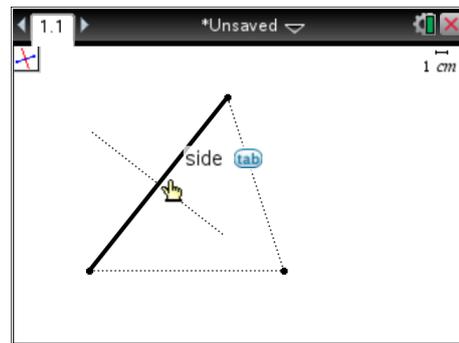


Use the **shapes** menu to construct a **triangle**. The mouse changes to a pencil. Make an acute angled triangle that is approximately half the height of the screen. Click on the mouse pad to place the vertices (corners) of the triangle. (Shown opposite)



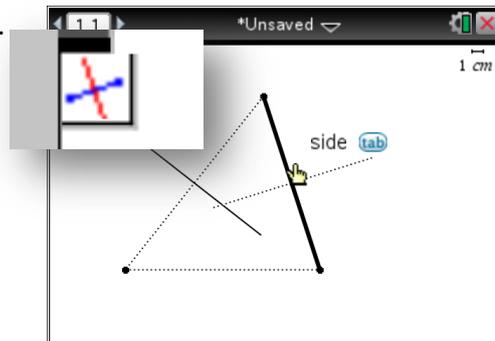
Use the **construction** menu to draw a **perpendicular bisector** on each side of the triangle.

Move the mouse over one of the triangle sides, the word 'side' appears as a prompt, the side of the triangle is highlighted and the perpendicular bisector appears dotted. Click on the side to construct the perpendicular bisector.



Repeat the above step for the other two sides of the triangle. The perpendicular bisector tool is still active therefore click on each of the remaining sides of the triangle to construct each perpendicular bisector.

To release the tool press **[esc]**



Perpendicular Bisector – This term has two parts:

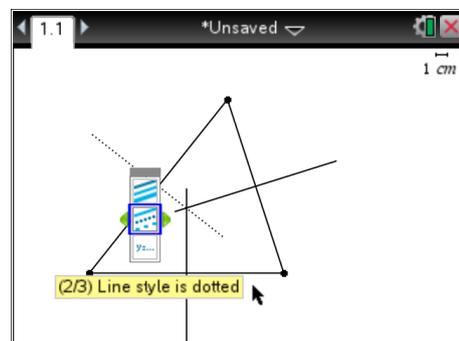
Perpendicular = Right angle

Bisector = Cut into two equal parts

To help keep the construction lines visually separate from the sides of the triangle, make the perpendicular bisectors 'dotted'.

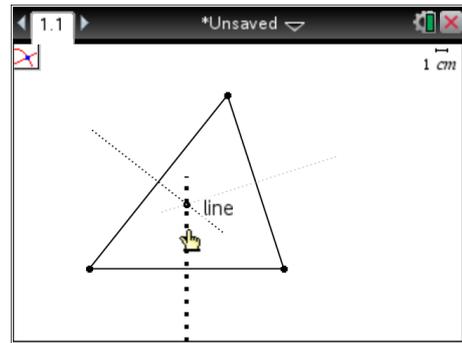
Move the mouse over the perpendicular bisector, a screen tip appears: 'line'. Press **[Ctrl]** followed by **[menu]** and select **attributes**.

Select the **dotted** line; repeat this process for the other two perpendicular bisectors.



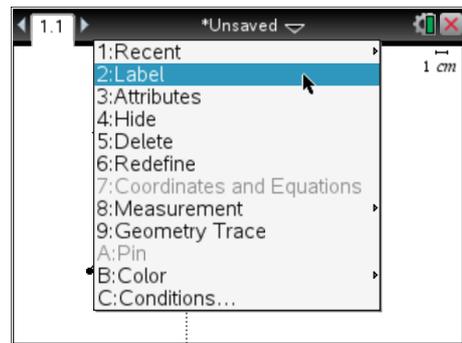
The intersection of the perpendicular bisectors can be marked by a point. From the **points and line** menu select **intersection point(s)**. Select two of the perpendicular bisectors to create the point of intersection.

Press **[esc]** to release the intersection point tool.

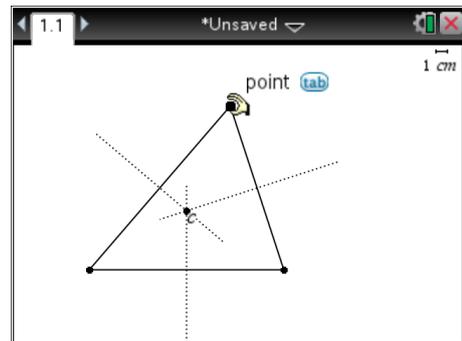


Move the mouse over the top of this new point and press **[Ctrl]** followed by **[menu]** to access the context menu.

Select **label** and mark the point of intersection: C.



Move the mouse over one of the triangle vertices. When it turns into a hand, hold the mouse down for a few moments to grab the point. Move the vertex of the triangle and watch point C as the triangle is changed.



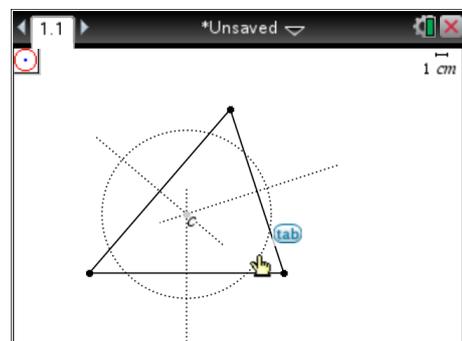
1. Do **all** three perpendicular bisectors meet at a single point?
2. How does the shape of the triangle affect the location of point **C**? (Comment on specific cases)
3. Do you think point **C** represents the centre of the triangle? (Justify your answer)

A circle can be drawn with the centre at point **C** and passing through one of the vertices of the triangle.

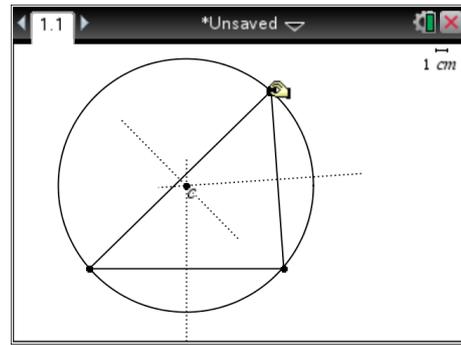
From the **shapes** menu select **circle**.

Click on point **C** followed by one of the triangle's vertices.

After the circle has been drawn, press **[esc]** to release the circle tool.



Drag the vertices of the triangle around the screen and observe how the circle changes.



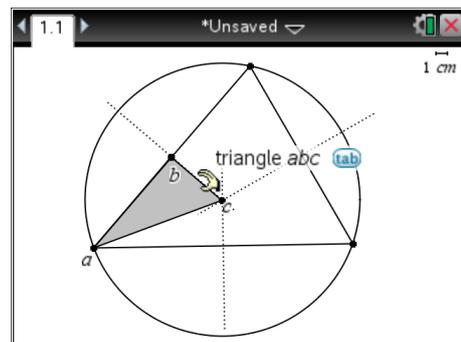
4. What does the circle tell you about the distance from point **C** to the vertices of the triangle?

Observation to Proof

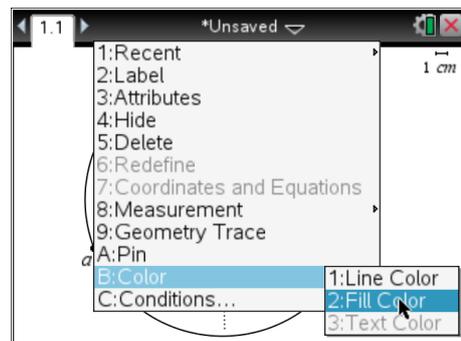
“It is by logic that we prove, but by intuition that we discover” [Henri Poincaré 1908]. Manipulating the triangle and observing the circle provides a degree of intuition about what may be true. Proof provides irrefutable evidence that our observations will always hold true.

Use the **shapes** menu to draw another triangle. Place the vertices according to the image shown opposite:

- (a) Vertex of the triangle
- (b) Intersection of perpendicular bisector and side of triangle
- (c) ‘Centre’ of triangle



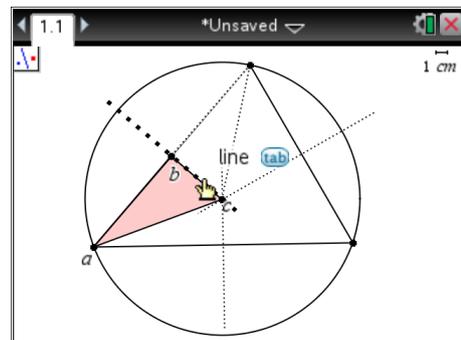
To make the triangle stand out it can be shaded (coloured). Move the mouse over the top of this new triangle and press **[Ctrl]** followed by **[menu]** and select **Colour – Fill Colour**.



This triangle can be reflected. From the **transformation** menu select **reflection**.

Click on the new triangle ($\triangle abc$) followed by the perpendicular bisector that defines one of the sides of the triangle. (Shown opposite)

Colour in the reflected triangle.



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5. Drag the vertices of the original triangle and observe the movement of $\triangle abc$ and its reflection. What do you notice about these two triangles?
 6. Use your knowledge of congruent triangles and reflections to prove that $\triangle abc$ and $\triangle a'b'c'$ (reflection) are congruent.
 7. Use congruent triangles to prove that point C is equidistant from each of the original triangle's vertices.
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