

Factor Game



Student Activity

7 8 9 10 11 12



TI-Nspire



Activity



Student



120 min

Instructions – How to Play

The factor game involves two players. Player one starts by selecting a number between 1 and 49 inclusive. The selected number is added to player one's score, provided the number has at least one *proper factor*¹ remaining on the board. The selected number is immediately removed from the board. Player two automatically scores the sum of all the remaining proper factors, these factors are then removed from the board. It is now player two's turn to select a number.

Player two selects a number from those remaining on the board. Player two scores this number, provided at least one proper factor remains. The selected number is immediately removed from the board. Player one scores the sum of all the remaining proper factors which are then removed from the board.

If any player selects a number that does not have any proper factors remaining on the board, the selection is deemed invalid. When a player selects an invalid number they score zero points for that turn, so too their opponent, however play is then transferred to the opponent.

The game ends when neither player can make a valid selection.

Example:

Player 1: The number 44 is selected. Proper factors of 44 are: {1, 2, 4, 11, 22}. As all of these numbers are currently on the board the selection is valid. The number 44 is immediately removed leaving player two to score: $1 + 2 + 4 + 11 + 22 = 40$ points. The numbers: 1, 2, 4, 11, 22 and 44 are now all removed from play. It is now player two's turn to select a number.

Player 2: The number 33 is selected. Proper factors of 33 are: {1, 3, 11}, however only the number 3 remains on the board. As a proper factor is still in play the number 33 is valid so player two scores 33 points, taking their total to: $40 + 33 = 73$ points. Player one scores the sum of the remaining proper factors: 3 bringing their total to: $44 + 3 = 47$ points. The numbers now removed from the board include: {1, 2, 3, 4, 11, 22, 33, 44}. It is now player one's turn again.

Calculator & Board

Use counters and the board on the next page. A TI-Nspire file is also available called: "Factor Game Two Player". Load this file onto your calculator and run the FactorGame program from the [VAR] list on page 1.2. Even if you are using the calculator, it is worth tracking pieces on the board so that you are more conscious of the factors as they are identified.

¹ Proper Factor Example: Factors of 6: {1, 2, 3, 6}. Proper factors of 6: {1, 2, 3}. The original number is ignored when referring to proper factors. In some cases unity (1) is also ignored. For the purposes of this game, the number 1 is included as a proper factor.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

Question & Discussion Points

Question: 1.

Which number would be better to start with: 46 or 39? Justify your answer and include supporting calculations.

Question: 2.

Which number is the best starting number? Justify your answer and include supporting calculations.

Question: 3.

Which number, if any, would provide a higher score for your opponent if selected first?

Question: 4.

If the number 49 is selected first, what numbers will never be removed from the board?

Question: 5.

Will the combined score of player one and two be the same for every game?

Question: 6.

Write down any strategies you developed whilst playing the game.

Human vs Computer (Calculator)

Open up the TI-Nspire file: Factor Game Single Player. Start the game as per the previous document. For this game 'humans' go first, very polite. The aim of course is to beat the computer. Scores are done automatically.

Question: 7.

Play the factor game 5 times and record the scores for each game. Who won the most, computer or human?

Question: 8.

Thinking just one move in advance; and selecting the best number for each move, what is the ideal sequence of numbers?

Question: 9.

Using an example, show that 'thinking only about one move in advance' is not a sufficient strategy.

Question: 10.

Alex played the computer and won by a total of 43 points. Is it possible to beat the computer by more than 43 points? Record your winning margin and the sequence of selections that lead to this margin. Note that when a game is over it is possible to use the cursor to move through all the selections for the game including progressive scores.