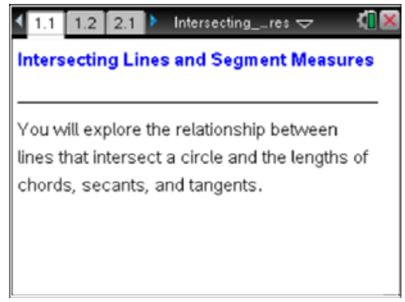


Math Objectives

- Students will investigate that if two chords of a circle intersect, the product of the segment lengths of one chord equals the product of the segment lengths of the other chord.
- Students will investigate that if two secant segments are drawn to a circle from an exterior point, then the product of the lengths of one secant segment and its external secant segment equals the product of the lengths of the other secant segment and its external secant segment.
- Students will investigate that if a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the tangent segment length equals the product of the lengths of the secant segment and its external secant segment.
- Students will investigate that if two segments from the same exterior point are tangent to a circle, then they are congruent.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- chord
- secant
- tangent

About the Lesson

- This lesson involves manipulating 2 intersecting lines that intersect with a circle at 1 or 2 points.
- As a result students will:
  - Infer the relationship between the segment lengths of each chord.
  - Observe the relationship between the products of the segment lengths of each chord.
  - Observe lengths and infer relationships regarding the segment measures of a secant-secant intersection, a secant-tangent intersection, and a tangent-tangent intersection.

TI-Nspire™ Navigator™ System

- Quick Poll
- Screen Capture

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

Intersecting_Lines_and_Segment_Measures_Student.pdf

Intersecting_Lines_and_Segment_Measures_Student.doc

TI-Nspire document

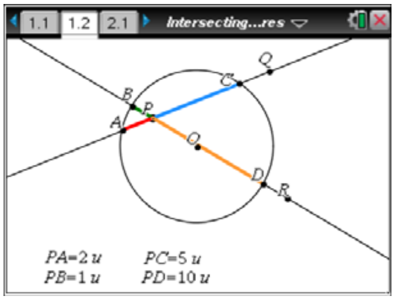
Intersecting_Lines_and_Segment_Measures.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging the point, check to make sure that they have moved the arrow until it becomes a hand (☝). Press ctrl [click] to grab the point and close the hand (✊). When finished moving the point, press esc to release the point.

Move to page 1.2.



1. Point P is the intersection of $\overleftrightarrow{AQ}$ and $\overleftrightarrow{BR}$. Drag point P to various locations inside the circle.
   a. What special type of segment is $\overline{AC}$? What two segments form $\overline{AC}$?

      Answer: $\overline{AC}$ is a chord. It is formed by $\overline{PA}$ and $\overline{PC}$.

   b. What special type of segment is $\overline{BD}$? What two segments form $\overline{BD}$?

      Answer: $\overline{BD}$ is a chord. It is formed by $\overline{PB}$ and $\overline{PD}$.

TI-Nspire Navigator Opportunity: *Screen Capture*
See Note 1 at the end of this lesson.

2. Drag point P to various locations outside the circle. Rotate $\overleftrightarrow{AQ}$ and $\overleftrightarrow{BR}$ by dragging points Q and R so that both lines intersect the circle. What special type of segments are $\overline{PC}$ and $\overline{PD}$?

   Answer: $\overline{PC}$ and $\overline{PD}$ are secant segments.

3. While point P is outside the circle, drag point Q until points A and C are at the same location.
   a. $\overline{PA}$ and $\overline{PC}$ are now the same segment. What special type of segment is this?

      Answer: a tangent segment

Teacher Tip: It takes a little practice to get points A and C to coincide.

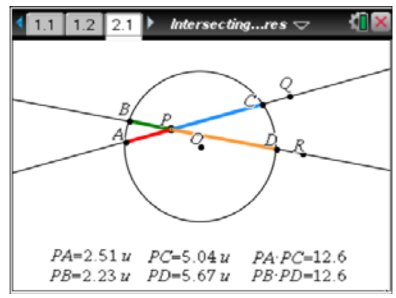
b. What special type of segment is still formed by $\overline{PD}$?

Answer: $\overline{PD}$ is still a secant segment.

4. While point *P* is outside the circle and points *A* and *C* are at the same location, drag point *R* until points *B* and *D* are at the same location. $\overline{PB}$ and $\overline{PD}$ are now the same segment. What special type of segment is this?

Answer: a tangent segment

Move to page 2.1.



$\overleftrightarrow{AQ}$ and $\overleftrightarrow{BR}$ intersect circle *O.* The measures of $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$ are shown at the bottom of the screen. The product of $\overline{PA}$ and $\overline{PC}$, and the product of $\overline{PB}$ and $\overline{PD}$ are also shown.

Rotate $\overleftrightarrow{AQ}$ by dragging point *Q*. Rotate $\overleftrightarrow{BR}$ by dragging point *R*.

5. The product of the segment lengths of each chord is given. What is the relationship between the products of the segment lengths of each chord?

Answer: The product of the segment lengths of one chord equals the product of the segment lengths of the other chord.

Teacher Tip: In some locations due to rounding issues, the products may not be exactly equal. This is a good discussion to have with students. Encourage students to first look at "integer" values.

Teacher Tip: This sometimes is referred to as "short X long" = "short X long" because each chord consists of a "short" segment and a "long" segment. If point *P* coincides with the center of the circle, then this generalization does not apply, because all segments would then be radii of the circle and therefore all segments would be congruent.

Teacher Tip: If students are struggling to see the relationship, have them consider the general patterns: sums (as in angle sums), differences, products, ratios (as in lengths in similar triangles), and so on.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

6. Drag points *P, Q*, and *R* so that segments $\overline{PC}$ and $\overline{PD}$ are secant segments. $\overline{PA}$ and $\overline{PB}$ are called external secant segments because they lie outside the circle. What is the relationship between the products of the lengths of the external secant segments and the secants?

   Answer: The product of 1 secant length and the length of its external segment equals the product of the other secant length and the length of its external segment.

   > **Teacher Tip:** Students sometimes believe that this is the product of the external segment and the "internal" segment (the chord) rather than the whole secant. Be sure to clarify this with students.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

7. Drag point *Q* so $\overline{PC}$ is a tangent segment. What is the relationship between the square of the length of $\overline{PC}$ and the product of the lengths of $\overline{PD}$ and $\overline{PB}$?

   Answer: The square of the length of the tangent segment equals the product of the lengths of the secant segment and its external secant segment.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 4 at the end of this lesson.

8. Drag point *R* so that $\overline{PD}$ is a tangent segment. What is the relationship between the lengths of $\overline{PC}$ and $\overline{PD}$?

   Answer: Two tangent segments intersecting at the same exterior point have the same length. As a result, the two tangent segments are congruent.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 5 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- If two chords of a circle intersect, then the product of the measures of the segments of one chord equals the product of the measures of the segments of the other chord.
- If two secant segments are drawn to a circle from an exterior point, then the product of the measures of one secant segment and its external secant segment equals the product of the measures of the other secant segment and its external secant segment.
- If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent segment equals the product of the measures of the secant segment and its external secant segment.
- If two segments from the same exterior point are tangent to a circle, then they are congruent.

Any of these results can be proven using similar triangles. If time and interest permit, encourage students to draw additional segments and name the similar triangles.

TI-Nspire Navigator

Note 1

Question 1, *Screen Capture:* Once students have moved point *P* to a different location in the circle, take a *Screen Capture* and have students verbalize any observations regarding the measures of the segments. Students will be given the product of the segments on page 2.1, but you might have some students who recognize that relationship early on in the lesson if given the opportunity to see the Screen Captures.

Note 2

Question 5, *Quick Poll:* Send students the following *Open Response Quick Poll* using the circle on page 2.1 in position for problem 5. If $PA = 12$, $PC = 3$, and $PB = 4$, then what is the measure of PD?

Answer: $PD = 9$

Note 3

Question 6, *Quick Poll:* Send students the following *Open Response Quick Poll* using the circle on page 2.1 in position for problem 6. If $PA = 3$, $PB = 2$, and $BD = 7$, then what is the measure of AC?

Answer: $AC = 3$

Note 4

Question 7, *Quick Poll:* Send students the following *Open Response Quick Poll* using the circle on page 2.1 in position for problem 7. If $PA = 4$ and $PB = 2$, then what is the measure of BD?

Answer: $BD = 6$

Note 5

Question 8, *Quick Poll:* Send students the following *Open Response Quick Poll* using the circle on page 2.1 in position for problem 8. If $PA = 2x + 5$ and $PB = 3x – 4$, then what is the measure of PA?

Answer: $PA = 23$