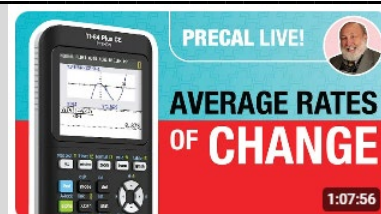




**Thursday Night Precalculus  
November 30, 2023**

In this AP Precalculus Live session, we will explore average rates of change.



 Difference Quotients and Average Rates of Change

**About the Lesson**

- This Teacher Notes guide is designed to be used in conjunction with the AP Precalculus Live session and Student Problems document that can be found on-demand:
  - <https://www.youtube.com/live/hwPPPgbyrms?si=8kk18sB-jH53hWLQ>
  - Please note that not all problems/content from the Student Problem Sheet is covered in the video component. Student/Teacher Notes are also useful without students viewing the “Live Session” but can be enriched by that resource.*
- This session involves difference quotients and average rates of change.
- Students should be able to use the TI-84 to visualize secant lines and to calculate the slopes of secant lines.
-  **Class Discussion:** Use these questions to help students communicate their understanding of the problem. These questions are presented in the *Live* video as well.

**Materials:**
*Student document*

- problems\_11\_30\_23

*Solutions*

- problems\_solutions\_11\_30\_23

*YouTube*

- <https://www.youtube.com/live/hwPPPgbyrms?si=8kk18sB-jH53hWLQ>

**AP Precalculus Learning Objectives**

- 1.2.A: Compare the rates of change at two points using average rates of change near the points.
- 1.3.A: Determine the average rates of change for sequences and functions, including linear, quadratic, and other function types.
- 1.3.B: Determine the change in the average rates of change for linear, quadratic, and other function types.
- 1.4.A: Identify key characteristics of polynomial functions related to rates of change.

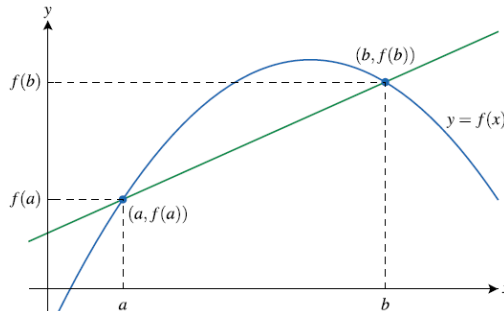
Source: AP Precalculus Course and Exam Description, The College Board

**Introduction – A Geometric Interpretation of Average Rate of Change**

Average Rate of Change is the change in  $y$  divided by the change in  $x$ .

$$\text{Average Rate of Change} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$$

The average rate of change finds the slope of a secant line from  $(a, f(a))$  to  $(b, f(b))$ .

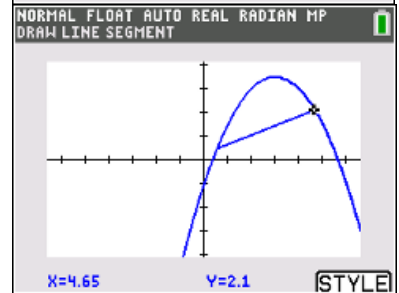
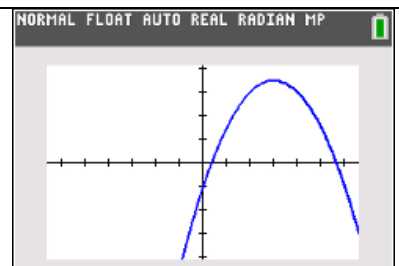


Graph  $y_1 = 3x - 1 - \frac{x^2}{2}$  using Zoom Decimal.

To graph a secant line, use  $\boxed{2\text{nd}}\boxed{\text{PRGM}}$  and select 2: Line(.

This method gives us a visual but the points may not be on the graph of the function.

**Technology Tip:** Zoom Decimal is preferred because the screen of the calculator is not square. The Trace values are nice.



Suppose we want to calculate the slope of the secant line from  $x = 1$  to  $x = 5$ .

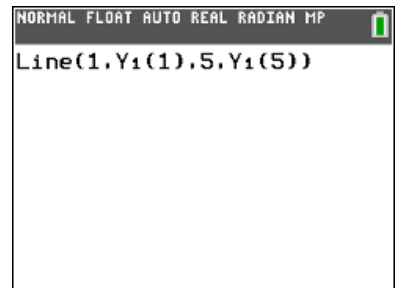
Zoom Decimal will erase the line drawn.

Go to a Calculator screen.

Store 1 to A and 5 to B.

Access  $\boxed{2\text{nd}}\boxed{\text{PRGM}}$  and select 2: Line(

Type Line(1,Y1(1),5,Y1(5)).

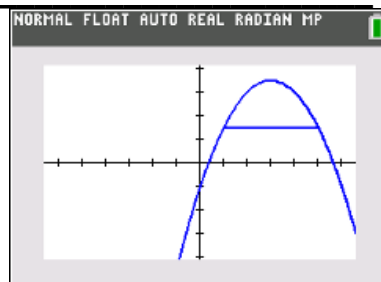


**Technology Tip:** In the Draw menu, ClrDraw will clear the drawn lines.

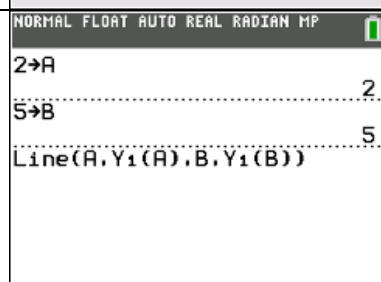
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The average rate of change is 0. It is the slope of the line.  
Use Trace to confirm that the  $y$ -values are the same for  $x = 1$   
and  $x = 5$ .

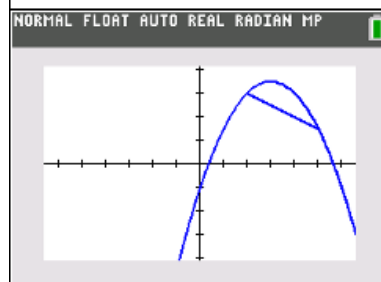


Go to a Calculator screen.  
Store 2 to A and 5 to B.  
Access  $\boxed{2\text{nd}}\boxed{\text{PRGM}}$  and select 2: Line(  
Type Line(A,Y1(A),B,Y1(B)).

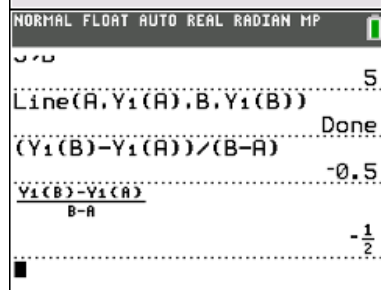


**Technology Tip:** Storing different values for A and B gives a  
quick way to visualize the secant line.

Type  $(Y1(B) - Y1(A))/(B - A)$  or use a short cut to obtain a  
fraction in MathPrint. Select  $\boxed{\alpha}\boxed{X,T,\theta,n}$  and use the fraction  
template.



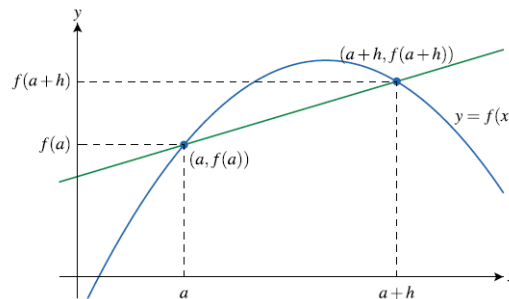
**Technology Tip:** Using the fraction template in MathPrint  
produces a slope of the secant line in fraction form.



**Introduction – A Geometric Interpretation of a Difference Quotient**

Difference Quotient is the measure of the average rate of change of a function over an interval  $h$ .

$$\text{Difference Quotient} = \frac{f(x+h) - f(x)}{h}$$



**Teacher Note:** The form of the difference quotient used most often in AP Precalculus is Average Rate of Change  $= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(b) - f(a)}{b - a}$ . The form used in Problem 1 is a preview of the development of a derivative in AP Calculus.

**Problem 1. (a) – (d)**

For each of the following functions, simplify the expression  $\frac{f(x+h) - f(x)}{h}$ ,  $h \neq 0$  as far as possible.

(a)  $f(x) = 3x^2 - 5x$

**Sample Solution:**

Refer to the Teacher Solutions Document for the full solution to this problem.



**Class Discussion:**

What is the average rate of change for consecutive equal-length intervals? Do you see a pattern in the given table?

Interval	Difference Quotient
$[a, a+h]$	$6a + 3h - 5$
$[a+h, a+2h]$	$6a + 9h - 5$
$[a+2h, a+3h]$	$6a + 15h - 5$
$[a+3h, a+4h]$	

**Possible Answers:** The difference quotient for the interval  $[a+3h, a+4h]$  is  $6a + 21h - 5$ . The pattern is to add  $6h$  for each consecutive equal-length interval.

(b)  $f(x) = \sqrt{x^2 - 1}$

**Sample Solution:**

Refer to the Teacher Solutions Document for the full solution to this problem.

$$(c) f(x) = \frac{1}{x^2}$$

**Sample Solution:**

Refer to the Teacher Solutions Document for the full solution to this problem.

$$(d) f(x) = \frac{x}{1+x^2}$$

**Sample Solution:**

Refer to the Teacher Solutions Document for the full solution to this problem.

**Problem 2**

The number of pounds (in millions) of lobster caught by Maine commercial fisherman is given by  $P(t)$ , where  $t$  is measured in years. Selected values for  $P(t)$  are given in the table.

$t$	2010	2012	2014	2016	2018	2020	2022
$P(t)$	255.8	318.3	306.9	302.9	285.8	205.1	107.1

Find the average rate of change in pounds of lobsters caught (i) from 2016 to 2018; (ii) from 2018 to 2020.

Indicate units of measure. What do your answers suggest about the change in the number of pounds of lobster caught in recent years?

**Sample Solution:**

Refer to the Teacher Solutions Document for the full solution to this problem.

**Teacher Note:** We encourage students and teachers to avoid abbreviations, including AROC for average rate of change. Students should present both a difference and a quotient when finding an average rate of change. Either of these expressions would be acceptable for

demonstrating this:  $\frac{P(2018) - P(2016)}{2018 - 2016} = \frac{285.8 - 302.9}{2018 - 2016}$

**Teacher Note:** The interpretation of the average rate of change is important in both AP Precalculus and AP Calculus. Including units of measure whenever working with average rates of change in a contextual problem is encouraged.

Use the Stats editor to enter the table given in the problem.

We'll use [L3] to find the successive differences in

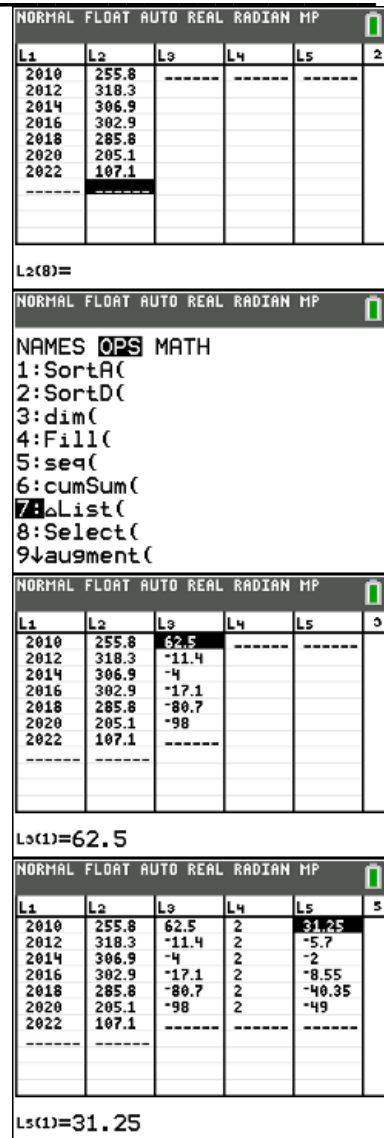
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TEACHER NOTES

Highlight [L3] and go to list, then go over to OPS. Select 7:  
 $\Delta$ List( and enter [L2].

In [L4] we will do the same thing using  $\Delta$ List([L1]).

To find every successive average rate of change, highlight [L5]  
and enter [L3]/[L4].



The screenshots show the following steps on a TI-84 Plus CE calculator:

- Screen 1:** Shows list L1 with values: 2010, 2012, 2014, 2016, 2018, 2020, 2022. List L2 has values: 255.8, 318.3, 306.9, 302.9, 285.8, 205.1, 107.1. The cursor is on L2(8).
- Screen 2:** Shows the OPS menu with  $\Delta$ List( highlighted.
- Screen 3:** Shows list L1 with values: 2010, 2012, 2014, 2016, 2018, 2020, 2022. List L2 has values: 255.8, 318.3, 306.9, 302.9, 285.8, 205.1, 107.1. List L3 has values: 62.5, -11.4, -4, -17.1, -80.7, -98. The cursor is on L3(1).
- Screen 4:** Shows list L1 with values: 2010, 2012, 2014, 2016, 2018, 2020, 2022. List L2 has values: 255.8, 318.3, 306.9, 302.9, 285.8, 205.1, 107.1. List L3 has values: 62.5, -11.4, -4, -17.1, -80.7, -98. List L4 has values: 2, 2, 2, 2. List L5 has values: 31.25, -5.7, -2, -8.55, -40.35, -49. The cursor is on L5(1).

**Problem 3**

A particle moves along the horizontal number line. Its position at time  $t \geq 0$  is given by  $s(t) = t^2 - 7t + 2$  where  $t$  is measured in seconds and  $s$  is measured in meters.

- (a) Find the average rate of change in the particle's position from  $t = 0$  to  $t = 8$  seconds.
- (b) Use your answer in part (a) to determine if the particle is to the left or the right of its initial position at time  $t = 8$ .

**Sample Solution:**

The average rate of change is  $\frac{s(8) - s(0)}{8 - 0} = 1$  meters per second.



**Class Discussion:**

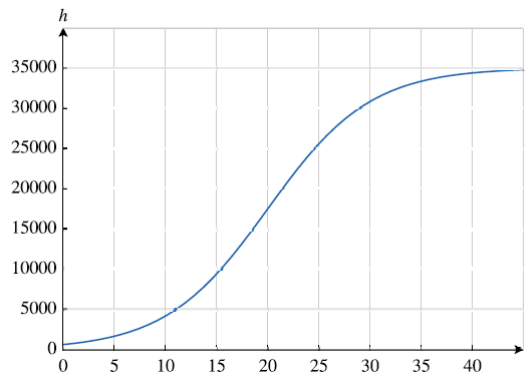
*How can we use the average rate of change in part (a) to answer the question in part (b)?*

**Possible Answers:** Since the average rate of change over the given interval is positive, the particle is to the right of its initial position. The average rate of change means that on average the particle moved 1 meter to the right each second.

Refer to the Teacher Solutions Document for the full solution to this problem.

**Problem 4**

The figure shows the graph of the altitude of a plane ( $h$ , in feet) from takeoff,  $t = 0$  minutes, to  $t = 45$  minute.



Use the graph to determine on which five-minute interval, 0-5, 5-10, 10-15, etc., the average rate of change in height is greatest.



**Class Discussion:**

*How can we use the graph to answer this question? If we had a function, we could evaluate the functions values to find the average rates of change.*

**Possible Answers:** Use a ruler and draw in the secant lines. Choose either the 15-20 or 20-25 five-minute interval.

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TEACHER NOTES

Enter  $Y1(x) = \frac{35000}{1 + e^{-0.2(x-20)}}$  and  $Y2(x) = \frac{Y1(x+5) - Y1(x)}{5}$

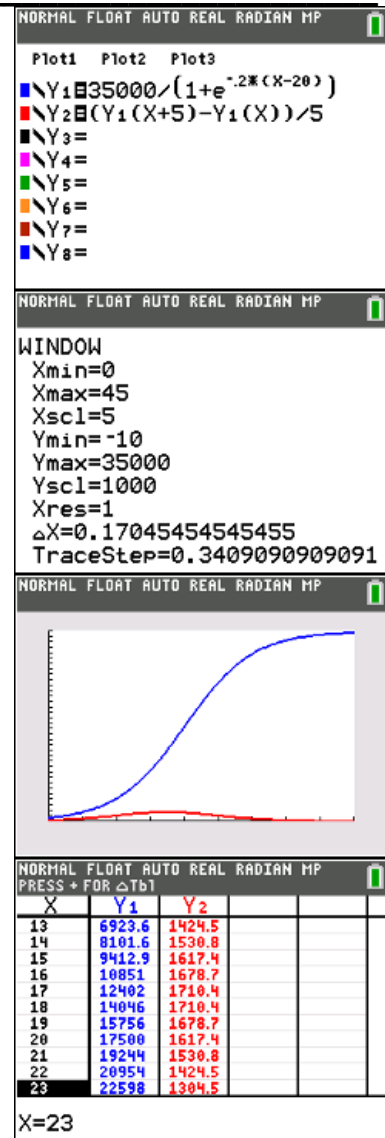
Notice that  $Y2(x)$  is the average rate of change of the given function.

To display a similar graph, use the window in the screen shot to the right.

The red graph represents the average rates of change. With the Xscl = 5, it appears that the maximum is in the 5-minute interval 15-20.

Trace along the Y2 graph to see the values of the average rates of change.

The table would also confirm that the greatest rate of change is in the 5-minute interval 15-20.

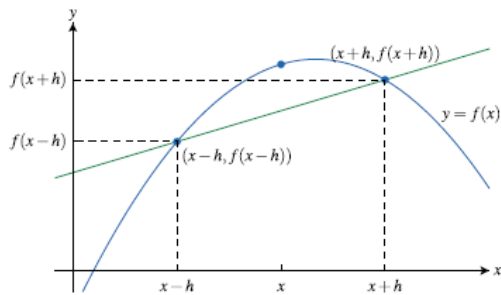


**Introduction – A Geometric Interpretation of a Symmetric Difference Quotient**

Symmetric Difference Quotient =  $\frac{f(x+h) - f(x-h)}{2h}$

**Teacher Note:** The symmetric difference quotient may appear in AP Calculus in the development of a derivative. It is not part of the AP Precalculus curriculum.





**Note:** The solutions of the following problems in 5 are not discussed in the video.

**Problem 5 (a) – (b)**

For each of the following functions, simplify the expression  $\frac{f(x+h) - f(x-h)}{2h}$

(a)  $f(x) = 2x + 8$

(b)  $f(x) = x^2 + 3x + 4$

**Sample Solution:**

Refer to the Teacher Solutions Document for the full solution to these problems.

**Supporting Activities**

Precalculus: Coming Soon!

**Wrap Up**

Upon completion of the discussion, the teacher should ensure that students understand:

- The

For more videos from the AP Precalculus Live series, visit our playlist

[https://www.youtube.com/playlist?list=PLQa\\_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI](https://www.youtube.com/playlist?list=PLQa_6aWmaC6B-5h5n2Cr5h3G2ZPfJ0HGI)

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