First Derivative Test

Math Objectives

- Students will relate the first derivative of a function to its critical points and identify which of these critical points are local extrema.
- Students will visualize why the first derivative test works and how it is used to determine local minima and maxima.
- Students will look for and make use of structure. (CCSS Mathematical Practice)
- Students will construct viable arguments and critique the reasoning of others. (CCSS Mathematical Practice)

Vocabulary

- first derivative
- critical point
- local maximum, local minimum, extrema

About the Lesson

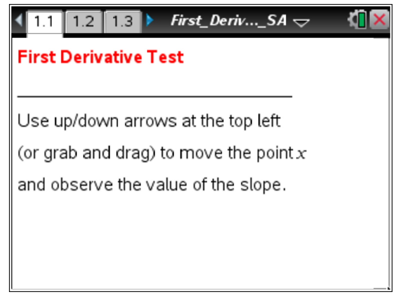
- This lesson involves visualizing the connections between the first derivative of a function, critical points, and local extrema.
- As a result, students will:
- Develop an understanding of the first derivative test.
- Explore a sequence of functions, some by moving a tangent line along the function graph and noting changes in the first derivative of a function near its critical points.
- Build on their familiarity with the concept of the derivative at a point as the local slope of the function graph at that point.

TI-Nspire™ Navigator™ System

- Send out the *First_Derivative_Test.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Materials:

Student Activity

- First_Derivative_Test_Student.pdf
- First_Derivative_Test_Student.doc

TI-Nspire document

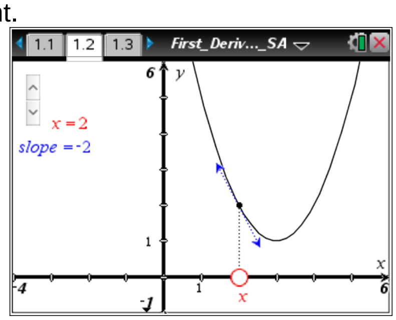
- First_Derivative_Test.tns

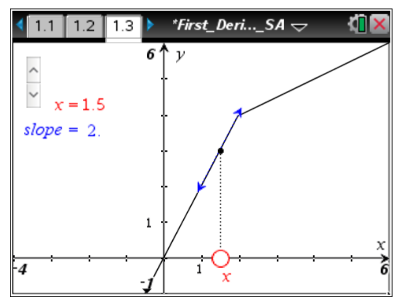
Discussion Points and Possible Answers

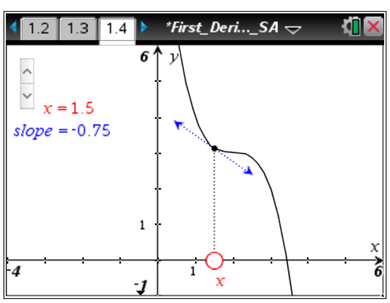
Move to page 1.2.

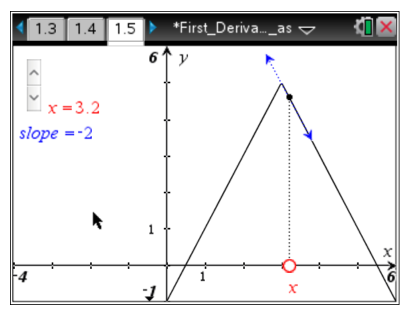
> **Tech Tip:** If students experience difficulty dragging a point, make sure they have not selected more than one point. Press esc to release points. Check to make sure that they have moved the cursor (arrow) until it becomes a hand (👆) getting ready to grab the point. Also, be sure that the word point appears. Then select ctrl to grab the point and close the hand (✊). When finished moving the point, select esc to release the point
>
> **Tech Tip:** To change the value of *x*, tap on the point to highlight it. Then, begin sliding it.

1. Each page of problem 1 in the document has a graph of a function with a single critical point.









a. Grab the white point on the *x*-axis and move it to see the slope of the tangent line change as you move along each graph. Complete the table below.

<u>**Answer:**</u> Completed table is below.

Function graph on page	Critical point	Local max, local min, or neither	Intervals where slope of tangent line is positive	Intervals where slope of tangent line is negative
1.2	$x = 3$	local min	$x > 3$	$x < 3$
1.3	$x = 2$	neither	$x < 2, x > 2$	none

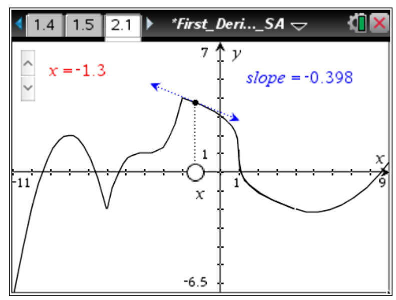
1.4	***x* = 2**	**neither**	**none**	***x* < 2, *x* > 2**
1.5	***x* = 3**	**local max**	***x* < 3**	***x* > 3**

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

b. Summarize your findings for the four graphs.

Answer: Student responses should relate the first derivative to slopes of the tangent line as an indication of where a function is increasing or decreasing. Students should note that a change in sign from positive to negative (as in 1.4) indicates that the function values are increasing until reaching a relative maximum at the critical point and then are decreasing. Similarly, a sign change from negative to positive indicates that the function values are decreasing, reach a relative minimum, and then are increasing. No sign change indicates a function is continually increasing (as in 1.5) or decreasing (1.4) in the local neighborhood of the critical point, and therefore there is no local maximum or minimum.

Move to page 2.1.



2. The graph shown here is that of a function with six different critical points indicated on the *x*-axis (by black dots). Use the up/down arrows at the upper left (or grab and drag) to move the white point along the *x*-axis and see the slope of the tangent line (when it exists).

Teacher Tip: The function graph shown on page 2.1 of the TI-Nspire document is crafted piecewise out of several functions. Because of this complexity, it takes a few moments for the graph to appear on the handheld. Once the graph appears, the up/down arrows in the upper left probably provide the easiest means for moving the point *x*. The tangent line will disappear entirely at the cusp ($x = -6$) and the corner ($x = -2$). The tangent line will appear vertical at the critical point $x = 1$, but the slope will be undefined there.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 2 at the end of this lesson.

a. Explain why each of these marked points is a critical point, and classify each as the location of a local minimum, local maximum, or neither.

Answer: Completed table is below.

Critical point	Reason why it is a critical point	Location of local max, local min, or neither	Describe any change of sign of f' at $x = a$
$x = -8$	$f'(-8) = 0$	local max	Change from positive to negative
$x = -6$	$f'(-6)$ does not exist	local min	Change from negative to positive
$x = -4$	$f'(-4) = 0$	neither	No change in sign
$x = -2$	$f'(-2)$ does not exist	local max	Change from positive to negative
$x = 1$	$f'(1)$ does not exist	neither	No change in sign
$x = 5$	$f'(5) = 0$	ocal min	Change from negative to positive

b. Use what you have learned to complete the definition of the first derivative test below:

Suppose **f** is continuous at the critical point *a*:

- If the first derivative **f′** changes sign from positive to negative at *a*, then **f**(*a*) is a local maximum.
- If the first derivative **f′** changes sign from negative to positive at *a*, then **f**(*a*) is a local minimum.
- If the first derivative **f′** does not change sign at *a,* then **f** has neither a local maximum nor a local minimum at *a*.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- How to identify the critical points of a function given its graph.
- Why the local minima or maxima of a function occur at its critical points.
- Why not every critical point is a local minima or maxima.
- How the first derivative test can be used to determine if a critical point is a local minimum, local maximum, or neither.

Note: The assumption of continuity is another idea that should be addressed in the lesson wrap up. All of the functions presented in this lesson are continuous, a condition for the first derivative test. Although most functions students will encounter will be continuous, this is an important condition to note. Students could be challenged to consider what might happen if this condition were relaxed. Is it then possible for $\mathbf{f}$ to have a critical point a such that $\mathbf{f}(a)$ is a local minimum, but the derivative $\mathbf{f}'(x) > 0$ for $x < a$ and $\mathbf{f}'(x) < 0$ for $x > a$? If so, what might it look like?

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Note 1

Question 1a, *Quick Poll*: You may want to use a Quick Poll at this point to assess student understanding of critical points and local maxima and minima with quick questions based on the completed table.

Note 2

Question 2, *Class Capture*: As students are exploring the graph, use Class Capture to ensure students are able to move the point. Scroll through the screens for students to encourage discussion of the different critical points.

Note 3

Whole Document, *Quick Poll*: At the conclusion of the activity, you can assess student understanding of the lesson and the First Derivative Test using a Quick Poll with leading questions from the activity.