Math Objectives

- Students will change the parameters of the exponential function $y = a \cdot \text{base}^{b(x-h)} + k$ to discover that:
  - Changes in k result in vertical translations.
  - Changes in h result in horizontal translations.
  - Changes in a result in vertical stretches and compressions (dilations), and reflections across the x-axis.
  - Changes in b result in horizontal stretches and compressions (dilations), and reflections across the y-axis.
- Students will look for and make use of structure (CCSS Mathematical Practice).

Vocabulary

- translation
- dilation
- compression
- stretch
- reflection
- transformation

About the Lesson

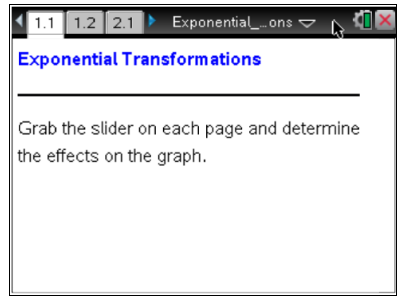
- This lesson involves graphing exponential functions of the form $y = a \cdot \text{base}^{b(x-h)} + k$.
- As a result, students will:
  - Manipulate given parameters and make conjectures about the relationships between the parameters' values and their effects on the resulting exponential function's graph.
  - Test their newly-learned knowledge and determine the exponential function for a given graph.

TI-Nspire™ Navigator™

- Use Class Capture to examine patterns that emerge.
- Use Teacher Software to review student documents.
- Use Quick Poll to share student responses.
- Use Live Presenter for student demonstrations.

Activity Materials

Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Exponential_Transformations_Student.pdf
- Exponential_Transformations_Student.doc

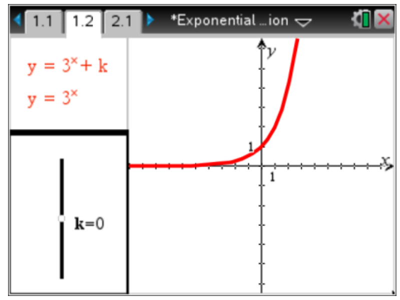
TI-Nspire document

- Exponential_Transformations.tns

Discussion Points and Possible Answers

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 1 at the end of this lesson.

Move to page 1.2.



For this activity, the function used is $y = a \cdot 3^{b(x-h)} + k$. This activity's investigations also work for any base *b* such that *b* > 0 and *b* ≠ 1.

1. a. What effect does dragging the *k*-value have on the parent function $y = 3^x$? What happens algebraically to the point (0, 1) in terms of *k* as the graph is translated up or down?

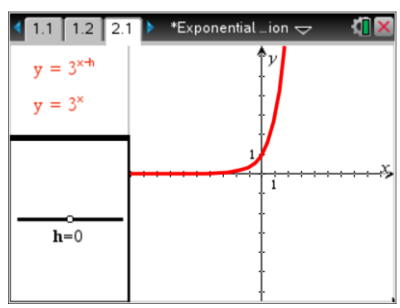
   Sample answer: Increasing *k*-values result in vertical translations up and decreasing *k*-values result in vertical translations down. Answers will vary. For example, if *k* = 4, when *x* = 0, $3^0 + 4 = 5$, the translated point goes from (0, 1) to (0, 5).

   b. Name the transformation, including its distance and direction, when the function $y = 3^x$ changes to $y = 3^x + 2$. How does the point (0, 1) change?

   Answer: A vertical translation up of 2 units. This is because 2 is added to every *y*-coordinate from the original function, so (0, 1) becomes (0, 3).

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)* and *Class Capture*
See Note 2 at the end of this lesson.

Move to page 2.1.



2. Change the *h*-value by grabbing and dragging the slider.

   a. What happens to the equation and graph when *h* > 0?

   Answer: If you drag the slider to the right, the equation changes to an exponent of *x* – *h*. Increasing *h*-values result in horizontal translations right and decreasing *h*-values result in horizontal translations left.

b. Christina says that the point (0, 1) on the parent function translates to (–2, 1) when she drags the *h*-value to –2 because the *y*-value is being multiplied by –2. Is her explanation mathematically correct? Explain. Change the *h*-value and confirm your explanation by grabbing and dragging the slider.

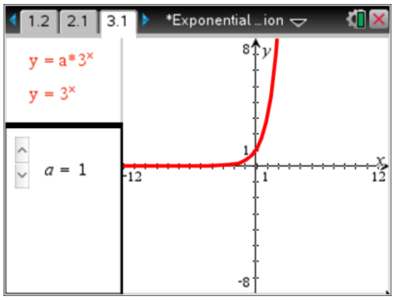
Answer: No, her explanation is not mathematically correct. When the *h*-value is moved to –2, the new function becomes $y = 3^{x+2}$. An *x*-value of –2 has a corresponding *y*-value of 1, so the point (0, 1) has been translated to the left to (–2, 1). It has nothing to do with multiplication.

c. Name the transformation, including its distance and direction, when the function $y = 3^x$ changes to $y = 3^{x-2}$.

Answer: A horizontal translation right of 2 units.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)* and *Class Capture*
See Note 3 at the end of this lesson.

Move to page 3.1.



3. Change the *a*-value by clicking on the arrows.
   a. When the *a*-value is 0.5, explain why the point (1, 3) becomes the transformed point (1, 1.5).

   Answer: Values of *a* greater than 1 result in vertical stretches, while values between –1 and 1 result in vertical compressions, and negative values result in reflections over the *x*-axis. When *a* = 0.5, $0.5 \cdot 3^1 = 1.5$, so the point goes from (1, 3) to (1, 1.5).

   b. What happens to the point (1, 3) when the function changes from $y = 3^x$ to $y = 2 \cdot 3^x$? What transformation occurred?

   Answer: The graph becomes vertically stretched by a factor of 2. The point (1, 3) becomes (1, 6).

Teacher Tip: Although *a* = 0 is skipped in the slider, it might be a good discussion to have with students. The function is now *y* = 0, which lies on top of the *x*-axis.

Teacher Tip: Ask students to compare the following two functions: $y = 3^{x+1}$ and $y = 3^1 \cdot 3^x$. While both of these functions are different transformations (a dilation and a horizontal translation), they are equivalent functions. Use rules of exponents to show that $3^1 \cdot 3^x = 3^{x+1}$.

Tech Tip: Use the Scratch Pad to graph $3^1 \cdot 3^x$ and 3^{x+1} to demonstrate their equivalence.

Teacher Tip: A more general word to use for stretching and compressing is *dilation.* A dilation of scale factor $\frac{1}{4}$ is a compression, while a dilation of scale factor 4 is a stretch.

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)* and *Class Capture*
See Note 4 at the end of this lesson.

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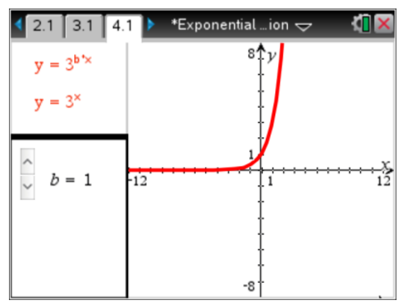

4. Change the *b*-value by clicking on the arrows.
   a. When $b < 0$, what happens to the point (0, 1)? If $b < 0$, what happens to the graph?

      <u>Answer:</u> If you drag the slider until *b* is negative, the negative *b*-values result in reflections over the *y*-axis.

   b. What other effects does the *b*-value have on the graph?

      <u>Answer:</u> Values greater than 1 result in horizontal compressions, and values between 1 and –1 result in horizontal expansions and negative values result in reflections over the *y*-axis.

   c. Suppose the function changes from $y = 3^x$ to $y = 3^{2x}$. Describe the transformation that occurs.

      <u>Answer:</u> The graph becomes horizontally compressed relative to the *y*-axis by a factor of 2.

Teacher Tip: The slider does not go to $b = 0$, but this might be a good discussion to have with students. When happens if $b = 0$?

TI-Nspire Navigator Opportunity: *Quick Poll (Open Response)* and *Class Capture*
See Note 5 at the end of this lesson.

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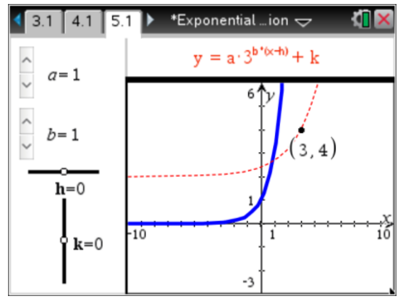

5. Apply what you have learned and change the values of *h* and *k* (by dragging their sliders) and of *a* and *b* (by clicking their arrows) so that in the displayed domain, the solid graph is transformed to the dashed graph. It will say *Correct!* when you have done it correctly.

   Write the function you arrived at here. Describe your thought process of getting to the answer.

   Answer: $y = 2 \cdot 3^{0.5(x-3)} + 2$. Student thought processes will vary.

   Teacher Tip: Encourage students to discuss what transformations need to be done and if the order of the transformations is important. Different transformations might produce the same graphs. If this happens, this would provide a great opportunity to discuss properties of exponents.

6. David says that positive *a*-values greater than 1 cause the function to stretch vertically. Is he correct? Explain.

   Answer: David is not correct. He will be correct for some values of *a* (like if *a* changes from 3 to 5), but what if *a* changes from 5 to 3? Those are still positive values, but the graph will actually be compressed. Instead, David should say that increasing values of *a* cause the function to stretch.

7. Leon says that changing $y = 3^x$ to $y = 3^{x+4}$ results in its graph having a horizontal translation to the right of 4 units. Is Leon correct? Why or why not?

   Answer: Leon is not correct. The correct translation is 4 units to the left. This is a common misconception. To help Leon with his thinking, it is better to decide what *x*-value makes the exponential expression *x* + 4 equal to zero. This *x*-value will be the translation.

8. a. Write the function that transforms $y = \sqrt{x}$ horizontally to the left 5 units and has a vertical dilation factor of 3.

   Answer: $y = 3\sqrt{x+5}$

b. Write the function that transforms $y = |x|$ with a vertical translation up 3 units.

Answer: $y = |x| + 3$

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand that given the exponential function $y = a \cdot \text{base}^{b(x-h)} + k$:

- Changes in *k* result in vertical translations.
- Changes in *h* result in horizontal translations.
- Changes in *a* result in vertical stretches and compressions, and reflections across the *x*-axis.
- Changes in *b* result in horizontal stretches and compressions (dilations), and reflections across the *y*-axis.

Assessment

1. Describe mathematically the effect that changing the function $y = 2^x + 5$ to $y = 2^x - 3$ has on the resulting graph.

   Answer: A vertical translation down of 8 units.

2. Describe mathematically the effect that changing the function $y = 2^{x-3}$ to $y = 2^{x+7}$ has on the resulting graph.

   Answer: A horizontal translation left of 10 units.

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Note 1

Question 1a, *Live Presenter*: Consider using *Live Presenter* to demonstrate how to drag the slider.

Note 2

Question 1b, *Quick Poll* (*Open Response*) and *Class Capture*: Send an Open Response Quick Poll asking students to submit their answers to question 1b.

If students had difficulty, take a Class Capture of page 1.2. As a class, discuss the relationship between various *k*-values and the resulting graphs.

Note 3

Question 2c, *Quick Poll* (*Open Response*) and *Class Capture*: Send an Open Response Quick Poll, asking students to submit their answers to question 2c.
If students had difficulty, take a Class Capture of page 2.1. As a class, discuss the relationship between the various *h*-values and the resulting graphs.

Note 4

Question 3b, *Quick Polls* (*Open Response*) and *Class Capture*: Send an Open Response Quick Poll, asking students to submit their answers to question 3b.
If students had difficulty, take a Class Capture of page 3.1. As a class, discuss the relationship between the various *a*-values and the resulting graphs.

Note 5

Question 4c, *Quick Polls* (*Open Response*) and *Class Capture*: Send an Open Response Quick Poll, asking students to submit their answers to question 4c. If students had difficulty, take a Class Capture of page 4.1. As a class, discuss the relationship between the various *b*-values and the resulting graphs.