

Dominance Matrices



Answers

7 8 9 10 11 12



TI-Nspire



Investigation



Student



30 min

Are all wins created equal?

Most team sports involve some sort of round-robin type tournament arrangement. As more teams join a competition, the season either needs to get longer or some teams will not get to play each other with the same frequency; in these situations some teams may face a tougher draw than others. The Australian Football League (AFL™) involves 18 teams that compete in the home and away season over 22 rounds. Each team plays every other team in the competition, however this leaves 5 more games, so some teams must play each other twice. At the end of the season the total number of wins and losses is recorded for each team, but are all these wins created equal?

In this investigation you will compare the results of the top four teams based on their form as they played one another throughout the season. The official AFL ladder is available from the AFL website:

<http://www.afl.com.au/ladder>

The top four positions at the end of the 2016 home and away season were as follows:

1. Sydney
2. Geelong Cats
3. Hawthorn
4. GWS Giants

The first¹ encounters for each team have been recorded in the table below:

Team: Sydney	Round	Result
Sydney vs GWS Giants	3	Sydney
Hawthorn vs Sydney	9	Sydney
Sydney vs Geelong Cats	16	Sydney

Team: Geelong Cats	Round	Result
Hawthorn vs Geelong Cats	1	Geelong
GWS Giants vs Geelong Cats	2	GWS
Sydney vs Geelong Cats	16	Sydney

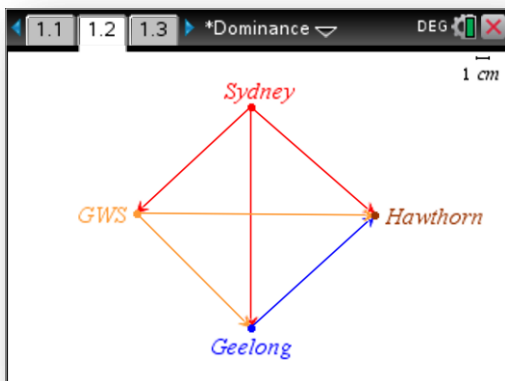
Team: Hawthorn	Round	Result
Hawthorn vs Geelong Cats	1	Geelong
GWS Giants vs Hawthorn	6	GWS
Hawthorn vs Sydney	9	Sydney

Team: GWS Giants	Round	Result
GWS Giants vs Geelong Cats	2	GWS
Sydney vs GWS Giants	3	Sydney
GWS Giants vs Hawthorn	6	GWS

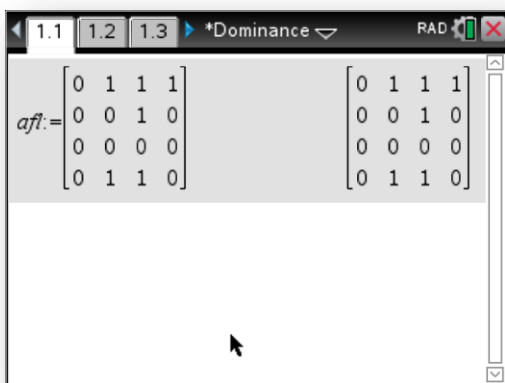
¹ **First Encounter:** Some teams played the top teams more than once, only the first encounter has been recorded, regardless of when it occurred.

Questions

1. Draw a directed network to represent the outcomes of these games.

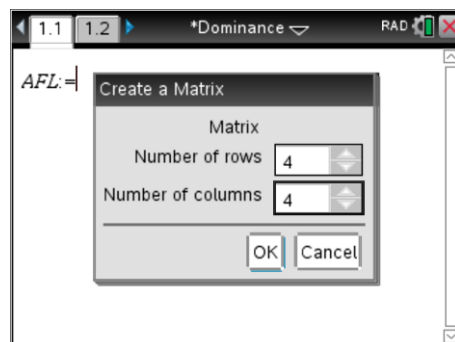


2. Construct a matrix to represent the network with the first row representing the team that finished on top of the ladder, the second row representing the team that placed second on the ladder and so forth.



Open the TI-nspire file “Dominance”.

Use the matrix template to define the matrix from Question 2 (above). Store the matrix in a variable called: **AFL**

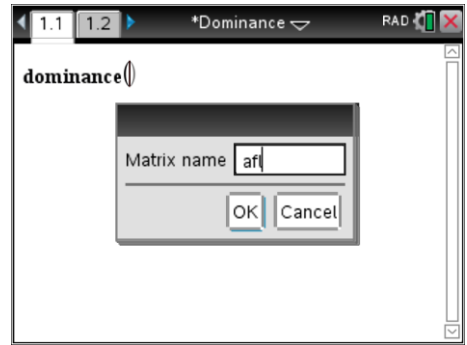


Once the matrix has been entered the dominance program can be used to create the dominance vector.

Press the [VAR] key, select the dominance program then press [ENTER] to run it.

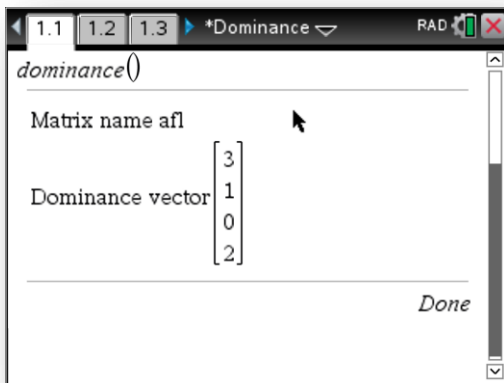
A prompt will appear, enter the matrix name: AFL

Press [ENTER] to see the dominance vector.



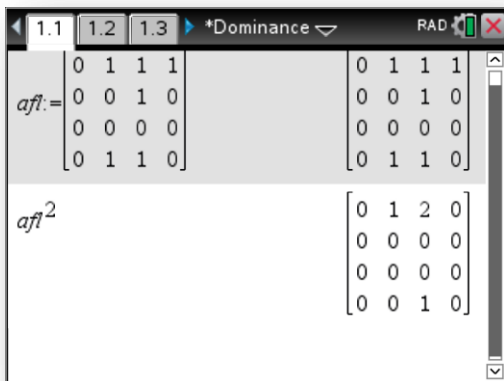
Questions

3. Explain how the matrix produced from the dominance program relates to the match results.



Each row represents the number of games won by Sydney, Geelong, Hawthorn and GWS respectively.

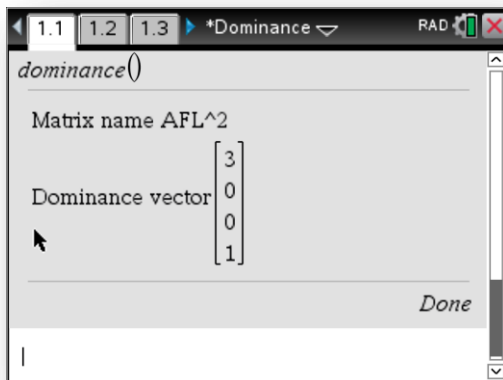
4. Write down the matrix result for AFL^2 .



5. In the first encounters of the 2016 season, Sydney defeated GWS Giants who in turn defeated Hawthorn. Sydney also defeated Geelong Cats who in turn defeated Hawthorn. How is this level of dominance displayed in the second order dominance matrix: AFL²?

$AFL^2 [1\ 3] = 2$ In the second order dominance matrix this corresponds to Sydney beating GWS Giants who have beaten **two** other teams.

6. Run the dominance program again, this time enter AFL² when prompted for the matrix name. Order the teams according to this result and interpret the outcome.



Sydney (row 1) is on 3 'points' as they have beaten one team that has beaten **two** other teams (GWS) and another team (Geelong) that has beaten **one** other team. Their win against Hawthorn does not factor into their 3 points since Hawthorn did not beat any of the other teams in this group.

Geelong (row 2) is on 0 'points', even though they won one match, it was against Hawthorn who did not beat anyone else, so for the second order dominance matrix this win doesn't count.

Hawthorn (row 3) is on 0 'points', they didn't beat any teams.

GWS (row 4) is on '1' point since they beat Geelong who had beaten Hawthorn.

So the second order dominance matrix puts Geelong and Hawthorn as level, students should see this as unfair and serves as a good discussion point with regards to balancing the value of the first and second order matrices.

7. Suppose the AFL decides to change the ranking system on the ladder to take into account that all wins may not be equal. If the new ranking is based on the following:

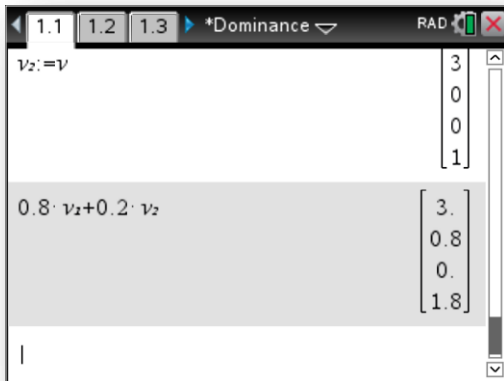
$$Ranking = 0.8V_1 + 0.2V_2$$

$V_1 =$ Dominance Vector 1

$V_2 =$ Dominance Vector 2

Determine the team rankings based on this model.

After the dominance program is run for AFL to produce the dominance vector, the result is automatically is stored in V, this can then be stored in V_1 . The program can then be run for AFL² and the resulting vector stored in V_2 . It is then possible to generate the ranking.

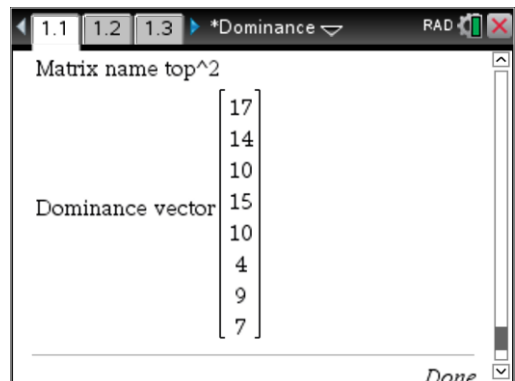
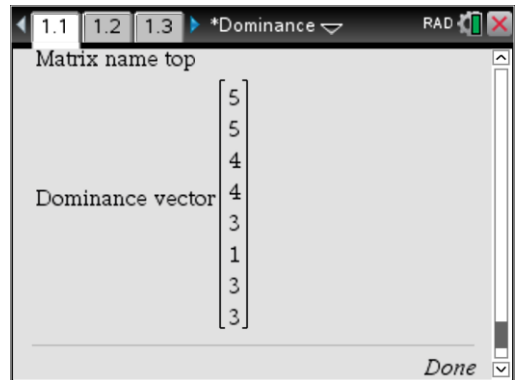


Teacher Notes:

The dominance matrix for the top 8 (first encounters) shows an interesting set of results...

After a very close season the first order dominance vector shows that the top 4 are very close and with the exception of West Coast (Row 6) the bottom half of the top 8 are very close.

The second order dominance vector shows that the top team (Sydney) was a clear winner, but Adelaide appears to have beaten a lot of teams that have beaten other teams. It also shows that Hawthorn (Row 3) appeared to struggle against some of the better teams.



Applying the proposed ranking system shows that the top 4 should remain in the top 4, however Hawthorn and GWS should really change around.

This ranking system produces the following order for the top 8:

1. Sydney
2. Geelong
3. GWS
4. Hawthorn
5. Adelaide
6. Western Bulldogs
7. North Melbourne
8. West Coast Eagles

