Parametric Equations

Answers

7 8 9 10 **11** **12**

TI-Nspire Investigation Student 30 min

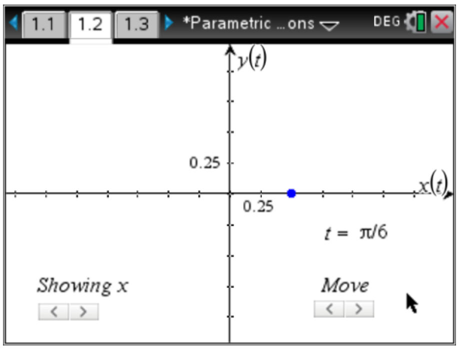
Introduction

What is a parametric equation and why do we need them? Parametric equations consist of two or more equations that depend on a single entity. Parametric equations are often used to study motion, the movement of a point along a line can be described by a function $x(t)$ while a point moving in a plane may be described by a pair of parametric equations : $x(t)$ and $y(t)$. Vertical motion can be incorporated by including a third equation: $z(t)$. The location of the object is determined by the parameter *t* which in this case refers to time. There are several advantages to working with equations parametrically, in the case of motion the parameter provides information about 'when' not just 'where', furthermore, each direction can be calculated independently which often results in a significant simplification of the calculations.

Equation in a Plane

Open the TI-*n*spire document: "Parametric Equations". Navigate to page 1.2 and adjust, as necessary, the slider in the bottom left corner of the Graph application so that it says "Showing x".

The "move" slider in the bottom right corner can be used to adjust the parameter *t* step by step or it can be animated to observe a more fluent change in *t*.



To animate a slider, place the mouse over the slider, press **Ctrl** + [**Menu**] and select **Animate**. To stop the animation, place the mouse over the slider, press **Ctrl** + [**Menu**] and select **Stop Animate**.

Question: 1.

Adjust the 'move' slider and use it to determine an approximate value for *x* for each of the following values of *t*:

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	0	1	0	-1	0

Question: 2.

Given $x(t)$ is a simple trigonometric function, determine the most likely function.

The simplest trigonometric function is: $x(t) = \sin(t)$

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Question: 3.

Adjust the viewing slider to: "showing y". Adjust the 'move' slider and use it to determine an approximate value for y for each of the following values of t:

T	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Y	1	0	-1	0	1

Question: 4.

Given $y(t)$ is a simple trigonometric function, determine the most likely function.

The simplest trigonometric function is: $y(t)=\cos(t)$

Question: 5.

Adjust the viewing slider to: "showing x & y". Animate the "move" slider and describe the path of the point controlled by the combined movement of x and y.

The path defined by the pair of parametric equations is a circle of radius 1 centred at the origin. The circle starts (t = 0) at the point (0, 1) and moves in a clockwise direction.

Question: 6.

Adjust the viewing slider to: "Showing Graph". What is the equivalent Cartesian equation for this graph?

The equation can be produced from prior knowledge: $x^2+y^2=1$ or by $\left(x(t)\right)^2+\left(y(t)\right)^2=1$

Navigate to Problem 2 on page 2.2 and answer the following questions.

Question: 7.

Adjust the 'move' slider and use it to determine an approximate value for x for each of the following values of t:

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	1	0	-1	0	1

Question: 8.

Given $x(t)$ is a simple trigonometric function, determine the most likely function.

The simplest trigonometric function is: $x(t)=\cos(t)$

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Question: 9.

Adjust the viewing slider to: "showing y". Adjust the 'move' slider and use it to determine an approximate value for y for each of the following values of t:

t	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
y	0	1	0	-1	0

Question: 10.

Given $y(t)$ is a simple trigonometric function, determine the most likely function.

The simplest trigonometric function is: $y(t)=\sin(t)$

Question: 11.

Adjust the viewing slider to: "showing x & y". Animate the "move" slider and describe the path of the point controlled by the combined movement of x and y.

The path defined by the pair of parametric equations is a circle of radius 1 centred at the origin. The circle starts (t = 0) at the point (1, 0) and moves in a counter-clockwise direction.

Question: 12.

Adjust the viewing slider to: "Showing Graph". What is the equivalent Cartesian equation for this graph?

The equation can be produced from prior knowledge: $x^2+y^2=1$ or by $\left(x(t)\right)^2+\left(y(t)\right)^2=1$

Question: 13.

Use the Pythagorean identity for trigonometric functions to show why the Cartesian equations on pages 1.2 and 2.2 are the same.

Page 1.2

$x(t)=\sin(t) \qquad y(t)=\cos(t)$

$$\left(x(t)\right)^2+\left(y(t)\right)^2=\sin^2(t)+\cos^2(t)$$

$$x^2+y^2=1$$

AND

Page 2.2

$x(t)=\cos(t) \qquad y(t)=\sin(t)$

$$\left(x(t)\right)^2+\left(y(t)\right)^2=\cos^2(t)+\sin^2(t)$$

$$x^2+y^2=1$$

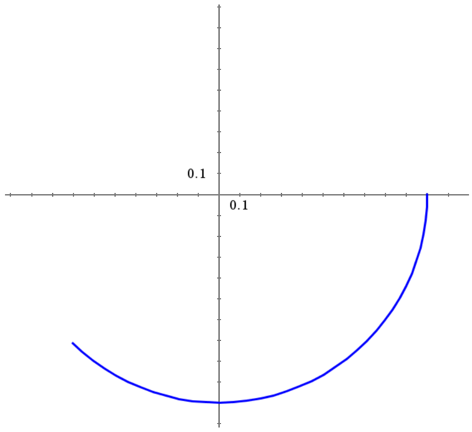
Question: 14.

While the Cartesian equations on pages 1.2 and 2.2 are the same, explain how the parametric equations provide different information.

The parametric equations provide 'direction' through the parameter t. The difference is particularly important if there are domain restrictions on the parameter.

Question: 15.

The graph shown opposite is a parametric graph of $x(t)$ and $y(t)$ for $t: \frac{3\pi}{4} \le t \le \frac{3\pi}{2}$ suggest simple trigonometric functions for both $x(t)$ and $y(t)$.



There are many possible answers for this question. The simplest equation set is to consider the minimum value for *t* as $\frac{3\pi}{4}$ which produces a negative value for both *x* and *y*. The graph then proceeds in a counter-clockwise direction which means a sign table for both *x* and *y* will help determine appropriate trigonometric functions.

$\frac{3\pi}{4} \le t \le \pi$	$-\frac{\sqrt{2}}{2} \le x \le 0$	$-\frac{\sqrt{2}}{2} \le y \le -1$
$\frac{\pi}{2} \le t \le \frac{3\pi}{2}$	$0 \le x \le 1$	$-1 \le y \le 0$

From the table it can be seen that the *y* values do not change sign over the domain: $\frac{3\pi}{4} \le t \le \pi$.

Cosine does not change sign over this domain and the signs align, therefore the function for *y* can be produced as: $y(t) = \cos(t)$. This leaves $x(t) = \mp\sin(t-h)$ where $x(t) = -\sin(t)$ as one option or $x(t) = \sin(t-\pi)$ would be another.

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