Angles in Quadrilaterals

Teachers Teaching with Technology™
Professional Development from Texas Instruments

ACMMG202

Answers & Teacher Notes

7 **8** 9 10 11 12

TI-Nspire | Navigator | Student | 50 min

Objective

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning.

Equipment

For this activity you will need:

- TI-Nspire
- TI-Nspire file: "Angles in Quadrilaterals" (tns)
- TI-Navigator system (Optional)

  When using the optional TI-Navigator system, answers in RED are corrected automatically.

Problem 1 – Properties of Rhombi

You will begin this activity by looking at angle properties of rhombi. On page 1.3, you are given rhombus *READ* and the measure of angles *R*, *E*, *A*, and *D*.

Question: 1.

Move point *E* to four different positions and collect the measures of *R*, *E*, *A*, and *D* and record your measurements in the table below.

Position	*R*	*E*	*A*	*D*
1	∠DRE = ∠DAE	∠AER = ∠ADR	∠DAE = ∠DRE	∠ADR = ∠AER
2	Individual answers will vary but follow the equality pairs above. Note also that ∠DRE + ∠REA = 180° and ∠REA + ∠EAD = 180°			
3				
4				

Question: 2.

Consecutive angles of a rhombus are supplementary.

Question: 3.

Opposite angles of a rhombus are congruent.

Author: Texas Instruments

Next, you will look at the properties of the angles created by the diagonals of a rhombi. On page 1.7, you are given rhombus *CARD* and the measure of angles *CSA*, *ASR*, *RSD*, and *DSC*.

Question: 4.

Move point *C* to four different positions. Angles formed by the intersection of the two diagonals of a rhombus are right angles (90^{o}).

On page 1.10, you are given rhombus *RHOM* and the measure of all angles created by the diagonals of the rhombus.

Question: 5.

The diagonals of a rhombus bisect the vertices.

Problem 2 – Properties of Kites

You will begin this problem by looking at angle properties of kites. You are given kite *KING* and the measure of angles *K*, *I*, *N*, and G.

Question: 6.

Move point *I* to two different positions and point *K* to two different positions and collect the measures of *K*, *I*, *N*, and *G* and record your measurements in the table below.

Position	*K*	*I*	*N*	*G*
1	$0^{o} < \angle GKI < 180^{o}$	$\angle KGN = \angle KIN$	$0^{o} < \angle GKI < 180^{o}$	$\angle KGN = \angle KIN$
2				
3				
4				

Question: 7.

What do you notice about the opposite angles of a kite?
One pair of opposite angles are congruent (equal).

Next, you will look at the properties of the angles created by the diagonals of a kite. On page 2.5, you are given kite *BLUE* and the measure of angles *BSL*, *LSU*, *USE*, and *ESB*.

Question: 8.

Move point *L* to four different positions. Angles formed by the intersection of the two diagonals of a kite are right angles (90^{o}).

On page 2.8, you are given rhombi *KITE* and the measure of all angles created by the diagonals of the rhombus.

Question: 9.

Move point *K* to four different positions. What do you notice about the angles created by the diagonals of a kite? The non-congruent angles are bisected by the diagonal.

Author: Texas Instruments

Problem 3 – Properties of Trapezoids

In this problem, you will look at angle properties of trapezoids. You are given trapezoid *TRAP* and the measure of angles *T*, *R*, *A*, and *P*.

Question: 10.

Move point *R* to four different positions and collect the measures of *T*, *R*, *A*, and *P* onto the table below.

Position	*T*	*R*	*A*	*P*
1	∠RTP + ∠ART = 180°	∠RTP + ∠ART = 180°	∠RAP + ∠APT = 180°	∠RAP + ∠APT = 180°
2				
3				
4				

Question: 11.

What do you notice about the angles of a trapezoid?

Angles pairs formed on each of the parallel sides are supplementary.

Problem 4 – Beyond Observation (Extension)

Students in year 8 can be extended by providing opportunities for simple proofs such as: "Formulate proofs involving triangle congruency and angles properties" – ACMMG243 [Year 10]. Accessing these types of questions provides students an opportunity to demonstrate some skills above the expected level.

Parallelogram:

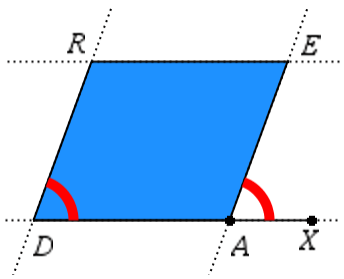
"A quadrilateral with pairs of opposite sides parallel".

Parallelograms have many properties that are a consequence of this definition. In problem 4 a parallelogram has been constructed. On page 4.1 the angle properties are explored through a series of steps. Follow these steps then answer the questions below.

Question: 12.

Name and describe the relationship between each angle pair.

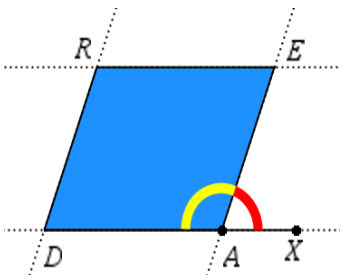
a)



Corresponding angles

∠EAX = ∠RDX

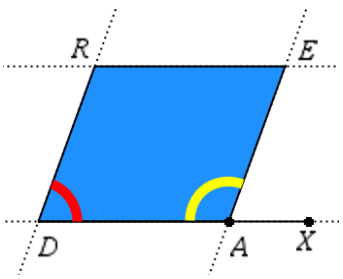
b)



Supplementary angles

∠EAX + ∠EAD = 180°

c)

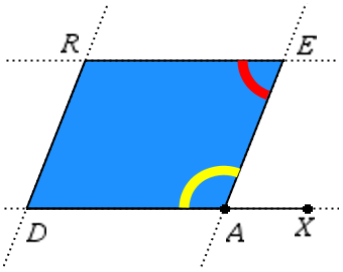


Supplementary angles

∠EAX + ∠RDX = 180°

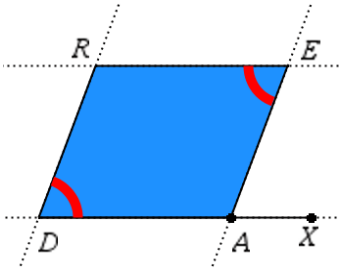
d)

e)



Supplementary angles

$\angle EAD + \angle AER = 180^{\circ}$



Opposite angles are congruent

$\angle AER = \angle ADR$

The interactive diagram on page 4.2 provides guided steps, to help prove that opposite sides of a parallelogram are equal in length.

Question: 13.

Use the interactive diagram to help formulate a proof to show that the opposite sides of a parallelogram are equal.

RE and *DE* are parallel	Parallelogram
$\angle ARE = \angle RAD$	Alternate angles (RA is transversal)
$\angle DRA = \angle EAR$	Alternate angles (RA is transversal)
RA is common side	
$\Delta AER \equiv \Delta ADR$	Congruent triangles ASA
Side ER = Side AD	Congruent triangles
Side DR = Side AE	Congruent triangles

Therefore opposite sides of a parallelogram are equal.