

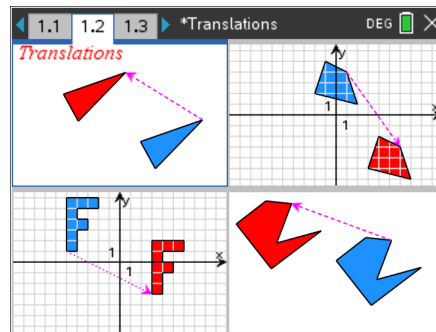


Translations Lesson

Transformational Geometry is a way to study geometry by focusing on geometric “movements” or “transformations” and observing/studying properties about these figures.

There are four geometric transformations:

< Reflections < **Translations** < Rotations < Dilations



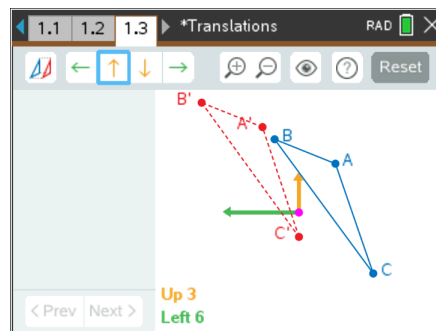
Play - Investigate - Explore - Discover

PIED

In the figure to the right, $\triangle ABC$ is translated up 3, to the left 6.

$\triangle ABC$ is called the pre-image while $\triangle A'B'C'$ is called the image (of translation).

$\triangle A'B'C'$ is read “triangle A prime, B prime, C prime.”



Download and install the red TI-Nspire student software and the Translations TNS file from the website where you obtained this document.

Then you can interact with these figures, too. If you decide not to download the software, or if you cannot, you can still do this activity along with the video.

A **conjecture** is an opinion or conclusion based on what is observed.

1. What conjecture(s) can you make based upon what you observed about a triangle and its image after being translated?

2. What is another word or phrase for what a translation does?

3. $\triangle PQR$ is typically called the _____ while $\triangle P'Q'R'$ is called the _____ (of translation).

$\triangle P'Q'R'$ is read _____.

4. a) If a triangle is translated, what appears to be true about the angles of the pre-image and image triangle? (please word your answer properly)

b) If a triangle is translated, what appears to be true about the sides of the pre-image and image triangle? (please word your answer properly)



Because the corresponding angles and the corresponding sides of the pre-image and image triangles are congruent (have equal measures), the triangles are congruent.

Therefore, a translation is called an **isometry**. An isometry is a transformation that does not change a figure's shape or size. A translation is also referred to as a **rigid motion** because it moves an object but preserves its shape and size (congruence).

We also say that a translation is a **distance-preserving** and an **angle-preserving** transformation.

5. Is a reflection an isometry? Explain.

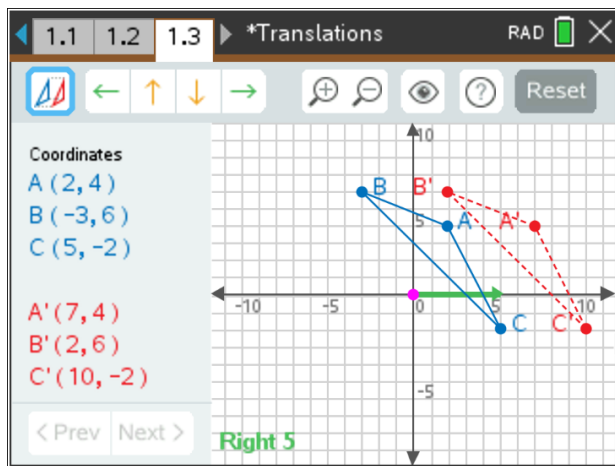
6. a) If a triangle is translated, what appears to be true about the perimeters of the pre-image and image triangle?

b) If a triangle is translated, what appears to be true about the areas of the pre-image and image triangle?

Grids and Coordinates

7. a) Translate $\triangle ABC$ to the right 5 units.

Draw your answer on the grid below.



b) Write the ordered pair for each point:

A _____ B _____ C _____

A' _____ B' _____ C' _____

c) Grab and move the vertices of the pre-image triangle.

Write the *new* ordered pair for each point:

A _____ B _____ C _____

A' _____ B' _____ C' _____

d) Using the pattern observed in the coordinates, if a point on the pre-image triangle has coordinates $(1, 2)$, what are the coordinates of its corresponding point on the image triangle?

That is, $(1, 2) \rightarrow$ _____ ' \rightarrow ' means "maps to"

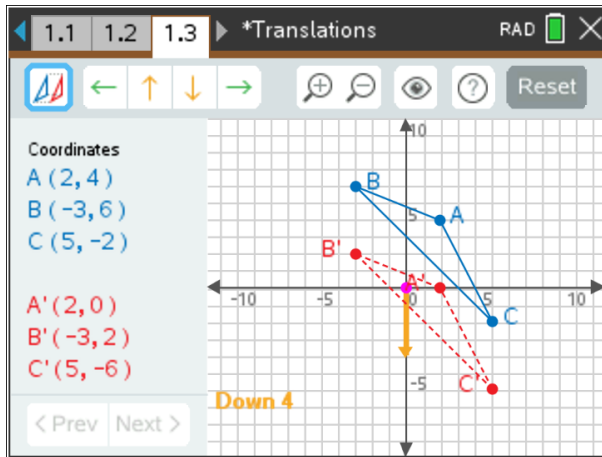
e) Similarly, the point $(-3, 7)$ would be translated to? That is, $(-3, 7) \rightarrow$ _____

f) Generalize the pattern. If a point on the pre-image triangle has coordinates (x, y) , what are coordinates of its corresponding point on the image triangle? That is $(x, y) \rightarrow$ _____



8. a) Translate $\triangle ABC$ down 4 units.

Draw your answer on the grid below.



b) Write the ordered pair for each point:

A _____ B _____ C _____

A' _____ B' _____ C' _____

c) Grab and move the vertices of the pre-image triangle.

Write the *new* ordered pair for each point:

A _____ B _____ C _____

A' _____ B' _____ C' _____

d) Using the pattern observed in the coordinates, if a point on the pre-image triangle has coordinates $(1, 2)$, what are the coordinates of its corresponding point on the image triangle?

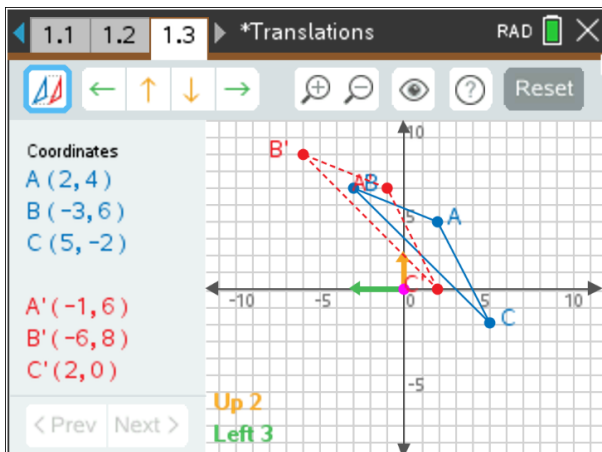
That is, $(1, 2) \rightarrow$ _____ ' \rightarrow ' means "maps to"

e) Similarly, the point $(-3, 7)$ would be translated to? That is, $(-3, 7) \rightarrow$ _____

f) Generalize the pattern. If a point on the pre-image triangle has coordinates (x, y) , what are coordinates of its corresponding point on the image triangle? That is $(x, y) \rightarrow$ _____

9. a) Translate $\triangle ABC$ to the left 3 units and up 2 units.

Draw your answer on the grid below.



b) Write the ordered pair for each point:

A _____ B _____ C _____

A' _____ B' _____ C' _____

c) Grab and move the vertices of the pre-image triangle.

Write the *new* ordered pair for each point:

A _____ B _____ C _____

A' _____ B' _____ C' _____



d) Using the pattern observed in the coordinates, if a point on the pre-image triangle has coordinates $(1, 2)$, what are the coordinates of its corresponding point on the image triangle?

That is, $(1, 2) \rightarrow$ _____ ' \rightarrow ' means "maps to"

10. Given: $\triangle DEF$ is translated to the right 4 units and down 2 units.

a) If D has coordinates $(5, 7)$, what are the coordinates of D'? _____

b) If E has coordinates $(-3, -7)$, what are the coordinates of E'? _____

c) If F has coordinates (a, b) , what are the coordinates of F'? _____

d) If E' has coordinates $(1, 6)$, what are the coordinates of E? _____

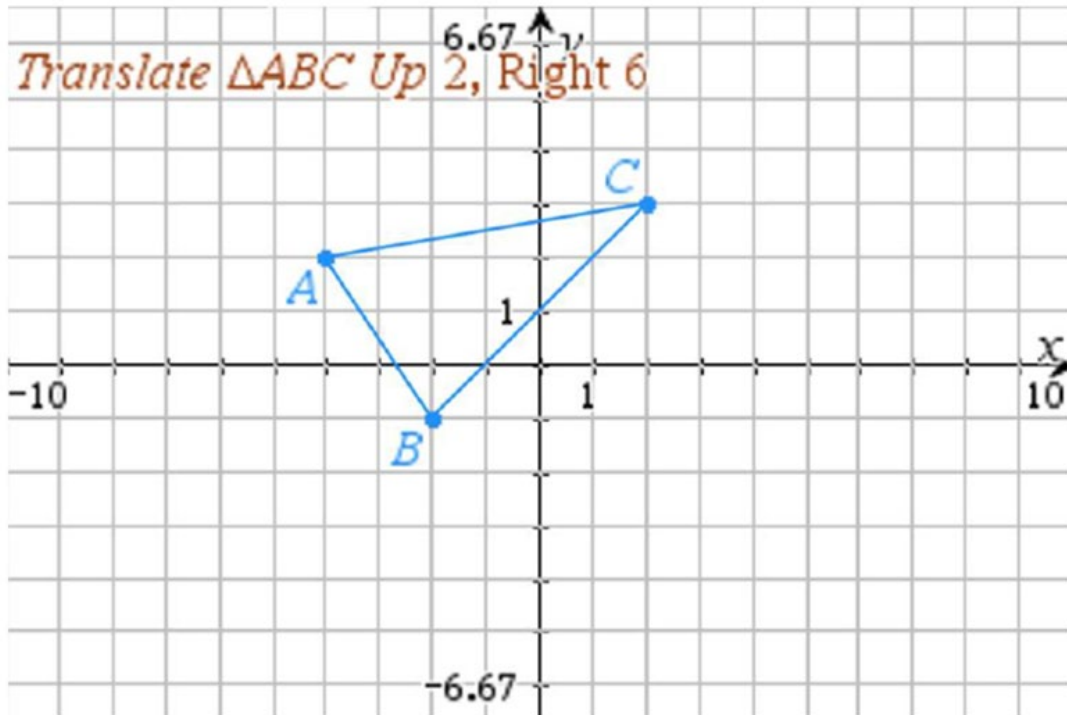
e) If D' has coordinates (p, q) , what are the coordinates of D? _____

Translate by Hand

11. Translate $\triangle ABC$ up 2 units, right 6 units, using a straightedge.

Label the vertices appropriately and show the 3 dashed segments that connect corresponding vertices.

a)



b) List the coordinates of each of the 6 vertices:

A: _____ B: _____ C: _____

A': _____ B': _____ C': _____



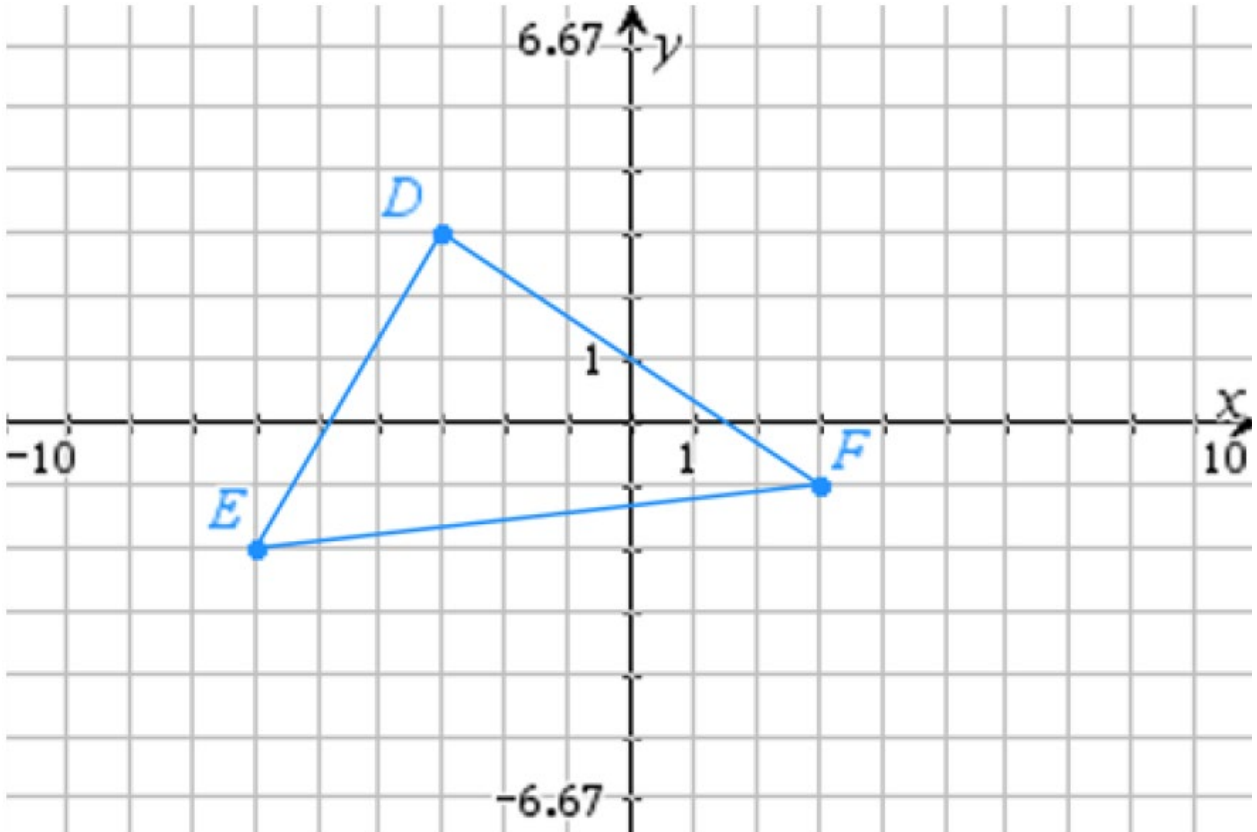
c) If (x, y) is a point on $\triangle ABC$, what are the coordinates of its image on $\triangle A'B'C'$? _____

d) If (g, h) is a point on $\triangle A'B'C'$, what are the coordinates of its pre-image on $\triangle ABC$? _____

12. Translate $\triangle DEF$ down 3 units, right 5 units, using a straightedge.

Label the vertices appropriately and show the 3 dashed segments that connect corresponding vertices.

a)



b) List the coordinates of each of the 6 vertices:

D: _____ E: _____ F: _____

D': _____ E': _____ F': _____

c) If (x, y) is a point on $\triangle DEF$, what are the coordinates of its image on $\triangle D'E'F'$? _____

d) If (g, h) is a point on $\triangle D'E'F'$, what are the coordinates of its pre-image on $\triangle DEF$? _____



Properties of Corresponding Sides of Translated Triangles

13. Translate $\triangle ABC$ up 3 units and to the left 6 units, using a straightedge.

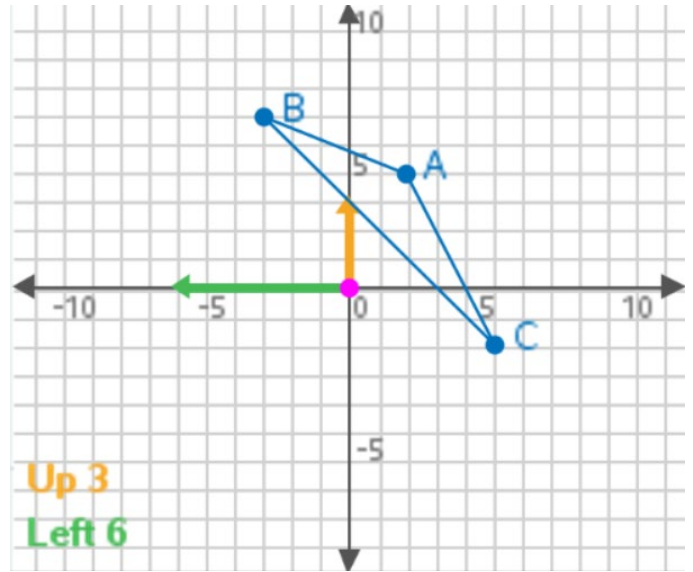
a) Look at corresponding sides \overline{AB} and $\overline{A'B'}$.

We have already established that these two segments have the same length.

What else appears to be true about these two segments?

What about \overline{BC} and $\overline{B'C'}$?

What about \overline{CA} and $\overline{C'A'}$?



b) It appears that each pair of corresponding sides is parallel.

If segments (lines) are to be parallel, what must be true about their slopes?

c) Calculate the slope of each pair of corresponding sides. Record your answers as fractions.

Slope of \overline{AB} = _____ . Slope of $\overline{A'B'}$ = _____ .

Slope of \overline{BC} = _____ . Slope of $\overline{B'C'}$ = _____ .

Slope of \overline{CA} = _____ . Slope of $\overline{C'A'}$ = _____ .

d) Based upon the results in part c above, is each pair of corresponding sides parallel?

e) This is not enough evidence to prove this conjecture for all triangles. We need to investigate more examples. Let's use the technology to do this.



14. Translate $\triangle ABC$ down 4 units and to the right 5 units, using a straightedge.

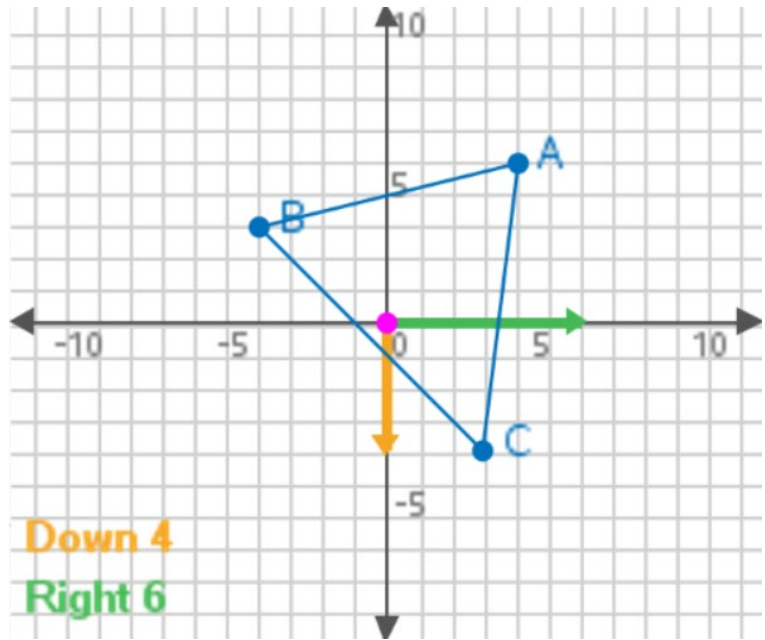
a) Look at corresponding sides

\overline{AB} and $\overline{A'B'}$.

We have already established that these two segments have the same length. What else appears to be true about these two segments?

What about \overline{BC} and $\overline{B'C'}$?

What about \overline{CA} and $\overline{C'A'}$?



b) Calculate the slope of each pair of corresponding sides. Record your answers as fractions.

Slope of \overline{AB} = _____.

Slope of $\overline{A'B'}$ = _____.

Slope of \overline{BC} = _____.

Slope of $\overline{B'C'}$ = _____.

Slope of \overline{CA} = _____.

Slope of $\overline{C'A'}$ = _____.

c) Based upon the results in part c above, is each pair of corresponding sides parallel?

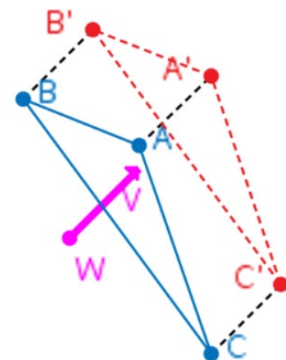
Translate by Vector

A **vector** is a directed line segment which has both length and direction.

15. In the figure at the right, $\triangle ABC$ is translated by **vector** \overline{WV} .

Look at the dashed segments, $\overline{AA'}$, $\overline{BB'}$, $\overline{CC'}$, and the **vector** \overline{WV} .

Two things seem to be true about vector \overline{WV} and these three dashed segments. Write two conjectures below.





16. Using the figure to the right, calculate the slopes of the following segments. Write your answers as fractions.

Note: $m(\overline{CD})$ means the slope of segment CD

a) $m(\overline{BA}) = \underline{\hspace{2cm}}$ $m(\overline{B'A'}) = \underline{\hspace{2cm}}$

$m(\overline{BC}) = \underline{\hspace{2cm}}$ $m(\overline{B'C'}) = \underline{\hspace{2cm}}$

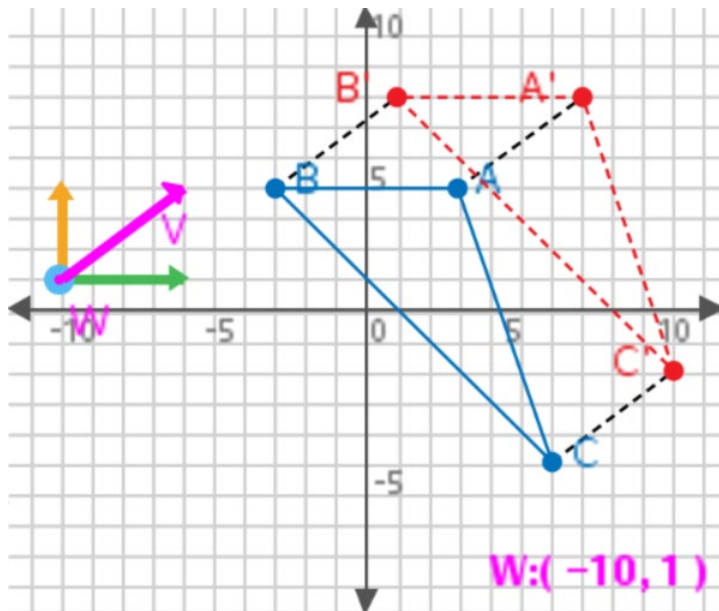
$m(\overline{AC}) = \underline{\hspace{2cm}}$ $m(\overline{A'C'}) = \underline{\hspace{2cm}}$

b)

$m(\overline{AA'}) = \underline{\hspace{2cm}}$ $m(\overline{BB'}) = \underline{\hspace{2cm}}$

$m(\overline{CC'}) = \underline{\hspace{2cm}}$ $m(\overline{WW}) = \underline{\hspace{2cm}}$

c) Name the segments that are parallel.



17. Translate $\triangle DEF$ by vector \overline{WW} using a ruler. Show these as dashed segments: $\overline{DD'}$, $\overline{EE'}$, $\overline{FF'}$.

