String Graphs – Part 2

Teacher Notes & Answers

7   8   9   **10**   11   12

TI-Nspire   Investigation   Student   45 min

Aim

- Determine the family of straight lines that can be used to stitch the lines: $y = x$ and $y = -x$
- Determine the corresponding parametric equation for the family of straight lines.
- Explore and generalise for $y = mx$ and $y = -mx$

https://bit.ly/Yr10StringGraphsPart2

Determining Equations

Start a **new document** and insert a **Graphs application**.

Use the **[Menu]** to adjust the window settings:

Xmin = -10

Xmax = 10

Ymin = -1

Ymax = 12.3

Graph the lines $y = x$ and $y = -x$.

Save your new document as: **STRINGS 2**

It is worthwhile changing the attributes of these lines to: *dotted.*

Place the mouse (cursor) over the line $y = x$, text such as "Graph f₁" or "Graph f₂" will be displayed.

Press: **[Ctrl] + [Menu]**

Select: **Attributes**

Here you can change the line to dotted and change the way the line is labelled such as $f_1(x) = x$, $y = f_1(x)$ or $y = x$.

A series of straight line graphs will be constructed to form a string pattern.

The first straight line graph passes through the points:

(-10, 10) & (1, 1)

The result is shown opposite. Use the following questions to help determine the equation for this line and all subsequent lines.

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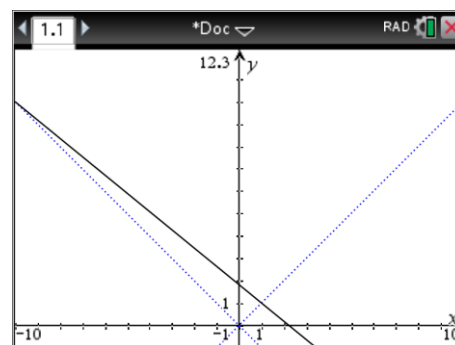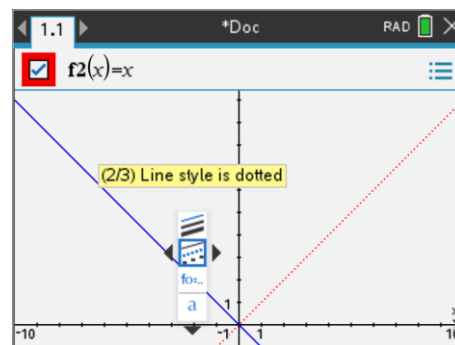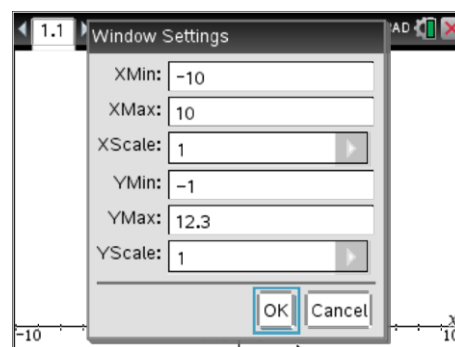
The equation to any straight line can be expressed in the form: $y = mx + c$

In this task it may be useful to express the equations in the form: $y = m(x - h) + k$

$$m = \text{gradient} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

(h, k) = represents the coordinates of a point that the line passes through

Question: 1.

Determine the equation of this first line, passing through the points: (-10, 10) & (1, 1)

a)   Calculate the gradient of the first line. $m = \dfrac{-9}{11}$

b)   Choose one of the two given points for the line to pass through and hence write down the equation in the form: $y = m(x - h) + k$.

Answers may vary: Selected Point (1, 1) Equation $y = \dfrac{-9}{11}(x - 1) + 1$

c)   Verify your equation from part (b) by substituting the other point's coordinates to show they also satisfy this equation.
Example:
If (1, 1) was used to generate the equation in (b), check that (1, 1) satisfies the equation.

$$10 = \frac{-9}{11}(-10 - 1) + 1$$

$$10 = \frac{-9}{11} \times \frac{-11}{1} + 1$$

$$10 = 9 + 1$$

d)   Expand and simplify the equation from part (b) to write it in the form: $y = mx + c$.

$$y = \frac{-9}{11}x + \frac{9}{11} + 1$$

$$y = \frac{-9}{11}x + \frac{9}{11} + \frac{11}{11}$$

$$y = \frac{-9}{11}x + \frac{20}{11}$$

Once the first line is completed, try the second line.

The second straight line graph passes through the points:

    (-9, 9) & (2, 2)

As more graphs are added it may be desirable to remove the equation labels.

    Settings > **Automatically hide plot labels**

Graphs & Geometry Settings

Display Digits: Float 3

Graphing Angle: Radian

Geometry Angle: Degree

☑ Automatically hide plot labels

☑ Show axis end values

☐ Show tool tips for function manipulation

Restore | Make Default | OK | Cancel

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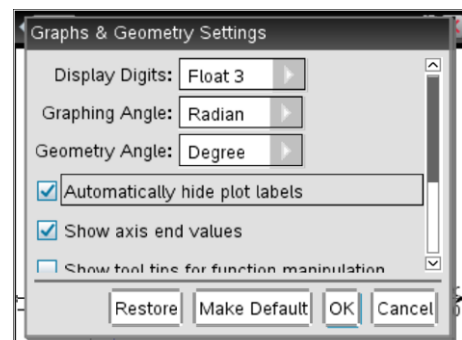
Question: 2.

Determine the equation of the line, passing through the points: (-9, 9) & (2, 2).

Answers may vary depending on equation format and selected point. $y = \dfrac{-7}{11}(x-2)+2$

Question: 3.

Determine the gradient and y – intercept for the remaining straight lines in this family of lines. Record your results using exact fractional values. Graph all 10 equations on the same set of axes.

Eqn. No.	Point 1	Point 2	Gradient	Equation
1	(-10, 10)	(1, 1)	$\dfrac{-9}{11}$	$y = \dfrac{-9}{11}(x-1)+1$ or $y = \dfrac{-9}{11}x+\dfrac{20}{11}$
2	(-9, 9)	(2, 2)	$\dfrac{-7}{11}$	$y = \dfrac{-7}{11}(x-2)+2$ or $y = \dfrac{-7}{11}x+\dfrac{36}{11}$
3	(-8, 8)	(3, 3)	$\dfrac{-5}{11}$	$y = \dfrac{-5}{11}(x-3)+3$ or $y = \dfrac{-5}{11}x+\dfrac{48}{11}$
4	(-7, 7)	(4, 4)	$\dfrac{-3}{11}$	$y = \dfrac{-3}{11}(x-4)+4$ or $y = \dfrac{-3}{11}x+\dfrac{56}{11}$
5	(-6, 6)	(5, 5)	$\dfrac{-1}{11}$	$y = \dfrac{-1}{11}(x-5)+5$ or $y = \dfrac{-1}{11}x+\dfrac{60}{11}$
6	(-5, 5)	(6, 6)	$\dfrac{1}{11}$	$y = \dfrac{1}{11}(x-6)+6$ or $y = \dfrac{1}{11}x+\dfrac{60}{11}$
7	(-4, 4)	(7, 7)	$\dfrac{3}{11}$	$y = \dfrac{3}{11}(x-7)+7$ or $y = \dfrac{3}{11}x+\dfrac{56}{11}$
8	(-3, 3)	(8, 8)	$\dfrac{5}{11}$	$y = \dfrac{5}{11}(x-8)+8$ or $y = \dfrac{5}{11}x+\dfrac{48}{11}$
9	(-2, 2)	(9, 9)	$\dfrac{7}{11}$	$y = \dfrac{7}{11}(x-9)+9$ or $y = \dfrac{7}{11}x+\dfrac{36}{11}$
10	(-1, 1)	(10, 10)	$\dfrac{9}{11}$	$y = \dfrac{9}{11}(x-10)+10$ or $y = \dfrac{9}{11}x+\dfrac{20}{11}$

A single equation can be determined to graph all 10 equations by using a parameter (*t*) for the equation number. Study each of your equations above and compare with the equation 'number'.

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Question: 4.

Use the following questions to help find the general equation in the form: $y = \dfrac{a}{b}(x-h)+k$ where a, b, h and k are expressions in terms of t.

a) Determine an expression for a in terms of t. $\qquad$ $a = 2t - 11$

b) Determine an expression for b in terms of t. $\qquad$ $b = 11$

c) Determine an expression for h and k in terms of t. $\qquad$ $h = t$ and $k = t$

d) Write down the general equation for the family of straight lines in the form: $y = \dfrac{a}{b}(x-h)+k$.

$$y = \frac{2t-11}{11}(x-t)+t$$

Note: Some students may use points on the line $y = -x$ however these will simplify to the same.

e) Write down the general equation for the family of straight lines in the form: $y = mx + c$.

$$y = \frac{2t-11}{11}x + \frac{2t(11-t)}{11}$$

f) Verify your equations by substituting a range of values for t and comparing with the corresponding original equation.

Answers will vary.

Examples:
$$y = \frac{2t-11}{11}x + \frac{2t(11-t)}{11}\bigg| t = 1 \qquad\qquad y = \frac{2t-11}{11}x + \frac{2t(11-t)}{11}\bigg| t = 2$$

$$y = \frac{2-11}{11}x + \frac{2(11-1)}{11} \qquad\qquad y = \frac{4-11}{11}x + \frac{4(11-2)}{11}$$

$$y = \frac{-9}{11}x + \frac{20}{11} \qquad\qquad y = \frac{-7}{11}x + \frac{36}{11}$$

So far you have mostly used " $y=$ " to express a relationship between y and x.

Function notation is very powerful and convenient way to represent and express relationships. We can write $y = f(x)$ which means y is equal to a rule defined by $f(x)$.

The straight line $y = 2x + 1$ can be written as: $f(x) = 2x + 1$

If we wish to determine the value of y when $x = 3$ we can write this as: $f(3)$

In the next section we define our rule in terms of the variable x and parameter t using function notation: $f(x,t)$

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Insert a Calculator application and define t as the set of integers: $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Define your general equation in terms of the variable x and parameter t.

Once you have defined $f(x,t)$ you can check any of your previous equations by entering: $f(x,1)$ or $f(x,2)$ ... for equations 1, 2 ... in Question 3.

Return to the Graphs application on page 1.1.

Press Tab or Ctrl + G to navigate to the graph entry line.

Enter the rule (shown opposite) as $f(x,t)$ then press **[Enter]** to see the family of straight line graphs.

You can explore the extended family of straight lines by experimenting with the range of values for t.

Here are some examples to try:

- $f(x, t+10)$
- $f(x, t-10)$
- $f\left(x, \dfrac{t}{2}\right)$

Note: Do these in addition to your original graph of $f(x,t)$.

To see the full effect, zoom out using the zoom out tool in the Window / Zoom menu and place the magnifying glass close to the centre of the screen.

Question: 5.

Describe the shape of the curve formed by the extended family of straight lines.

Curve appears to be a parabola. Students may compare the 'shape' to familiar objects such as 'curved mirrors' or better, a satellite dish.

Note: Students may enter the equation $y = x^2$ and then dilate and translate this function by dragging it around the calculator screen.

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Extension – Becoming Flexible

For each extension section, use a 'New Problem' in your TI-nspire file. The New Problem means that you can use the same variables as the first problem without losing all your previous work. So for Question 6, your Graphs application will be on Page 2.1.

Question: 6.

The curve is now created by symmetrically stitching points on the lines $y = -mx$ and $y = mx$.

a. Determine the equations for each of the straight lines formed as the lines: $y = -2x$ and $y = 2x$ are stitched together. The points being stitched in this section of the investigation include: (-10, 20) to (1, 2); (-9, 18) to (2, 4); (-8, 16) to (3, 6) … Tabulate your results just like you did in the Question 3 and define a new general function for this family of straight lines.

The points being stitched in this investigation include: (-10, 20); (-9, 18); (-8, 16) …

The general equation for the straight lines: $y = \dfrac{2(2t-11)}{11} x - \dfrac{4t(t-11)}{11}$

b. Determine the equation for the curve defined by the envelope formed as the lines: $y = -\dfrac{x}{2}$ and $y = \dfrac{x}{2}$ are stitched together.

Determine the general equation for the curve defining the envelope.

The points being stitched in this investigation include: (-10, 10m); (-9, 9m); (-8, 8m) …

The general equation for the straight lines: $y = \dfrac{(2t-11)}{22} x - \dfrac{2t(t-11)}{22}$

The equation for the envelope: $y = \dfrac{(x^2 + 120)}{44}$

c. An additional parameter can be used for the family of straight lines so as to stitch to the general equations: $y = mx$ and $y = -mx$. In this case the general equation would be defined as: $f(x, t, m)$. This part of the extension is not for the faint hearted!

Students need to start with the general solution in this case and validate their final answer using their results from the first part of the investigation (m = +/- 1) and then also with parts (a) and (b) of the extension (above).

Points to be stitched lie on the lines: $y = mx$ and $y = -mx$. This means the points to be stitched would include: (-10, 10m) to (1, m); (-9, 9m) to (2, 2m) …

The gradient for the first line would therefore be: $\dfrac{10m - m}{-10 - 1} = \dfrac{-9m}{11}$

The gradient for the second line would be: $\dfrac{9m - 2m}{-9 - 2} = \dfrac{-7m}{11}$

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This pattern should look familiar! The general expression for the gradient would therefore be:
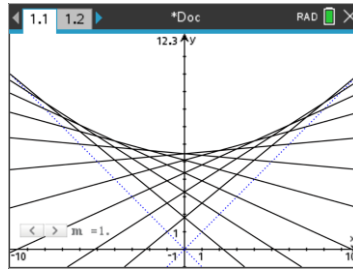
Gradient: $\dfrac{m(2t-11)}{11}$

The translational form of the straight line provides a relatively *straight* forward approach to finding the overally general equation.

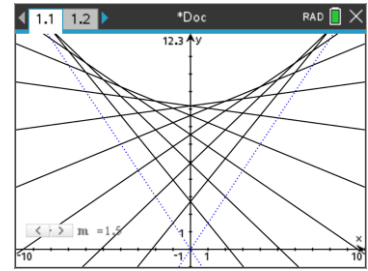$$f(x,t,m) = \frac{m \cdot (2t-11)}{11}(x-t) + m \cdot t$$

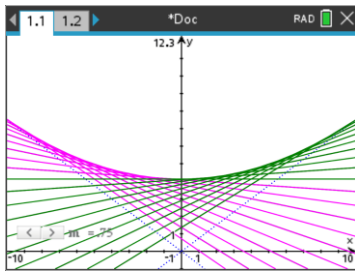The screen shots below use the function above with t = {1, 2, 3 … 10}. The value of m is defined by the slider.
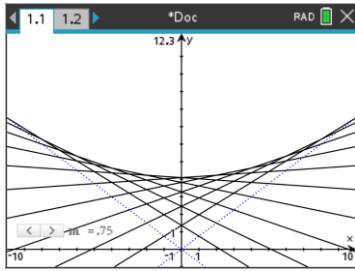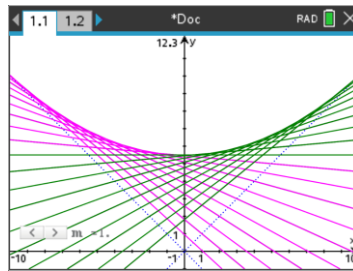

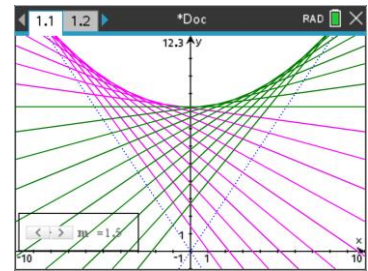M = 0.75 & T = {1, 2, 3, …10}

M = 1 & T = {1, 2, 3, …10}

M = 1.5 & T = {1, 2, 3, …10}

M = 0.75 & T = {0.5, 1, 1.5 … 10}*

M = 1 & T = {0.5, 1, 1.5 … 10}*

M = 1.5 & T = {0.5, 1, 1.5 … 10}*

*You cannot plot 20 lines at once using a single parameter. Instead, plotting $f\left(x, \dfrac{t}{2}, m\right)$ and

$f\left(x, \dfrac{t+10}{2}, m\right)$ produces all the required lines.

A slider has been used for the value of m, this way the lines $y = mx$ and $y = -mx$ also adjust as the slider is changed.

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