Building Concepts: Solving Equations

Lesson Overview

In this TI-Nspire lesson, students draw upon tools learned in previous work and focus on making strategic choices in solving linear equations.

Solving linear equations with rational number coefficients can combine a variety of methods.

Learning Goals

1. Solve a linear equation of the form $ax + b = cx + d$ and interpret the solution in terms of the equation and, when appropriate, the context;
2. identify equations that are identities or those for which no solution is possible;
3. recognize when one solution strategy might be more efficient or elegant than another.

Prerequisite Knowledge

Solving Equations is the twelfth lesson in a series of lessons that explores the concepts of expressions and equations. In this lesson, students decide which tool would be best to solve linear equations. This lesson builds on the concepts of previous lessons. Prior to working on this lesson, students should have completed *Building Expressions*, *Equations and Operations,* and *Building Expressions with Two Variables*. Students should understand:

- the associative and commutative properties of addition and multiplication;
- how to interpret and write an expression;
- the concept of the distributive property

Vocabulary

- **linear equation:** an algebraic equation in which each term is either a constant or the product of a constant and a variable
- **solution:** a means of solving a problem
- **identity:** an equation where the right and left sides of the equation are equivalent expressions

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Solving Equations_Student.pdf
- Solving Equations_Student.doc
- Solving Equations.tns
- Solving Equations_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet also can be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

In *Equations and Operations, Using Structure to Solve Equations*, and *Visualizing Linear Equations Using Mobiles*, students have created a toolbox for reasoning about and solving equations. Students can use structure to find missing factors and addends as well as perform solution-preserving moves. This lesson provides a framework that allows students to choose the tools they want to use to solve an equation that has a general linear expression on each side of the equal sign. They inspect an equation and select an approach, then identify a series of steps that are carried out by the TNS activity. Students consider the efficiency and elegance of multiple approaches for finding the solution for a given equation. (Note that "elegant" solutions might vary depending on the situation and the student, for example, recognizing that $3(x-2)=5(x-2)$ implies $x=2$ because the only way the product of 5 and something equals the product of 3 and that same quantity is when the value of the quantity is 0.) As students chart out different pathways to finding a solution, the ultimate goal is to enable them to become fluent and flexible in solving linear equations that can be reduced to the form $ax+b=cx+d$.

The process is similar to that used in computer algebra systems, where the technology carries out manipulation commands. The emphasis here is on students making decisions about which process will lead to a solution and why one choice might be more advantageous than another in a given situation. Using the technology to do the computational work frees the student to focus on making strategic choices using the tools they learned in earlier lessons.

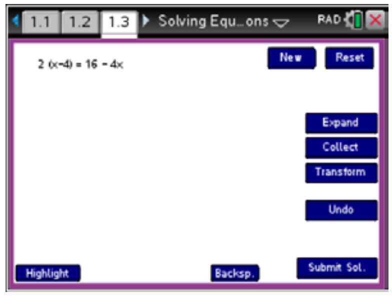
Note: The solution-preserving operations with respect to equations are addition and multiplication. Solving equations of the form $ax=b$ is approached by multiplying both sides of the equation by the multiplicative inverse of *a* rather than dividing by *a*. This allows equations of the form $\left(\frac{a}{c}\right)x=c$ to be approached using the multiplicative inverse of $\frac{a}{c}$, which is $\frac{c}{a}$, rather than having the term $\frac{1}{\left(\frac{a}{c}\right)}$.

Note: Because this TNS activity does much of the computational arithmetic in solving an equation, a strong emphasis is on having students "say their becauses" (Horn, 2012). From their work on using structure to solve equations, students should be encouraged to go from $5x=60$ to $x=12$ "because" they know that the product of 5 and 12 is 60 rather than multiplying both sides by $\frac{1}{5}$.

Part 1, Page 1.3

Focus: Empower students to make sense out of solving equations by flexibly using both solution-preserving moves and structure to transform equations to find a solution.

On page 1.3, an equation is displayed. Students use a variety of methods to arrive at a solution.



Expand carries out the distributive property.

Collect combines like terms.

Transform can be used to add a linear expression to both sides of an equation or to multiply both sides of an equation by a non-zero rational number.

Highlight then **Enter** and the arrow keys highlight parts of an equation, press **Enter** to select the highlighted portion.

Submit Sol. shows a blank in which a solution can be entered and displays options for no solution or all numbers.

New generates new equations at random.

Equations displays five preset equation choices.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles through on-screen options.

enter activates on-screen buttons or highlights.

ctrl enter displays the fraction template.

ctrl del resets the page.

Class Discussion

Answer each of the following. Be ready to explain how the method works and why you know you have the correct solution.

- ***Create an equation you would solve using the Highlight method at least twice.***
- ***Create an equation you would solve using the solution-preserving method at least twice.***

Answers will vary. Have several students share their equations for a whole-class discussion or pair students and have them trade equations with each other. A discussion question for the class might be: Could you have solved either equation with both methods? Why or why not?

Teacher Tip: You may want to do the next question as a class where you or a student demonstrates the process. The focus should be on having students predict the outcome of using the options **Expand, Collect, Transform, Highlight** to support their understanding of the mechanics involved in solving an equation.

Class Discussion (continued)

On page 1.3, use* New *to generate a new equation.

- ***Suggest an option to begin finding the solution for the equation and write down your predictions for the new equation that would result from using that option. What was your reason for making that choice?***

  Answers will vary. Be sure to emphasize both checking the prediction and giving a reason for making the choice—saying "because…"

- ***Check the result, then select an option for continuing to look for a solution, give a reason for your choice, and then predict what will happen next.***
- ***Repeat the process until you have a solution. Check your solution by selecting* Submit Sol. *and entering your solution value for* x.**

Reset page 1.3.

- ***Decide which of the possible options would be useful to begin solving the equation*** $\mathbf{2(x-4)=16-4x}$ **.**

  Answers will vary.

- ***Explain why you made that choice.***

  Answers may vary. **Expand** probably will seem the most reasonable for many students because there is nothing to collect and there is no common quantity that highlighting would reveal as a structure that could be used to reason about a missing factor or addend.

- ***Experiment with the other choices until you have a solution. Look at your partner's screen and see if you can explain what your partner did—for example, adding* 4x *to both sides because, like the mobiles, you can add the same shape to both sides, a solution-preserving move.***

  Answers will vary. Remind students to say "because ..."

Class Discussion (continued)

Mari wanted to solve the equation differently. She noted that all of the numbers in the equation were even, so she transformed both sides by multiplying by $\frac{1}{2}$.

- ***Why might Mari want to do this?***

  Answer: Because then she would have smaller numbers that would be easier to reason about.

- ***Carry out Mari's strategy. Carl wondered why the expression to the left side was not transformed to $1x - 2$ because if you multiplied the expression by $\frac{1}{2}$, you should multiply all of the values in the expression by $\frac{1}{2}$. What would you say to Carl?***

  Answers may vary: By the order of operations and the grouping property, you would multiply from left to right, so $\frac{1}{2}(2(x-4))$ would be $\left(\frac{1}{2}\cdot 2\right)(x-4)$ or $1(x-4)$.

- ***Can you use the Highlight method on the new equation? Why or why not?***

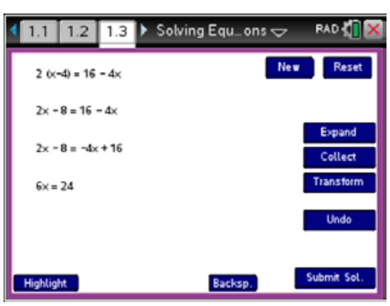
  Answer: It would be difficult to highlight because there are two terms on either side of the equal sign, and it would be difficult to think about a missing factor or addend.

- ***Finish solving the equation. Explain why your solution should be the same as the solution you found previously.***

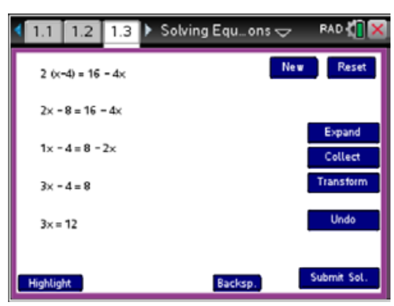
  Answer: None of the moves you took would "unbalance" the equation, so the solution should be the same no matter which sequence of steps you followed. Since you are solving the same equation, if you do not make any mistakes, regardless of the method, you should get the same solution.

Here are screen shots of different approaches to solving an equation. State the option that might have been used for each step. Give a reason why that option might be reasonable—a "because".

a.

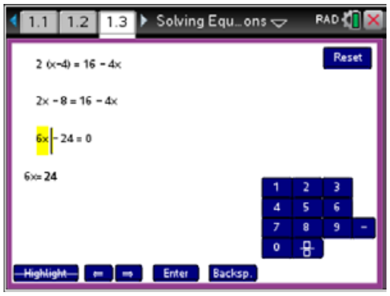


b.

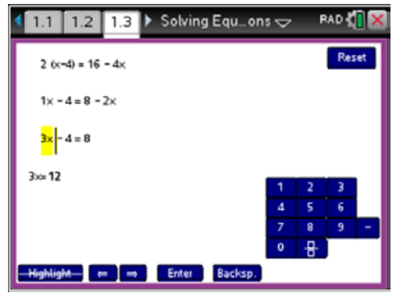


Class Discussion (continued)

c.



d.



Answers may vary:

In a, Expand, then Transform by adding 4*x* to both sides, then use Transform by adding 8 to both sides, then use Highlight to find $x = 4$.

In b, Expand, then Transform by multiplying by $\frac{1}{2}$, Transform by adding 2x to both sides, Transform by adding 4, then Highlight to get $x = 4$.

In c, Expand, Transform by adding 4x to both sides, then Highlight 6x to get $6x = 24$, and use Highlight to find $x = 4$.

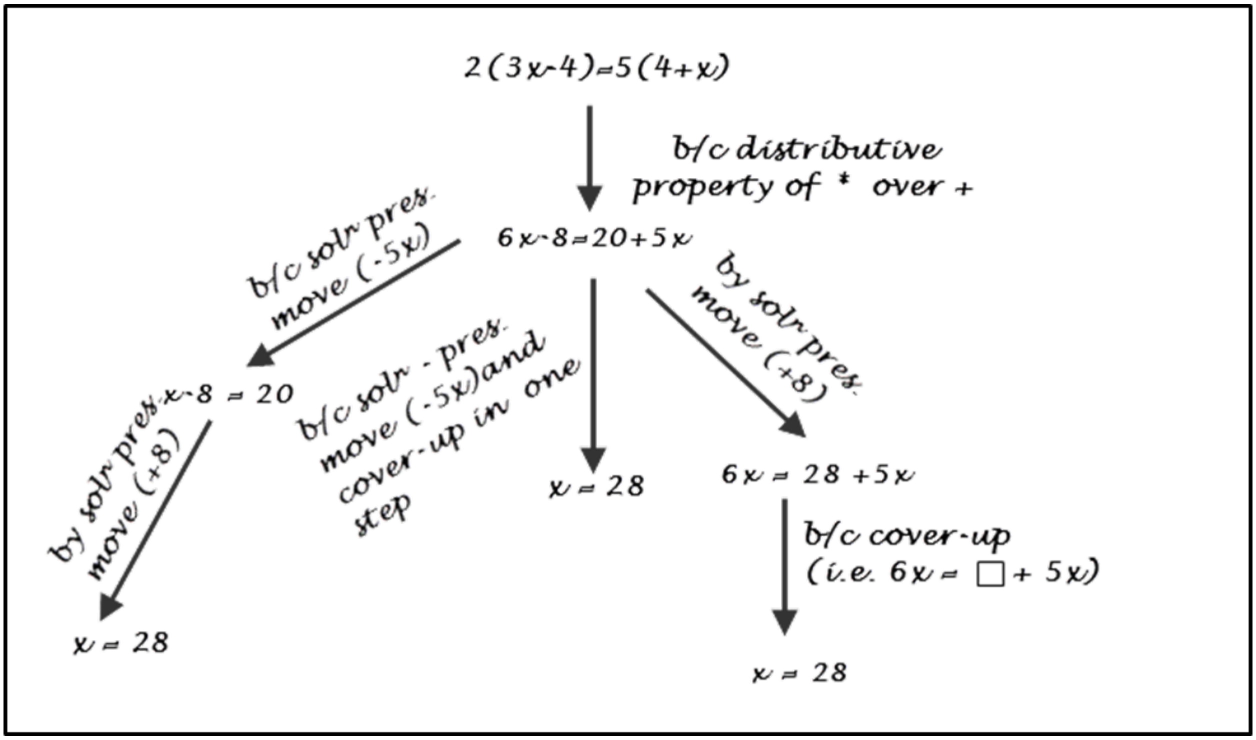
In d, Transform by multiplying both sides of the equation by $\frac{1}{2}$. Then Transform by adding 2x, and Highlight to get $3x = 12$, and Highlight to find $x = 4$.

The following questions introduce the notion of different "pathways" toward a solution, often a combination of highlighting and solution-preserving moves for transforming the equation.

> **Teacher Tip:** In order for students to discuss the first question in groups or as a class, you may wish to copy the pathway onto a whiteboard or display using a projector.

Class Discussion (continued)

Here is a chart of different pathways toward a solution to the equation $2(3x-4)=5(4+x)$.



- ***What do the b/c represent on the pathway steps?*** — Answer: b/c is short for "because" describing the reason why the move is OK to make.
- ***Describe one pathway.*** — Answers will vary. One example: Use the distributive property to get to the form $ax+b$ on the left, then use a solution-preserving move adding –5*x* to both sides to get $x-8=20$, then another solution-preserving move adding 8 to get $x=28$.
- ***Which of the pathways seems most efficient to you? Why?*** — Answers will vary. Some might choose the pathway that used the Highlight method $x-8=20$ because it is shorter.
- ***Mathematicians talk about "elegant" solutions—where elegant means insightful or clever; often involving a relatively short way and immediate approach to an equation. Did any of the pathways seem elegant to you? Explain your reasoning (say your "becauses").*** — Answers will vary.

Class Discussion (continued)

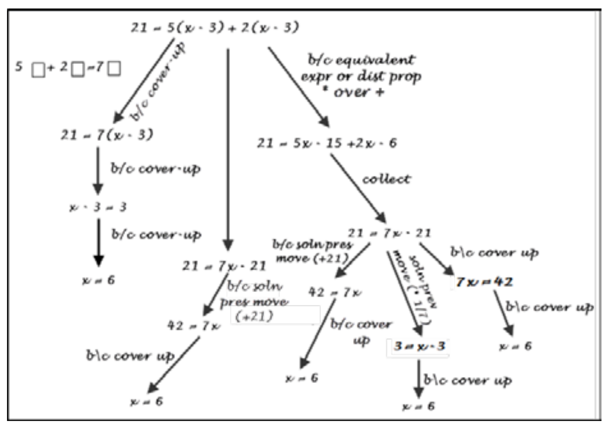
Go to* menu> Equations *and select* Equation 2*.
Work with a partner to create at least two different pathways for finding the solution. Predict what will happen before you take any action. Be ready to share your thinking, including when your prediction was wrong and why.

Select two or three students to demonstrate their pathways using the TNS activity and ask:

- ***Which of the pathways seems most efficient to you? Why?***

Answers will vary: For example, some students might see that $21 = 5(x-3) + 2(x-3)$ could be solved by writing $21 = 7(x-3)$ using the Highlight method and then applying the Highlight method again to get $3 = x-3$, so $x = 6$. Others might Expand and Collect to get to $21 = 7x - 21$, then use the Highlight method to get $7x = 42$, and others Transform by adding 21 to both sides. Be sure students check their work to be sure they actually have the solution.

Sample pathway:



- ***Did any of the pathways seem elegant to you? Explain your reasoning.***

  As a class, come to some understanding of how the pathways differ.

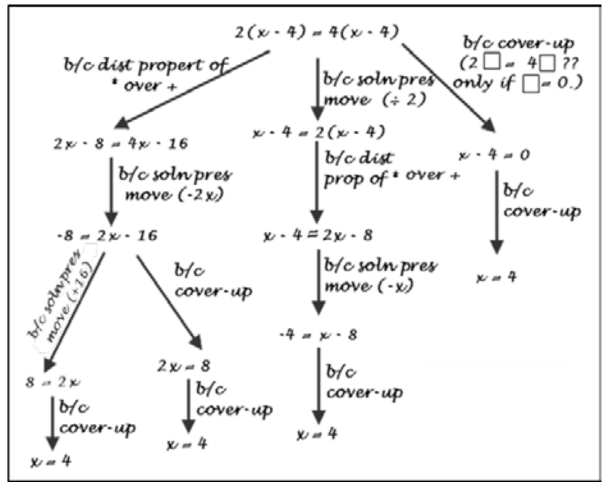
Answers will vary: The most "elegant" solution might be the first, which involves very little recording and a lot of reasoning; either of the other strategies could be considered efficient by the students.

Class Discussion (continued)

Teacher Tip: For the next question, you may want all students to do all three equations, but it might be useful to have several pairs do each of the equations, then have one pair share their pathway and note whether other pairs had any other possible pathways.

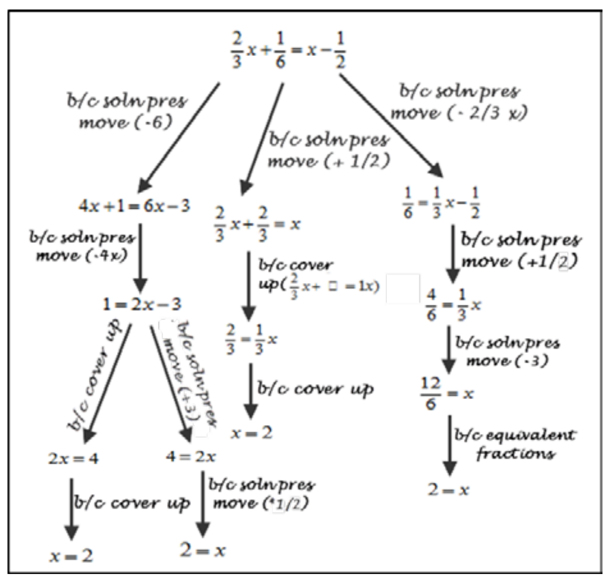
- ***Working with a partner, make a pathway chart for the solutions to Equation 3 in the menu.***

Equation 3 possible pathways:



- ***Working with a partner, make a pathway chart for the solutions to Equation 4 in the menu.***

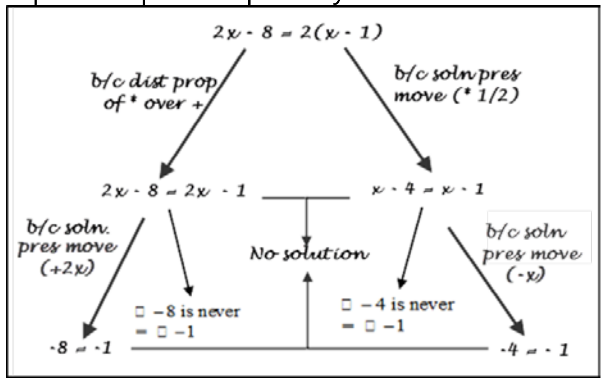
Equation 4 possible pathways:



Class Discussion (continued)

- ***Working with a partner, make a pathway chart for the solutions to Equation 5 in the menu.***

Equation 5 possible pathways:



- ***For each of the equations, which of the pathways seems most efficient to you? Why?***

Equation 3: Answers will vary: An efficient pathway might be Expand, Collect, then use the Highlight strategy.

Equation 4 can be solved using a variety of strategies. Students should contrast the pathway that begins by multiplying by 6 to eliminate the fractions with the pathway that begins by using Transform to add $-\frac{2}{3}x$ or $-x$ to both sides. The first allows students to use the cover-up strategy much earlier than the second, unless students can reason well with rational numbers.

Equation 5 leads to the statement that $-2 = 5$ or some other impossible situation, leading to the conclusion that this equation will not be true for any value of the variable. An efficient solution would be to recognize this as soon as the equation is expanded, because $2x + (\ \ ) = 2x + [\ \ ]$ cannot happen when ( ) and [ ] are not the same.

Class Discussion (continued)

- ***Did any of the pathways seem elegant to you? Explain your reasoning (say your "becauses").***

Equation 3 can be solved "elegantly" by recognizing that

$2(\ \ ) = 4(\ \ )$ only if ( ) = 0, so $x = 4$. Expanding and collecting in a variety of ways are possible pathways.

Equation 4: An elegant solution is to recognize that the relation between sum of two-thirds of *x* and a number and the other side with the sum of one *x* and a number. Adding $\frac{1}{2}$ to both sides results in the equation $\frac{2}{3}x + \frac{2}{3} = 1x$, and thus, reasoning that $\frac{2}{3}x + (\ \ )$ has to equal a whole *x*, ( ) is $\frac{1}{3}x$, so $\frac{2}{3} = \frac{1}{3}x$, and *x* is 2.

Equation 5: An elegant solution would be to recognize this as soon as the equation is expanded, because $2x + (\ \ ) = 2x + [\ \ ]$ cannot happen when ( ) and [ ] are not the same.

Student Activity Questions—Activity 1

1. When solving equations—or doing any mathematics—it helps to "look before you leap". For each of the following, think about which strategy would be an efficient and useful way to begin solving the equation. Explain the reasoning behind your choice of strategy.

a. $\mathbf{2(x-3) = 5(2x+1)+3}$

Answers may vary. Expand and Collect to get an equation of the form $ax + b = cx + d$ $(2x - 6 = 10x + 8)$ because the equation has no obvious structure to help my thinking; then Transform by adding $-2x$ to both sides.

b. $\mathbf{14 = 4 - 5x}$

Answers may vary. Highlight the 5*x* and find the missing addend because it makes finding the solution easy—what subtracted from 4 makes 14 and the answer is -10, and if so, *x* is -2.

c. $\mathbf{2x - 17 = 4x + 25}$

Answers may vary. Transform by adding $-2x$ or $-4x$ to both sides to get $-17 = 2x + 25$ because there is no clear way that thinking about structure can help. Then you can use the Highlight method or Transform using a –25.

Student Activity Questions—Activity 1 (continued)

d. $\mathbf{6x = 28 + 5x}$

Answers may vary. An elegant solution might be to Highlight the 28 and reason that 1*x* has to be 28 because $5x + 1x = 6x$. Note that this pushes back on a misconception that the Highlight method only works for equations of the form $ax + b = c$, while also illuminating the danger of trying to establish "rules" for when certain methods should be used.

e. $\mathbf{7 - 3x = 7 - 3(x - 2)}$

Answers may vary. By inspecting the structure, you can see that $7 - 3x$ will be on both sides, but there will be an additional constant on the right side, which cannot happen, so there is no solution.

f. $\mathbf{\frac{3}{8}x - 2 = 5 + \frac{1}{4}x}$

Answers may vary. Transform by multiplying by 8 to get $3x - 16 = 40 + 2x$, because that makes it easier to use the Highlight method at a later stage. Then, you can almost reason that $x = 56$ by noting that 3*x* is on one side and 2*x* on the other, so the difference would be 1*x* and then just think about the numbers.

2. **Why is it important to look for efficient and even elegant solutions?**

Answers will vary. One of the reasons is to save time to spend more time thinking about harder things. Another reason is that it reduces the chance of computational errors—i.e., not having to use the distributive property of multiplication over addition in a situation such as $21 = 5(x - 3) + 2(x - 3)$.

3. **Use New to generate equations to find two equations you think will be interesting to solve by considering "look before you leap". Solve them and be ready to share your thinking with the class.**

Answers will vary. Students might work in pairs and have their partner solve their equations. They could nominate ones they think are really good for "looking before you leap". This strategy also could be called "Contemplate, then Calculate" in case students like to use "big words". If it is possible to screen capture, students could share with the entire class, taking turns being the presenter.

4. **An *identity* is an equation where the right and left sides of the equation are equivalent expressions. Which of each pair is an identity? Explain your reasoning. (If necessary, think about your work on *Building Expressions in Two Variables*.)**

a. $\mathbf{4(x - 2) = 4x - 8}$ **or** $\mathbf{4(x - 2) = 4x - 6}$

Answer: $4(x - 2) = 4x - 8$ is an identity since by the distributive property of multiplication over addition/subtraction, $4(x - 2)$ is equivalent to $4x - 8$.

b. $\mathbf{2x + 3(x - 6) = 5x - 6}$ **or** $\mathbf{x + 4(x + 2) = 3x + 2(x + 4)}$

Answer: $x + 4(x + 2) = 3x + 2(x + 4)$ is an identity since by the distributive property of multiplication over addition and collecting like terms, $x + 4(x + 2)$ is equivalent to $3x + 2(x + 4)$.

Student Activity Questions—Activity 1 (continued)

c. $\mathbf{15x - 10 = 15x + 10}$ or $\mathbf{15x - 10 = 5(3x - 2)}$

Answer: $15x - 10 = 5(3x - 2)$ is an identity since by the distributive property of multiplication over addition, $15x - 10$ is equivalent to $5(3x - 2)$.

5. **Sort the following equations into three categories: identities, those that have no possible solution, and those that have exactly one solution. Explain how you decided which equation went to which category.**

a. $\mathbf{3x - 5 = x + 2(x - 3)}$ b. $\mathbf{5(x - 3) - 2(x - 3) = 0}$ c. $\mathbf{5x - (x - 3) = 3 + 4x}$

d. $\mathbf{3(x - 4) - x = 2(x - 6)}$ e. $\mathbf{7x + 2(1 - x) = 2x + 4(x - 1)}$ f. $\mathbf{2(x + 2) + x = 3(x - 6)}$

Answers will vary. Students may solve the equations, search for the number of *x*'s on each side or use other ways to reason. Identities: c) and d); no solution: a) and f); and one solution: b) and e).

6. **Generate a new equation you like, let *x* be a number, and then describe in words what the equation would be. For example, "I'm thinking of a number so that the product of 3 and 2 less than the number is 21" would produce the equation $\mathbf{3(x - 2) = 21}$. See if your partner can write and solve your equation.**

Answers will vary.

Deeper Dive — Page 1.3

Marta spent* x *dollars for lunch. Lee spent* a *times as much as Marta, and Songe spent* b *dollars more than Marta. Lee and Songe spent the same amount. Use* New *to find an equation that could be used to model the problem.

- ***What are your values for* a *and* b?**

  Answers will vary. One example is $4x = x + 6$, where *a* is 4 and *b* is 6.

- ***What is the solution to your equation?***

  Answers will vary. The solution to the example above is that Marta spent $2.00.

Create an equation where a good strategy to solve the equation will be to start with

a. ***the Highlight method***

b. ***Transforming using a solution preserving move***

c. ***an "elegant" way of reasoning***

Answers will vary.

Deeper Dive — Page 1.3 (continued)

Given the equation $3(x+4)=2x+5(x-1)$, change at most two of the numbers so

- ***the equation will become an identity.***

  Answers will vary. One possible answer is to change the 2 to a -2 and the 4 to a $-\frac{5}{3}$.

- ***the equation will not have any numbers as solution.***

  Answers will vary. One possible answer is to change the 2 to a -2, and the equation will have a 3*x* on both sides and different constants, so one side can never be equal to the other for any replacement of *x* and there is no possible solution.

- ***the solution will be $x=0$.***

  Answers will vary. One possible answer is to change the 4 to a $-\frac{5}{3}$.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Place a number from 1 to 9 in the blanks to create an equation that has no solution: $8x - 3x + 2 - x = \_\_\_ x + \_\_\_$

   (Adapted from Smarter Balance Mathematics Practice Test, Item 1865)

   Answer: the first blank has to be a 4 and the second blank can be any number not equal to 2.

2. Which equation has the same solution as $4 - 2(x-5) = x - 19$?

   a. $2(x+5) = -8$

   b. $3(x-3) = 9$

   c. $x + 2 = 2x - 3$

   d. $3x - 4 = 2x + 7$

   PARCC Math Spring Operational Grade 8 End of Year release item M21742

   Answer: d. $\boldsymbol{3x - 4 = 2x + 7}$

3. What value of x makes the equation $3(x-6) - 8x = 2 + 5(2x+1)$ true?

   PARCC Math Spring Operational Grade 8 End of Year release item M2017

   Answer: $\boldsymbol{-\frac{7}{5}}$

4. If $15 + 3x = 42$, then x =

   a. 9

   b. 11

   c. 12

   d. 14

   e. 19

   NAEP 2007, Grade 8

   Answer: a. 9

5. If a bar of soap balances $\frac{3}{4}$ of a bar of soap and $\frac{3}{4}$ of a pound, how much does the bar of soap weigh?

   CCSSM Expressions and Equations progressions

   Answer: The bar of soap weighs 3 pounds.

Student Activity Solutions

In these activities, you will identify equivalent expressions involving rational numbers. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. When solving equations—or doing any mathematics—it helps to "look before you leap". For each of the following, think about which strategy would be an efficient and useful way to begin solving the equation. Explain the reasoning behind your choice of strategy.

   a. $2(x-3)=5(2x+1)+3$

   Answers may vary. Expand and Collect to get an equation of the form $ax+b=cx+d$ $(2x-6=10x+8)$ because the equation has no obvious structure to help my thinking; then Transform by adding $-2x$ to both sides.

   b. $14=4-5x$

   Answers may vary. Highlight the 5x and find the missing addend because it makes finding the solution easy—what subtracted from 4 makes 14 and the answer is -10, and if so, x is -2.

   c. $2x-17=4x+25$

   Answers may vary. Transform by adding $-2x$ or $-4x$ to both sides to get $-17=2x+25$ because there is no clear way that thinking about structure can help. Then, you can use the Highlight method or Transform using a –25.

   d. $6x=28+5x$

   Answers may vary. An elegant solution might be to highlight the 28 and reason that 1x has to be 28 because $5x+1x=6x$. Note that this pushes back on a misconception that the Highlight method only works for equations of the form $ax+b=c$, while also illuminating the danger of trying to establish "rules" for when certain methods should be used.

   e. $7-3x=7-3(x-2)$

   Answers may vary. By inspecting the structure, you can see that $7-3x$ will be on both sides, but there will be an additional constant on the right side, which cannot happen, so there is no solution.

   f. $\frac{3}{8}x-2=5+\frac{1}{4}x$

   Answers may vary. Transform by multiplying by 8 to get $3x-16=40+2x$, because that makes it easier to use the Highlight method at a later stage. Then, you can almost reason that $x=56$ by noting that 3x is on one side and 2x on the other, so the difference would be 1x, and then just think about the numbers.

2. Why is it important to look for efficient and even elegant solutions?

   Answers will vary. One of the reasons is to save time to spend more time thinking about harder things. Another reason is that it reduces the chance of computational errors—i.e., not having to use the distributive property of multiplication over addition in a situation such as $21=5(x-3)+2(x-3)$.

3. Use New to generate equations to find two equations you think will be interesting to solve by considering "look before you leap". Solve them and be ready to share your thinking with the class.

   Answers will vary. Students might work in pairs and have their partner solve their equations. They could nominate ones they think are really good for "looking before you leap". This strategy also could be called "Contemplate, then Calculate" in case students like to use "big words". If it is possible to screen capture, students could share with the entire class, taking turns being the presenter.

4. An *identity* is an equation where the right and left sides of the equation are equivalent expressions. Which of each pair is an identity? Explain your reasoning. (If necessary, think about your work on *Building Expressions in Two Variables*.)

   a. $4(x-2)=4x-8$ or $4(x-2)=4x-6$

      Answer: $4(x-2)=4x-8$ is an identity since by the distributive property of multiplication over addition/subtraction, $4(x-2)$ is equivalent to $4x-8$.

   b. $2x+3(x-6)=5x-6$ or $x+4(x+2)=3x+2(x+4)$

      Answer: $x+4(x+2)=3x+2(x+4)$ is an identity since by the distributive property of multiplication over addition and collecting like terms, $x+4(x+2)$ is equivalent to $3x+2(x+4)$.

   c. $15x-10=15x+10$ or $15x-10=5(3x-2)$

      Answer: $15x-10=5(3x-2)$ is an identity since by the distributive property of multiplication over addition $15x-10$ is equivalent to $5(3x-2)$.

5. Sort the following equations into three categories: identities, those that have no possible solution, and those that have exactly one solution. Explain how you decided which equation went to which category.

   a. $3x-5=x+2(x-3)$ b. $5(x-3)-2(x-3)=0$ c. $5x-(x-3)=3+4x$

   d. $3(x-4)-x=2(x-6)$ e. $7x+2(1-x)=2x+4(x-1)$ f. $2(x+2)+x=3(x-6)$

   Answers will vary. Students may solve the equations, search for the number of x's on each side, or use other ways to reason. Identities: c) and d); no solution: a) and f); and one solution: b) and e).

6. Generate a new equation you like, let *x* be a number, and then describe in words what the equation would be. For example, "I'm thinking of a number so that the product of 3 and 2 less than the number is 21" would produce the equation $3(x-2)=21$. See if your partner can write and solve your equation.

   Answers will vary.

