Building Concepts: Using Structure to Solve Equations

TEACHER NOTES

Lesson Overview

In this TI-Nspire lesson, students reason about the structure of expressions on either side of an equation as a way to relate the equation to the order of arithmetic operations and use this method to solve equations.

Visualizing the arithmetic structure of expressions in an equation can help in thinking about how to find the solution for the equation—a value or values that will make the equation true.

Learning Goals

1. Recognize equations whose structure is $a(\_\_)=b$ or $(\_\_)+a=c$ where $(\_\_\_)$ involves an unknown, *x*;
2. recognize that solving an equation of the form $ax=b$ or $x+a=c$ can be viewed as looking for a missing factor or missing addend, respectively;
3. use arithmetic structure to find solutions to equations that can be converted to the form $ax=b$ or $x+a=c$.

Prerequisite Knowledge

Using Structure to Solve Equations is the sixth lesson in a series of lessons that explore the concepts of expressions and equations. In this lesson students reason about the structure of expressions on either side of an equation. This lesson builds on the concepts of the previous lessons. Prior to working on this lesson students should have completed *What is an Equation?* and *Equations and Operations*. Students should understand:

- the associative and commutative properties of addition and multiplication;
- how to associate an addition problem with a related subtraction problem and a multiplication problem with a related division problem.

Vocabulary

- **expression:** a phrase that represents a mathematical or real-world situation
- **equation:** a statement in which two expressions are equal
- **solution:** a number that makes the equation true when substituted for the variable
- **factor:** a number that when multiplied with another results in a product
- **addend:** a number that is added to another number

Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

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Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Using Structure to Solve Equations_Student.pdf
- Using Structure to Solve Equations_Student.doc
- Using Structure to Solve Equations.tns
- Using Structure to Solve Equations_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided for additional student practice and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

Lesson 5, *Equations and Operations,* focused on "solution-preserving moves", operations that can be performed on equations such that any solution to the original equation will be a solution for the new equation. This lesson extends the strategies students might use to solve an equation of the form $x+b=c$ or $c=ax$. The lesson emphasizes how to reason about the structure of the expressions on either side of the equation as a way to relate the equation to arithmetic operations. Solving the equation $x+b=c$ can be thought of as finding a missing addend and solving $c=ax$ as finding the missing factor. For example, consider the equation $5(x+4)=45$. $5(\_\_)=45$ can be thought of as "what times 5 makes 45?". The response would indicate that the missing factor (____) has to have the value 9. This reduces the problem to $(\_\_)=9$ or substituting the sub-expression, $x+4$, what number added to 4 makes 9. The missing addend is 5, which leads to the conclusion that the value for *x* that makes the equation true is 5.

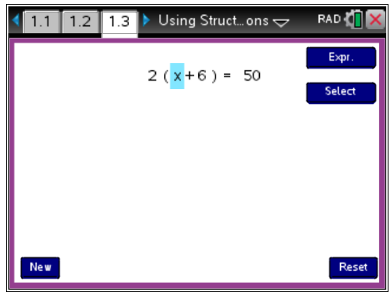
This approach can be related to the mathematical strategy of substitution—where the structural form of an expression may be simplified by viewing the expression in terms of a new variable or sub-expression. The expressions might be thought of as the results of composition of functions, and the sub-expressions are the "inner" functions involved in the composition. In formal language, if $f(x)=x+4$ and $g(x)=5x$, consider what value of *x* would make $g(f(x))=45$. Note this language is not part of this lesson.

Engaging students in thinking about the arithmetic structure of algebraic expressions can support their understanding of the process of solving equations as well as help them develop computational fluency. The process in the lesson involves visually checking whether a value satisfying the sub-expression equation satisfies the original equation.

The TNS activity and lesson can be used in different grades. The equations on page 1.3 in the TNS activity do not require knowledge of negative integers and Parts 1 and 2 of the lesson are written for students without that knowledge. The equations on page 1.5 and the questions in Part 3 do require knowledge of computation with positive and negative numbers.

Part 1, Page 1.3

Focus: Use the arithmetic structure of an expression in an equation to think about the arithmetic that is necessary to find a solution to the equation.



Page 1.3 shows equations of various forms.

Expr highlights part of the expression on one side of the equation.

Select displays that expression with a ?

Use the keypad to fill in the blank with the appropriate value.

Submit checks a value.

Del deletes a value in the blank.

Undo goes back to the previous step or returns to the original problem.

menu> Equations offers a choice of predetermined equations.

New generates random equations.

TI-Nspire Technology Tips

menu accesses page options.

tab cycles through onscreen buttons.

Right/Left arrow keys highlight parts of the expression.

del clears an entered value.

ctrl del resets the page.

Class Discussion

Teacher Tip: You may want to revise the directions depending on your students. You may want them to work in pairs, and after they have solved several using the “cover-up” method, do a quick check to see if they understand the process by having them solve an equation such as $3(x+5)=21$.

When you solve an equation, your task is to find the values from a specified set, if any, that make the equation true. A very useful strategy in all of mathematics is to solve a simpler problem with which you are comfortable and that is related to the original problem. By now, you should be comfortable with order of arithmetic operations.

- ***Given a value x, identify the operations in the left side of the equation*** $2(x+6)=50$ ***and the order in which they would be performed. According to the order of operations, what is the last operation you would perform on the left side of the equation?***

Answer: For the equation $2(x+6)=50$, multiplying by 2 and adding 6 are the two operations. Order of operations would require that you first add 6 to *x*, and then multiply by 2. The last operation would be multiplying by 2.

Class Discussion (continued)

- ***Use* Expr *to highlight or "cover" the expression $x+6$. Thinking about the arithmetic, what value would the highlighted expression have to be? Choose* Select*, type in your answer and* Submit*. Were you correct?***

  Answer: The highlighted expression would be $x+6$, so the arithmetic would be 2 times what number makes 50. The number would have to be 25.

- ***Write a question describing the arithmetic that is left to think about.***

  Answer: What do I add to 6 to get 25?

- ***Be sure the missing addend is highlighted.* Select *the addend and* Submit *a value that you think will make the equation true. How can you tell if you were right?***

  Answer: 19 is correct because when it is substituted for the *x* in the original equation, the equation is a true statement, $2(19+6)=50$.

Select* menu> Equations> Problem 2*.

- ***How is this equation different from the equation in the question above?***

  Answer: The parentheses are missing, which changes the order of the operations.

- ***Given a value of x, identify the operations in the left side of the equation $2x+6=50$ and the order in which they would be performed. According to the order of operations, what is the last calculation you would perform on the left side of the equation?***

  Answer: For the equation $2x+6=50$, the operations are adding 6 and multiplying by 2, and the last operation would be adding the 6.

- **Select *the expression $2x$. What value would the product have to be?* Submit *your answer to see if you are correct.***

  Answer: You would highlight 2*x,* and the value that makes that statement true is 44.

- ***What would the unknown factor in the multiplication problem have to be? How do you know that this factor is the solution to the equation?***

  Answer: $x=22$ is the solution because

  $$2(22)+6=50$$
  $$44+6=50$$
  $$50=50$$

Class Discussion (continued)

***Select* menu> Equations> Problem 3.**

- ***If you substitute a value for x, what is the last operation you would do to evaluate the right side of the equation? Use* Expr *or the arrow keys to choose the expression on which you would perform that calculation and* Select *or* Enter*. What arithmetic question could you ask yourself about the highlighted expression in terms of the equation? How would you answer your question?***

  Answer: The last operation would be to divide by 3, so the expression you would divide by 3 would be $2x$. What number divided by 3 makes 6 is a question I could ask. The number is 18.

- ***Fill in the answers in the TNS activity to finish the problem. What value for x makes the equation true? How do you know you are right?***

  Answer: $x = 9$ because $\frac{2(9)}{3} = \frac{18}{3} = 6$

- ***Select* Undo*. Suppose you highlighted the $\frac{x}{3}$ in the problem? What question could you ask about the arithmetic?***

  Answer: I could ask what number times 2 equals 6. The answer would be 3.

- ***What would be your next question? And what would be your solution?***

  Answer: The question would be what number divided by 3 equals 3, and the answer and solution is 9.

For each of the following items, tell whether the suggested expression would be a good first expression to highlight in the equation to help you find a value for x that makes the equation true? Explain your thinking. If the given suggestion does not seem like a good first expression, suggest an alternate expression to highlight.

- ***If the equation is $(4x-1)+10=49$, highlight the $(4x-1)$.***

  Answer: That will work, as the arithmetic will be finding what number added to 10 makes 49.

- ***If the equation is $15=5(x-4)$, highlight the x.***

  Answer: If you highlight the *x*, you do not have a clean arithmetic problem; if you highlight the $(x-4)$ the arithmetic becomes what multiplied by 5 makes 15.

- ***If the equation is $17=5x+2$, highlight the 5x.***

  Answer: Yes, because the arithmetic will be what is added to 2 to get 15.

Class Discussion (continued)

- ***If the equation is $\frac{1}{2}x+7=21$, highlight the 7.***

Answer: If 7 is highlighted, the arithmetic will be what added to $\frac{1}{2}x$ will be 21. Not knowing the value of *x*, it is hard to know what value to add. It will be easier if you highlight the $\frac{1}{2}x$, and then the arithmetic will be what added to 7 makes 21.

Reset ***to return to the first equation you solved***.

- ***You found $x=19$. Is this the only value for x that will make the equation true? How can you tell?***

Answers may vary. If *x* is even a bit larger than 19, $x+4$ will be larger than 25, and twice a number larger than 25 will be larger than 50. So no number larger than 19 can make the equation true. The same reasoning will show that no number smaller than 19 will make the equation true. The only value for *x* that makes the equation true is 19. Note that the same kind of reasoning was used in the Lesson *What is an Equation*?

- ***Li solved the equation $3(x-9)=39$ and got the solution $x=22$. Do you agree with Li? Explain why or why not.***

Answer: Li is correct because

$$3(22-9)=39$$
$$3(13)=39,$$
$$39=39$$

so 22 makes a true statement.

- ***Aliana disagreed with Li because she said the solution to the equation $2x+6=50$ was 22. What would you say to Aliana? Give an example to support your thinking.***

Answer: Aliana is incorrect because lots of equations can have the same solution. For example, $x+2=24$ and $2x=44$ and $x-1=21$ all have the same solution, $x=22$

Student Activity Questions—Activity 1

1. Tammy generated the equation $29=2(x-11)-3$.

a. She reasoned that if she highlighted $2(x-11)$ it would be like figuring out what number minus 3 is 29. Do you agree or disagree with Tammy? Explain your thinking.

Answer: I agree because it is like $29=\_\_-3$

Student Activity Questions—Activity 1 (continued)

b. **What equation would her reasoning produce? What arithmetic question could she then ask?**

Answers may vary. The equation would be $2(x-11)=32$. The question could be *What number times 2 makes 32?*

c. **Tim followed Tammy's advice and ended up with the equation $x-11=3$. What would you say to Tim?**

Answers may vary. Tim made an error. You can't just take a 2 off both sides; one 2 is the units digit in 32 and the other is a factor in the product $2(x-11)$).

d. **Tammy ended up with $x-11=16$. What would you say to Tammy?**

Answer: She is correct because $2(16)=32$. Then she could see that *x* has to be 27 because you are looking for a number that when you subtract 11 you get 16.

2. **Suppose you generated the equation $36=4(x-3)+8$.**

a. **Think about highlighting different parts of the expression on the right in the equation above. Which helps you think about an easier arithmetic problem?**

Answer: You can highlight the *x*, the $x-3$, the $(x-3)$ or the $4(x-3)$. Highlighting $4(x-3)$ first makes the arithmetic easiest.

b. **Use the highlight method to find a value for *x* that makes the equation true.**

Answer: $28=4(x-3)$; $7=(x-3)$; $10=x$, which is the value that makes the equation true.

3. **Work with a partner. Use *New* or *menu> Equations> New* to generate four different equations of the form $ax=b$; $ax+b=c$; $\frac{ax}{b}=c$; or any of those forms with the constant term on the left side of the equals sign. Find the value of *x* that makes the equation true for each equation. Write down the equations and how you found the value for *x*. Be ready to share your equations and solutions with the class.**

Answers will vary. For the equation $40=\frac{5x}{6}$ the value that makes the equation true is 48. First equation that I used was $40=5(\_)$ which meant that $\frac{1}{6}(x)=8$ which yielded $x=48$

4. **Select *New* or *menu> Equations> New* (Depending on your teacher's instructions, you may want to choose equations of the same form as those in problem 3.).**

a. **Find a value for *x* that makes the equation true using the highlight method.**

Answers will vary. For the equation $39=6x+3$ the value for *x* that makes the equation true is 6.

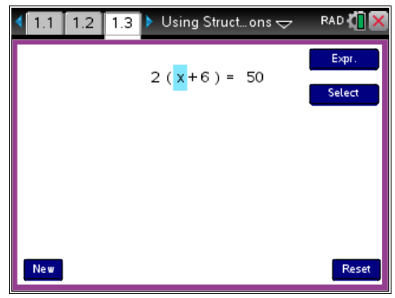
Student Activity Questions—Activity 1 (continued)

b. Write an explanation for someone who was absent from class explaining how you can use the file to find a value for *x* that makes your equation true. Share your explanation with a partner to see if they agree.

Answers will vary. For the above equation, I highlighted the term 6*x* and asked the question, “what plus 3 makes 39?” that meant that 6*x* had to be 36, and “what multiplied by 6 makes 36?” means that $x = 6$. Which is true since $39 = 6(6) + 3$.

Part 2, Page 1.3

Focus: Use the “cover up” or highlight method to find solutions to problems in context.



Class Discussion

The questions in this section involve contextual situations in which students relate an equation to the context and find the solution for the equation.

Have students…	**Look for/Listen for…**
Write an equation that fits the story, then find the solution using the highlight method. Check your answer with a partner.	
• ***How much did Sammy earn a day as a waiter if he gave $3 each day to the cook, worked five days a week and took home a total of $120 a week?***	Answer: If *x* is the amount Sammy earned each day, the equation would be $5(x-3) = 120$. Sammy earned $27 a day.
• ***Find the early morning temperature if the temperature had doubled by noon and then dropped 11º to 59º by dinner time.***	Answer: If *x* is the early morning temperature, the equation would be $2x - 11 = 59$. The early morning temperature was 35º.

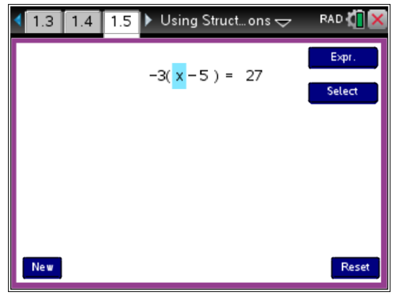
Student Activity Question—Activity 2

1. **Generate a new equation. Write a story for your equation. Then exchange your story with a partner and solve each other’s stories.**

   Answers will vary. For the equation $42 = 12 + 10(x-1)$ the story could be “Teagan spent $42 on his school pictures. The first sheet cost 12 dollars and every sheet after was $10, how sheets of pictures did he buy?”

Part 3, Page 1.5

Focus: Use the arithmetic structure of an expression to find a solution to an equation that can be converted to the form $ax = b$, $a + x = b$, or $ax + b = c$, where *a*, *b*, and *c* are rational numbers.



Page 1.5 functions in the same way as page 1.3, with a set of common equations in **menu> Equations**. The equations involve positive and negative numbers.

Class Discussion

Have students… | **Look for/Listen for…**

Remember the rules for finding the sums and products of negative numbers.

- ***What would you highlight in the equation on page 1.5? What question would this lead to about the arithmetic?***

Answer: $(x-5)$ so the question would be what number times -3 is a positive 27. The answer would be -9.

- ***Highlight and Select the next sub-expression. What question about the arithmetic would you have?***

Answer: What can you take 5 from to get -9? The answer would be -4.

Use* menu> Equations> Problem 2*.

- ***Describe what you can highlight to find the solution.***

Answers may vary. One approach is to highlight $-1(x-5)$ to consider what you divide by 3 to get -11. The answer would be -33. Then highlight the $x-5$, and the question would be what times -1 will be -33, and the answer is 33. Highlighting *x* leads to what number minus 5 leads to 33, and the answer and solution to the equation is 38.

- ***Explain why your solution is correct.***

Answer: 38 is correct because $\frac{-1(38-5)}{3} = -\frac{33}{3} = -11$.

Select* menu> Equations> Problem 3*.

- ***Stu highlighted the $(x-5)$ first. Do you think this is a good thing to highlight first? Why or why not?***

Answer: If you highlight the $(x-5)$ there are too many arithmetic questions left; like dividing by 3 and figuring out how to get -11. It will be hard to tell without a lot of thinking what you need to get when dividing.

Class Discussion (continued)

- ***What would you suggest as a good expression to highlight? Why?***

  Answer: Highlight $3(x-5)$ then you can ask what number do you need so that when you add 28, you get -11. It would be -39.

- ***Continue to highlight parts of the expression until you find the solution.***

  Answer: $x = 18$

- ***What operations will you perform with positive and negative numbers if you check the value you found for x?***

  Answer: Adding a positive and negative number and multiplying a positive and negative number.

Student Activity Questions—Activity 3

> **Teacher Tip:** The following sorting task is intended to engage students in thinking more generally about the process of solving an equation with a variable on one side.

1. Use *New* or *menu> Equations> New* to generate six equations.

a. Write down each equation and its solution. Work with a partner to separate the twelve equations you have into two to four groups according to some criteria that makes sense to you. Generate two more equations and see in which of your groups they could be placed. Be ready to share your thinking with the class.

Answers will vary. For example, one grouping might be where all of the numbers are even vs all odd vs a mix of even and odd:

All Even	All Odd	Mix of Even and Odd
$2x-6=2$	$9(x+7)=9$	$-26=2(x-5)+8$
$78=8x-2$	$5(x-1)=-45$	$-7=5(x-1)+8$
$6x-8=-2$	$\frac{3(x-9)}{7}=-3$	$5x-5=30$
	$9=9(x+5)$	$-11=\frac{2(x-6)}{3}$
		$-7=\frac{(x+6)}{10}$

Student Activity Questions—Activity 3 (continued)

Another example, grouping by forms $ax+b=c$, $a(x+b)=c$ and $a(x+b)+c=d$, might give:

Group 1:	Group 2	Group 3
$2x-6=2$	$9=9(x+5)$	$-26=2(x-5)+8$
$78=8x-2$	$5(x-1)=-45$	$-7=5(x-1)+8$
$5x-5=30$	$\frac{3(x-9)}{7}=-3$	
$6x-8=-2$	$-7=\frac{(x+6)}{10}$	
	$9(x+7)=9$	
	$-11=\frac{2(x-6)}{3}$	

Others might group those that had fractions separately. Some might group by those that have a negative number on one side of the equal sign vs those that have a positive number on one side.

b. Do you notice anything about the equations in any of your groups? About their solutions?

Answers will vary. Some might notice that if all of the numbers are even you can divide all of them by 2 and have a simpler equation to solve. Some might notice that having a common factor in general allows you to divide by that factor and make all of the values smaller numbers.

Deeper Dive

Can you solve the following equations using the highlight method? Why or why not?

- $\boldsymbol{5(3x+1)-11=39}$

Answer: Highlight $5(3x+1)$ to have $(5(3x+1))-11=39$, so $(5(3x+1))$ equals 50. Then highlight $5(3x+1)=50$ so $(3x+1)=10$ Highlight $(3x)+1=10$, so $3x=9$ and $x=3$.

- $\boldsymbol{2x+3=4x-5}$

Answer: This one will be difficult to solve using the highlight method because nothing that you can highlight will make the arithmetic easier.

- $\boldsymbol{5(x-4)+3(x-4)=8}$

Answer: Yes, if you highlight the $(x-4)$, you can see that you have $8(x-4)$ has to be 8, so the $(x-4)$ has to be 1. From here you can figure out the value for *x* that will make the equation true.

Deeper Dive (continued)

- ***$2x + 3x + 5 = 2x + 17$***

Answer: If you highlight the 2x on both sides, 2x $+ 3x + 5 =$ 2x + 17, you can tell that 3x in $3x + 5 = 17$ has to equal 17, then highlight the 3x in 3x $+5 = 17$ to figure out that $3x = 12$, then $x = 4$.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Which of the following equations has the same solution as the equation $2x+6=32$?

   a. $2x=38$

   b. $2x=26$

   c. $x+6=16$

   d. $2(x-3)=16$

   e. $2(x+3)=32$

   Adapted from NAEP 2009, Grade 8 Question ID: 2009-8M10 #1 M142001

   Answer: b. $2x=26$ and e. $2(x+3)=32$

2. Which of the following is double the solution for $18=\left(\frac{2}{3}\right)x$?

   a. 9 b. 24 c. 2 d. 54

   Answer: d. 54

3. Which of the following equations will have the largest number as a solution?

   a. $1.5x=30$ b. $x+1.5=30$ c. $0.5x=30$ d. $x-1.5=30$

   Answer: c. $0.5x=30$

4. Sophie is helping her band collect money to fund a field trip. The band decided to sell boxes of chocolate bars. Each bar sells for $1, and each box contains 30 bars. The band collected $1530 from chocolate bar sales. How many boxes did they sell?

   Adapted from Illustrative Mathematics 6 EE Chocolate Bar Sales

   Answer: 51 boxes

5. Stephanie bought a package of pencils for $1.75 and some erasers that cost $0.25 each. She paid a total of $4.25 for these items, before tax. Exactly, how many erasers did Stephanie buy?

   PARRC released item VF823888

   Answer: 10 erasers

6. The solution to the equation $-3x - 29 = -5$ will also be a solution to which of the following equations?

   a. $3x = -24$

   b. $-3x = -24$

   c. $3x - 30 = -6$

   d. $-3x - 30 = -6$

   Answer: a. $3x = -24$ and d. $-3x - 30 = -6$

7. Tom spent $14.50 bowling. He rented shoes for $2.50 and bowled 3 games. How much did it cost to bowl one game, if each game cost the same amount?

   Answer: $4.00

8. Mark spent $30 for 4 sandwiches and 4 bottles of water. Each sandwich cost the same, and a bottle of water cost $2. The equation $4(x+2) = 30$ can be used to model what Mark spent.

   a. Mark solved the equation as follows:

   $$\begin{aligned} 4(x+2) &= 30 \\ 4x + 2 &= 30 \\ 4x &= 28 \\ x &= 7 \end{aligned}$$

   so each sandwich cost $7.

   Did Mark solve the equation correctly? If so, justify each step of his work. If not, describe his error and explain what he should have done.

   Answer: Mark did not solve the equation correctly because he should have considered what number times 4 will be 30. Or he should have distributed the 4 to both the x and the 2 to begin the problem.

   b. Solve the equation.

   Answer: $4(x+2) = 30$; so $x + 2 = 7.5$, so $x = 5.5$. Each sandwich cost $5.50.

9. A student usually saves $20 a month. He wants to save $372 in a year. He writes the equation $12(20 + x) = 372$ to determine the additional amount he must save to reach his goal.

   a. Solve the equation.

   Answer: $x = \$11$

   b. Explain what the value for *x* represents in the context of the problem.

   Answer: He needs to save $11 more each month or a total of $31 to reach his goal.

Questions 7–9 adapted from PARCC

https://prc.parcconline.org/system/files/7th%20grade%20Math%20-%20PBA%20-%20Item%20Set.pdf

10. Entry to the fairgrounds is \$4.50. Each ticket for a ride is \$0.50.

    a. If Heidi spent \$12, write an equation showing how many tickets she bought.

    Answer: $\boldsymbol{12 = 4.50 + 0.5t}$ ***, where t is the number of tickets Heidi bought.***

    b. Solve your equation.

    Answer: $\boldsymbol{7.50 = .5t}$ ***,*** $\boldsymbol{t = 15}$ ***. Heidi bought 15 tickets.***

11. Sheldon solves the equation $\left(\frac{1}{2}\right)(x+6=7)$.

    a. Identify each incorrect step she makes.

    1. $\left(\frac{1}{2}\right)x + 6 = 7$
    2. $\left(\frac{1}{2}\right)x = 7 + 6$
    3. $\left(\frac{1}{2}\right)x = 13$
    4. $x = \frac{13}{2}$
    5. $x = 6\frac{1}{2}$

    Answer: Step 1 is wrong because you need to think $\boldsymbol{\frac{1}{2}}$ ***of what makes 7 or distribute the*** $\boldsymbol{\frac{1}{2}}$***; the answer is not 13. Step 2 is wrong because you have to think what added to 6 makes 7; the answer is 1 not 13. Step 4 is wrong because you have to think*** $\boldsymbol{\frac{1}{2}}$ ***of what is 13 and the answer is 26.***

    b. Solve the equation.

    Answer: $\boldsymbol{x = 8}$

Questions 10 and 11 Adapted from http://www.smarterbalanced.org/wp-content/uploads/2015/11/G7_Practice_Test_Scoring_Guide_Math.pdf

Building Concepts: Using Structure to Solve Equations

TEACHER NOTES

Student Activity Solutions

In these activities you will use arithmetic structure to find solutions to equations that can be converted to the form $ax = b$ or $x + a = c$. After completing the activities, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. Tammy generated the equation $29 = 2(x - 11) - 3$.

    a. She reasoned that if she highlighted $2(x - 11)$ it would be like figuring out what number minus 3 is 29. Do you agree or disagree with Tammy? Explain your thinking.

    Answer: I agree because it is like $29 = \_\_ - 3$

    b. What equation would her reasoning produce? What arithmetic question could she then ask?

    Answers may vary. The equation would be $2(x - 11) = 32$. The question could be What number times 2 makes 32?

    c. Tim followed Tammy's advice and ended up with the equation $x - 11 = 3$. What would you say to Tim?

    Answers may vary. Tim made an error. You can't just take a 2 off both sides; one 2 is the units digit in 32 and the other is a factor in the product $2(x - 11)$).

    d. Tammy ended up with $x - 11 = 16$. What would you say to Tammy?

    Answer: She is correct because $2(16) = 32$. Then she could see that x has to be 27 because you are looking for a number that when you subtract 11 you get 16.

2. Suppose you generated the equation $36 = 4(x - 3) + 8$.

    a. Think about highlighting different parts of the expression on the right in the equation above. Which helps you think about an easier arithmetic problem?

    Answer: You can highlight the x, the $x - 3$, the $(x - 3)$ or the $4(x - 3)$. Highlighting $4(x - 3)$ first makes the arithmetic easiest.

    b. Use the highlight method to find a value for *x* that makes the equation true.

    Answer: $28 = 4(x - 3)$; $7 = (x - 3)$; $10 = x$, which is the value that makes the equation true.

3. Work with a partner. Use **New** or **menu> Equations> New** to generate four different equations of the form $ax = b$; $ax + b = c$; $\frac{ax}{b} = c$; or any of those forms with the constant term on the left side of the equals sign. Find the value of *x* that makes the equation true for each equation. Write down the equations and how you found the value for *x*. Be ready to share your equations and solutions with the class.

   Answers will vary. For the equation $40 = \frac{5x}{6}$ *the value that makes the equation true is 48. First equation that I used was* $40 = 5(\_)$ *which meant that* $\frac{1}{6}(x) = 8$ *which yielded* $x = 48$

4. Select **New** or **menu> Equations> New** (Depending on your teacher's instructions, you may want to choose equations of the same form as those in problem 3.)

   a. Find a value for *x* that makes the equation true using the highlight method.

   b. Write an explanation for someone who was absent from class explaining how you can use the file to find a value for *x* that makes your equation true. Share your explanation with a partner to see if they agree.

   Answers will vary. For the above equation, I highlighted the term 6x and asked the question, "what plus 3 makes 39?" that meant that 6x had to be 36, and "what multiplied by 6 makes 36?" means that $x = 6$ *. Which is true since* $39 = 6(6) + 3$ *.*

Activity 2 [Page 1.3]

1. Generate a new equation. Write a story for your equation. Then exchange your story with a partner and solve each other's stories.

   Answers will vary. For the equation $42 = 12 + 10(x - 1)$ *the story could be "Teagan spent $42 on his school pictures. The first sheet cost 12 dollars and every sheet after was $10, how sheets of pictures did he buy?"*

Activity 3 [Page 1.5]

1. Use **New** or **menu> Equations> New** to generate six equations.

   a. Write down each equation and its solution. Work with a partner to separate the twelve equations you have into two to four groups according to some criteria that makes sense to you. Generate two more equations and see in which of your groups they could be placed. Be ready to share your thinking with the class.

   Answers will vary. For example, one grouping might be where all of the numbers are even vs all odd vs a mix of even and odd:

All Even	*All Odd*	*Mix of Even and Odd*
$2x-6=2$	$9(x+7)=9$	$-26=2(x-5)+8$
$78=8x-2$	$5(x-1)=-45$	$-7=5(x-1)+8$
$6x-8=-2$	$\frac{3(x-9)}{7}=-3$	$5x-5=30$
	$9=9(x+5)$	$-11=\frac{2(x-6)}{3}$
		$-7=\frac{(x+6)}{10}$

   Another example, grouping by forms $ax+b=c$, $a(x+b)=c$ *and* $a(x+b)+c=d$, *might give:*

Group 1:	*Group 2*	*Group 3*
$2x-6=2$	$9=9(x+5)$	$-26=2(x-5)+8$
$78=8x-2$	$5(x-1)=-45$	$-7=5(x-1)+8$
$5x-5=30$	$\frac{3(x-9)}{7}=-3$	
$6x-8=-2$	$-7=\frac{(x+6)}{10}$	
	$9(x+7)=9$	
	$-11=\frac{2(x-6)}{3}$	

   Others might group those that had fractions separately. Some might group by those that have a negative number on one side of the equal sign vs those that have a positive number on one side.

b. Do you notice anything about the equations in any of your groups? About their solutions?

Answers will vary. Some might notice that if all of the numbers are even you can divide all of them by 2 and have a simpler equation to solve. Some might notice that having a common factor in general allows you to divide by that factor and make all of the values smaller numbers.