Building Concepts: Dividing a Fraction by a Fraction

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson can be used to develop an understanding of dividing a fraction, $\frac{a}{b}$, by the fraction $\frac{c}{b}$. The approach is based on reasoning about fractions that have a common denominator. Suppose $\frac{a}{b}$ is divided by the fraction $\frac{c}{b}$. If two fractions have a common unit $\left(\frac{1}{b}\right)$, then the fraction $\left(\frac{a}{b}\right)\Big/\left(\frac{c}{b}\right)$ is the same as $a\left(\frac{1}{b}\right)\Big/c\left(\frac{1}{b}\right)$. This may also be written as $\frac{a}{c}$ times $1\left(\frac{1}{b}\right)\Big/1\left(\frac{1}{b}\right)$ which is equivalent to $\frac{a}{c}$ times 1; or $\frac{a}{c}$.

One way to think about *a* divided by *b* is to think about how many *b*'s are contained in *a*.

Learning Goals

Students should understand and be able to explain each of the following:

1. Dividing a fraction by a fraction can be done by finding common denominators for the fractions;
2. How to identify and interpret problems involving division of fractions, in particular that division of ***a*** by ***b*** can be thought of **a**'s the number of ***b***'s contained in ***a*** (i.e., $\frac{3}{2}$ divided by $\frac{1}{2}$ can be thought of as the number of $\frac{1}{2}$s contained in $\frac{3}{2}$);
3. Division of fractions can also be thought of as looking for a missing factor. Students use the relationship between multiplication and division to explain that a division problem can be rewritten as a multiplication problem (i.e., $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$ because $\frac{3}{4} \times \frac{8}{9} = \frac{2}{3}$. In general, $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$.)

Prerequisite Knowledge

Students should have had experience with the lessons *What is a Fraction?*, *Equivalent Fractions, Creating Equivalent Fractions* and *Mixed Numbers*. *Dividing a Fraction by a Fraction* is built on the concepts presented in the previous activities *Dividing a Fraction by a Whole Number* and *Dividing a Whole Number by a Fraction*. Encourage students to compare this generalized strategy for division of fractions to the strategies they used in the first two lessons. In particular, have them consider whether this strategy works for each of the two earlier cases. Many of the suggested questions involve improper fractions, which could be restated using mixed numbers. Prior to working on this lesson students should understand:

- how to restate improper fractions as mixed numbers.
- the relationship between multiplication and division.
- how to find the common denominator in a pair of fractions.

Vocabulary

- **identity element of multiplication:** number 1; if you multiply a number by the identity element, the result is that original number

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Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Dividing a Fraction by a Fraction_Student.pdf
- Dividing a Fraction by a Fraction_Student.doc
- Dividing a Fraction by a Fraction.tns
- Dividing a Fraction by a Fraction_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

Mathematical Background

This TI-Nspire™ lesson can be used to develop an understanding of dividing a fraction, $\frac{a}{b}$, by the fraction $\frac{c}{b}$. The approach is based on reasoning about fractions that have a common denominator. Suppose $\frac{a}{b}$ is divided by the fraction $\frac{c}{b}$. If two fractions have a common unit $\left(\frac{1}{b}\right)$, then the fraction $\left(\frac{a}{b}\right)\Big/\left(\frac{c}{b}\right)$ is the same as $a\left(\frac{1}{b}\right)\Big/c\left(\frac{1}{b}\right)$. This may also be written as $\frac{a}{c}$ times $1\left(\frac{1}{b}\right)\Big/1\left(\frac{1}{b}\right)$ which is equivalent to $\frac{a}{c}$ times 1; or $\frac{a}{c}$. In other words, **a** copies of the unit fraction over **c** copies of the unit fraction is the same as $\frac{a}{c}$ because the unit fraction over the unit fraction is equal to 1.

As an example, for either $\frac{9}{7}$ divided by $\frac{8}{7}$ or $\frac{9}{25}$ divided by $\frac{8}{25}$, each fraction shares a common denominator, so both are asking how many 8's are in 9—or 9 divided by 8—which is the same as $\frac{9}{8}$.

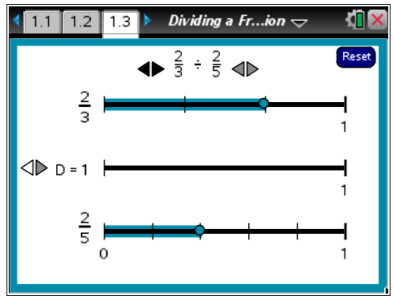
Mathematically, this approach can also be connected to creating equivalent fractions by multiplying the numerator and denominator by the same factor, which is essentially multiplying by the identity element, 1 (i.e. multiply $\left(\frac{3}{4}\right)\Big/\left(\frac{2}{3}\right)$ by $\frac{12}{12}$, which is 1, to get $\left(\frac{3}{4}(12)\right)\Big/\left(\frac{2}{3}(12)\right)=\left(\frac{36}{4}\right)\Big/\left(\frac{24}{3}\right)=\frac{9}{8}$.)

You may recall that Lesson 9 *Multiplying Fractions by a Whole Number* engaged students in exploring the product of whole number and a fraction, where the whole number could also be considered a scale factor of the fraction; i.e., $2\times\frac{3}{4}$ is seen as 2 sets of the fraction $\frac{3}{4}$. Students explored multiplication as the "fraction of" a quantity; i.e., $\frac{3}{4}\times\frac{2}{3}$ is seen as $\frac{2}{3}$ of the fraction $\frac{3}{4}$. From this perspective, $\frac{3}{4}$ is 3 copies of the unit fraction $\frac{1}{4}$. So, $\frac{2}{3}$ can be seen as scaling that set of copies: "What will $\frac{2}{3}$ of those 3 copies of $\frac{1}{4}$ be?"

This lesson can be used to make explicit the relationship between multiplication and division of fractions when division is approached using the idea of missing factor. For example, $\frac{2}{3}$ divided by $\frac{3}{4}$ can be thought of as, "What factor times $\frac{3}{4}$ will produce $\frac{2}{3}$?" It also provides a means of looking at multiplication of fractions using scaling and a number line—the notion of "fraction of" a quantity, where the fraction could be considered a scale factor.

Part 1, Page 1.3

Focus: Students use number lines to investigate division of fractions with numbers less than or equal to 1.



On page 1.3, use the horizontal arrows at the top of the page to set the denominators, ***b*** and ***d*** of the fractions in the division statement, $\left(\frac{a}{b}\right)\Big/\left(\frac{c}{d}\right)$. Dragging the dot on the top number line will set the numerator, ***a***, in the first fraction. In the same manner, dragging the dot on the bottom number line will set the numerator, ***c***, in the second fraction. Use the arrows next to D to partition the middle number line and find a common denominator for the two fractions.

The activity is limited to common denominators less than or equal to 24. When D is equal to a number that is a common denominator of the two fractions, the resulting equivalent fractions are displayed, along with the appropriate partitioning of the number line. Use the middle number line to help determine the solution of the division.

Note: The pink bar on the middle number line represents a new unit, based on the common denominator. For $\left(\frac{2}{3}\right)\Big/\left(\frac{2}{5}\right)$, the common denominator is 15, and the equivalent fractions are $\frac{10}{15}$ and $\frac{6}{15}$. The division $\left(\frac{10}{15}\right)\Big/\left(\frac{6}{15}\right)$ can be seen as $\left(10 \div \left(\frac{1}{15}\right)\right)\Big/\left(6 \div \left(\frac{1}{15}\right)\right)$; which is equivalent to $\frac{10}{6}$ or $1\frac{2}{3}$.

TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

> **Teacher Tip:** Be sure students understand how the interaction with the number lines supports the mathematics. Ask them how the number lines are connected to their thinking about division of fractions and the role of equivalent fractions in the process. This can lead to a productive discussion about the mathematical concepts. If students have gone through earlier activities with equivalent fractions including adding fractions with unlike denominators, they may be able to quickly identify common denominators. If this is the case, once they understand the process, they should use the number lines for confirmation of their thinking. The goal is to enable them to divide any two fractions without the support of the number lines and to be comfortable with their answers. You might want to encourage them to check their answers by thinking about the corresponding multiplication problem.

Class Discussion

> **Teacher Tip:** Students might need to work through the first two groups of problems as a class, then the third and fourth problems on their own in order to understand the role of the common denominator.

Have students…	**Look for/Listen for…**
For each of the following, explain whether the answer is more than 1, equal to 1, or less than 1 and why.	
• *the number of $\frac{3}{5}$s in $\frac{4}{5}$*	Answer: More than 1 because 3 copies of $\frac{1}{5}$ is less than 4 copies of $\frac{1}{5}$ when you have the same whole.
• *the number of $\frac{3}{5}$s in $\frac{3}{5}$*	Answer: Equal to 1 because they are equivalent
• *the number of $\frac{3}{5}$s in $\frac{2}{5}$*	Answer: Less than 1 because 3 copies of $\frac{1}{5}$ is more than 2 copies of $\frac{1}{5}$ when you have the same whole so not all of $\frac{3}{5}$ will fit into $\frac{2}{5}$.

Class Discussion (continued)

- ***the number of $\frac{2}{5}$s in $\frac{4}{5}$***

Answer: More than 1 because 4 copies of $\frac{1}{5}$ is larger than 2 copies of $\frac{1}{5}$ when you have the same whole so $\frac{2}{5}$ will fit into $\frac{4}{5}$ more than once.

Use the arrows at the top of page 1.3 to set the top number line with the fraction you want to divide and the bottom number line with the divisor. Remember that one way to think about a divided by b is to think about how many b's are contained in a. Use the number lines to find the following: $4 \div \frac{1}{3}$

- $\frac{4}{5} \div \frac{3}{5}$

  Answer: $\frac{1}{3}$

- $\frac{3}{5} \div \frac{3}{5}$

  Answer: 1

- $\frac{2}{5} \div \frac{3}{5}$

  Answer: $\frac{2}{3}$

- $\frac{4}{5} \div \frac{2}{5}$

  Answer: 2

✓ ***Sami says the answer to $\frac{7}{8} \div \frac{3}{8}$ is $2\frac{1}{8}$. Joan says it is $2\frac{1}{3}$. Who is correct and why?***

(Question #1 on the Student Activity sheet.)

Answer: Joan is correct. You are counting groups of $\frac{3}{8}$, and you have two groups of $\frac{3}{8}$ with only a partial group left over. That partial group is $\frac{1}{3}$ of the three copies of $\frac{1}{8}$ that you need to make another whole, so the fraction remaining is $\frac{1}{3}$ not $\frac{1}{8}$.

Class Discussion (continued)

- ***For each of the following, identify the groups you are counting for each divisor and explain what the denominator of any remainder, if there is one, will be:***

  a. $\frac{7}{4} \div \frac{3}{4}$ **b.** $\frac{7}{5} \div \frac{3}{5}$

  c. $\frac{7}{2} \div \frac{3}{2}$ **d.** $\frac{3}{2} \div \frac{7}{2}$

  Answers: For a, counting groups of $\frac{3}{4}$; For b, counting groups of $\frac{3}{5}$; For c, counting groups of $\frac{3}{2}$; For a-c the remainder will have a denominator of 3. For d, counting groups of $\frac{7}{2}$; the remainder will have a denominator of 7.

- ***Use the number lines to find the answers to the divisions in the previous question.***

  Answers: a. $2\frac{1}{3}$ b. $2\frac{1}{3}$ c. $2\frac{1}{3}$

  d. $\frac{3}{7}$

For each of the following, explain whether the answer is more than 1, equal to 1, or less than 1 and why.

- ***the number of $\frac{3}{7}$s in $\frac{3}{7}$***

  Answer: Equal to 1 because the fractions are the same.

- ***the number of $\frac{3}{7}$s in $\frac{4}{7}$***

  Answer: More than 1 because $\frac{4}{7}$ has more copies of $\frac{1}{7}$ than $\frac{3}{7}$

- ***the number of $\frac{4}{7}$s in $\frac{3}{7}$***

  Answer: Less than 1 because $\frac{3}{7}$ only has 3 copies of $\frac{1}{7}$ and you would need more to contain all of $\frac{4}{7}$.

- ***the number of $\frac{4}{7}$s in $\frac{2}{7}$***

  Answer: Less than 1 because $\frac{2}{7}$ only has 2 copies of $\frac{1}{7}$ and you would need more to contain all of $\frac{4}{7}$.

Class Discussion (continued)

Rewrite each of the problems from the question above as a division problem and use the number lines to find the answers.

- ***the number of $\frac{3}{7}$s in $\frac{3}{7}$***

$\frac{3}{7} \div \frac{3}{7} = 1$

- ***the number of $\frac{3}{7}$s in $\frac{4}{7}$***

$\frac{4}{7} \div \frac{3}{7} = \frac{4}{3}$

- ***the number of $\frac{4}{7}$s in $\frac{3}{7}$***

$\frac{3}{7} \div \frac{4}{7} = \frac{3}{4}$

- ***the number of $\frac{4}{7}$s in $\frac{2}{7}$***

$\frac{2}{7} \div \frac{4}{7} = \frac{1}{2}$

- ***Would you expect the number of $\frac{3}{4}$s in $\frac{7}{8}$ to be more than 1, less than 1 or equal to 1? Explain your reasoning.***

Answer: $\frac{7}{8}$ is more than $\frac{6}{8}$, which is equivalent to $\frac{3}{4}$ so there will be more than one $\frac{3}{4}$ in $\frac{7}{8}$.

- ***Would you expect the number of $\frac{9}{10}$s in $\frac{7}{8}$ to be more than 1, less than 1 or equal to 1?***

Answer: $\frac{9}{10}$ is larger than $\frac{7}{8}$ so the number of $\frac{9}{10}$ in $\frac{7}{8}$ will be less than 1.

- ***Which expression $\frac{3}{4} \div \frac{7}{8}$ or $\frac{7}{8} \div \frac{3}{4}$ correctly expresses, 'How many $\frac{3}{4}$ are in $\frac{7}{8}$?' Explain why.***

Answer: One way to think about a divided by b is to consider the number of b's in a. So the correct way to write the expression as a division problem is $\frac{7}{8} \div \frac{3}{4}$.

Class Discussion (continued)

If two fractions have a common denominator, it is easier to tell how many of one fraction is contained in the other because you have a way to compare. Use the middle number line on page 1.3 to find common denominators and help answer the questions.

- ***Find a common denominator for $\frac{2}{3}$ and $\frac{3}{8}$.***

Answer: 24

- ***Use the number lines to see how many $\frac{3}{8}$ are in $\frac{2}{3}$.***

Answer: Change each to equivalent fractions with denominators of 24; $\frac{9}{24}$ and $\frac{16}{24}$. There is 1 whole group of $\frac{9}{24}$ in $\frac{16}{24}$ and $\frac{7}{9}$ of a group left over.

Use the number lines to solve each:

- $\frac{3}{7} \div \frac{1}{3}$

  Answer: $\frac{9}{7}$ or $1\frac{2}{9}$

- $\frac{3}{7} \div \frac{2}{3}$

  Answer: $\frac{9}{14}$

- $\frac{1}{3} \div \frac{1}{7}$

  Answer: $\frac{7}{3}$ or $2\frac{1}{3}$

- $\frac{2}{3} \div \frac{1}{7}$

  Answer: $\frac{14}{3}$ or $5\frac{2}{3}$

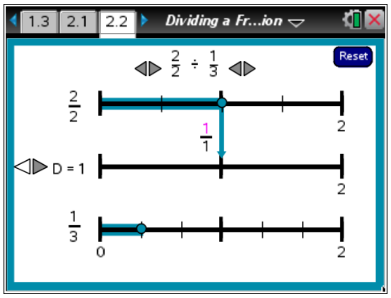
- $\frac{2}{3} \div \frac{2}{7}$

  Answer: $\frac{7}{3}$ or $2\frac{1}{3}$

Part 2, Page 2.2

Focus: Students will continue to explore division of fractions.

The primary difference between pages 1.3 and 2.2 is the extension of the number line to two units that allows division involving improper fractions less than or equal to 2. The fractions are set the same way as on page 1.3, and the arrows on the middle number line are used in the same manner as those on page 1.3. To reset the page, select **Reset** in the upper right corner.



Teacher Tip: Have students the review process they used to work through the first page of the activity. Remind them that they can use multiplication to check the answers of their division problems.

Class Discussion

Have students…	**Look for/Listen for…**
• ***How many $\frac{5}{6}$ s are in $\frac{4}{3}$?***	Answer: $\frac{24}{15}$ *or* $1\frac{3}{5}$
• ***How many $\frac{4}{3}$ s are in $\frac{5}{6}$?***	Answer: $\frac{5}{8}$
• ***Explain the difference in your answers to the two questions above.***	Answer: One is a divided by b and the other is b divided by a: the same as asking how many $\frac{1}{2}$ s in 4 (4 divided by $\frac{1}{2}$) and how many 4s are in $\frac{1}{2}$ ($\frac{1}{2}$ divided by 4).

Use the number lines to solve each:

How many $\frac{2}{3}$-pound servings can you get from $1\frac{1}{2}$ pounds of meat?	Answer: $\frac{3}{2}$ divided by $\frac{2}{3} = \frac{9}{4}$ or $2\frac{1}{4}$ servings.
✓ ***How many $\frac{3}{8}$-foot long small ribbons can you cut from a 2-foot long ribbon? From a $1\frac{2}{3}$ foot long ribbon?***	Answer: 2 divided by $\frac{3}{8} = 5\frac{1}{3}$ so 5 small ribbons. $\frac{5}{3}$ divided by $\frac{3}{8} = \frac{40}{9} = 4\frac{4}{9}$ or 4 small ribbons.

(Question #2 on the Student Activity sheet.)

Class Discussion (continued)

- ***What is the minimum length a ribbon needs to be in order for it to be cut into 6 smaller ribbons that each measure $\frac{3}{8}$ foot long?***

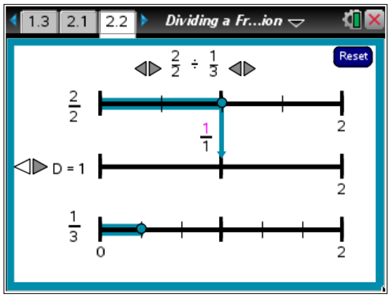
Answer: $6\times\frac{3}{8}=\frac{9}{4}$ or $2\frac{1}{4}$ feet.

- ***How long is a rectangular field that is $\frac{3}{4}$ mile wide and has area $1\frac{1}{2}$ square miles?***

Answer: $1\frac{1}{2}$ divided by $\frac{3}{4}=2$, so the field is 2 miles long.

Part 3, Page 3.2

Focus: Students can investigate the relationship between multiplication and division.



The arrows at the top of the page set a denominator and dragging the dot on the bar determines a fraction as a multiplier or scalor: i.e. $\frac{1}{2}$ "of" a quantity. The arrows at the bottom of the page set a denominator that partitions the bar into designated unit fractions. Dragging the dot at the end of the bar determines the quantity being multiplied (from 0 to 4). Note: The multiplier can extend past 1.

Class Discussion

Have students…	**Look for/Listen for…**
Look at the top number line.	
• ***How long is the bar associated with the number line? What fraction of the bar is shaded?***	Answer: The bar is 1 unit long, and $\frac{1}{2}$ is shaded.
• ***On the bottom number line, what fraction of the length from 0 to 4 is shaded?***	Answer: The distance from 0 to 2 is shaded which is $\frac{1}{2}$ of the distance from 0 to 4.
Select the fraction $\frac{1}{4}$ on the top number line.	
• ***What does the shaded part of the bottom bar represent?***	Answer: The shaded part represents one of the four units on the bottom number line or $\frac{1}{4}$ of the bar.
• ***How are the top and bottom bars the same?***	Answer: Both have the same fraction of the bars shaded, $\frac{1}{4}$.

- ***How are they different?***

Answer: The length of the top bar is only one unit, but the length of the bottom bar is 4 units.

Class Discussion (continued)

Create a bar that is $\frac{4}{3}$ shaded on the top.

- ***Explain what it means for a bar to be $\frac{4}{3}$ shaded.***

Answer: If the bar is the unit, $\frac{4}{3}$ is 1 unit and $\frac{1}{3}$ more of the unit so the whole bar is shaded plus $\frac{1}{3}$ of another bar the same size is shaded.

- ***Make the length of the bottom bar 2. Change D to find the shaded length of the bottom bar. Explain why your answer makes sense.***

Answer: The $\frac{8}{3}$ is the length of the shaded part of the bottom bar. This makes sense because the length of the top bar is 1 and $\frac{4}{3}$ of that bar is shaded; the bottom bar is twice as long, so the length of the shaded part should be twice as much as $\frac{4}{3}$ or $\frac{8}{3}$.

- ***Write an equation that describes the relation between the shaded areas of the two bars.***

Answer: $2\left(\frac{4}{3}\right)=\frac{8}{3}$

- ***Explain how you can use the file to find $\frac{3}{4}$ of 3.***

Answer: Set the top so that $\frac{3}{4}$ of the bar is shaded. Then set the bottom bar to have length 3. Change the partitions on the bottom until you find one where $\frac{3}{4}$ of the length of 3 is shaded. It would be $\frac{9}{4}$.

- ***Write an equation to describe the problem in the question above.***

Answer: $\frac{3}{4}(3)=9$

Which of the following do you agree with and why? (Use the file to justify your thinking.)

- ***Sam argued that $\frac{7}{4}$ of 2 was the same as 2 and $\frac{3}{4}$ of another 2.***

Answer: Sam is correct because you can see that a $\frac{7}{4}$ bar is the same as 1 whole unit + $\frac{3}{4}$ of another unit. When you are taking $\frac{7}{4}$ of 2 you have $(1+\frac{3}{4})$ of 2 means you need one whole 2 and $\frac{3}{4}$ of another 2.

Class Discussion (continued)

- ***Sallee argued that $\frac{7}{4}$ of 2 was the same as $\frac{14}{8}$.***

Answer: Sallee is not correct because the 2 does not affect the unit fraction but tells you how many of the unit fractions you have; that is $\frac{7}{4}$ is 7 copies of $\frac{1}{4}$ and because multiplication is commutative, you can say you want two sets of these 7 copies of $\frac{1}{4}$ or 14 copies of $\frac{1}{4}$ not of $\frac{1}{8}$.

- ***Spike stated that $\frac{7}{4}$ divided by $\frac{1}{2}$ was the same as two sets of $\frac{7}{4}$ or $\frac{14}{4}$.***

Answer: Spike is correct because $\frac{7}{4}$ divided by $\frac{1}{2}$ is the same as saying that, "$\frac{1}{2}$ is $\frac{1}{2}$ times some number. And $\frac{1}{2}\left(\frac{14}{4}\right)$ is $\frac{1}{2}$".

- ***Serena stated that $\frac{7}{4}$ divided by $\frac{1}{2}$ was the same as $\left(\frac{7}{4}\right)(2)$.***

Answer: Serena is correct because when you think about $\frac{7}{4}=\frac{1}{2}$ times x, then to solve for x you multiply both sides by 2.

✓ ***Use the file to find the missing value in each equation. Explain your reasoning.***

a. $\frac{3}{4}$ **of** $\frac{1}{2}$ **is what?**

b. $\frac{5}{2}$ **is** $\frac{3}{2}$ **of what?**

c. $\frac{2}{3}$ **of what is 3?**

(Question #3 on the Student Activity sheet.)

Answers: a) $\frac{7}{8}$ b) $\frac{5}{3}$ c) $\frac{6}{5}$

Explanations will vary. Students may set the top bar to represent the first fraction in the product (i.e., $\frac{3}{4}$ in problem a), move the dot on the bottom bar so that the length of the bar or the length of the shaded part is equal to the result, then find a denominator that will give the length of the shaded region on the bottom bar.

Class Discussion (continued)

Describe the difference between how you think about the two problems:

- ***$\frac{5}{6}$ is $\frac{1}{2}$ of what number?***

Answer: This is a missing factor problem that can be done by division.

- ***$\frac{5}{6}$ of $\frac{1}{2}$ is what number?***

Answer: In this problem both factors are given and you are looking for the product.

Use any pages in the file to help you answer the following. Which of the following is true. Explain your reasoning in each case.

- ***$\frac{7}{12}$ divided by $\frac{3}{4}$ is smaller than 1.***

Answer: True. $\frac{7}{12}$ is smaller than $\frac{9}{12}$.

- ***$\frac{7}{12}$ divided by $\frac{3}{4}$ is larger than $\frac{3}{4}$ divided by $\frac{7}{12}$.***

Answer: Think of $\frac{3}{4}$ as $\frac{9}{12}$ and compare $\frac{7}{12}$ divided by $\frac{9}{12}$ and $\frac{9}{12}$ divided by $\frac{7}{12}$. $\frac{7}{12}$ divided by $\frac{9}{12}$ is less than 1 and $\frac{9}{12}$ divided by $\frac{7}{12}$ is more than one so $\frac{3}{4}$ divided by $\frac{7}{12}$ is the larger.

- ***$\frac{11}{8}$ divided by $\frac{5}{3}$ is larger than 2.***

Answer: $\frac{11}{8}$ is smaller than $\frac{5}{3}$ so the answer is less than 1.

- ***$\frac{3}{8}$ divided by $\frac{1}{3}$ is smaller than 1.***

Answer: False. Using 24 as a common denominator, $\frac{9}{24}$ divided by $\frac{8}{24}$ will be more than 1 because $\frac{9}{24}$ is big enough for all of $\frac{8}{24}$ and a remainder.

Which is larger and why?

- ***$\frac{1}{5}$ divided by $\frac{1}{3}$ or $\frac{1}{3}$ divided by $\frac{1}{7}$.***

Answer: This problem compares $\frac{3}{5}$ to $\frac{7}{3}$; $\frac{7}{3}$ is larger than 1 and $\frac{3}{5}$ is smaller than 1 so $\frac{7}{3}$ is the larger of the two.

Class Discussion (continued)

- ***$\frac{1}{5}$ divided by $\frac{1}{3}$ or $\frac{1}{3}$ divided by $\frac{1}{5}$.***

  Answer: This problem compares $\frac{3}{5}$ to $\frac{5}{3}$; $\frac{5}{3}$ is larger because it is more than 1 and $\frac{5}{3}$ is not.

- ***Use the fact that $\frac{3}{4}\times\frac{8}{9}-\frac{2}{3}$ to explain the answer to $\frac{2}{3}\div\frac{3}{4}$.***

  Answer: Rewriting the multiplication problem as division would give you $\frac{2}{3}\div\frac{3}{4}-\frac{8}{9}$.

- ***Suppose $\frac{3}{8}\div\frac{3}{4}$ = some number N. Write this as a multiplication problem, then solve the problem for N.***

  Possible answer: $\frac{3}{8}=N\left(\frac{3}{8}\right)$. Multiplying both sides by the reciprocal of $\frac{3}{4}$ would produce

  $\frac{3}{8}\times\frac{4}{3}=N\left(\frac{3}{4}\right)\left(\frac{4}{3}\right) \rightarrow \frac{3}{8}\times\frac{4}{3}=N\times1 \rightarrow \frac{1}{2}=N$

- ***Use a related multiplication problem to find the solution to $\frac{4}{9}\div\frac{3}{2}$.***

  Answer:

  $\frac{4}{9}=Nx\left(\frac{3}{2}\right) \quad \frac{4}{9}\times\frac{2}{3}=N \rightarrow \frac{8}{27}=N$

Find the missing factor in each case:

- ***What would you multiply $\frac{1}{2}$ by to obtain 2?***

  Answer: 4

- ***What would you multiply $\frac{7}{9}$ by to obtain $\frac{1}{3}$?***

  Answer: $\frac{3}{7}$

- ***What would you divide $\frac{5}{4}$ by to obtain $\frac{3}{5}$?***

  Answer: $\frac{25}{12}$

Class Discussion (continued)

Have students…	Look for/Listen for…
Identify each as a division problem or a multiplication problem and explain your reasoning.	
• ***How much chocolate will each person get if 3 people share $\frac{1}{2}$ pound of chocolate equally?***	Answer: division; dividing a total among a number of people
• ***Teenagers were $\frac{7}{8}$ of the movie audience. $\frac{3}{4}$ of the teenagers were girls. How many girls were at the movie?***	Answer: Multiplication; Taking $\frac{3}{4}$ times the total number of girls.
• ***I have $\frac{4}{5}$ ton of sand. A pickup truck can move $\frac{1}{10}$ of a ton at a time. How many trips will it take to move all of the sand?***	Answer: Multiplication; taking $\frac{1}{10}$ of the original pile of sand.
• ***I have $\frac{4}{5}$ ton of sand. Three pickup trucks are available to move the sand. How much sand will each truck have to move?***	Answer: Division; dividing the $\frac{4}{5}$ ton among 3 trucks.
• ***Harry walks $1\frac{2}{3}$ miles in an hour at a constant pace. How long would it take him to walk 5 miles?***	Answer: Division; you need to find how many $1\frac{2}{3}$ miles are in 5 miles.
• ***Harry walks $1\frac{2}{3}$ miles in an hour at a constant pace. How far does he walk in $\frac{3}{4}$ of an hour?***	Answer: Multiplication; you need what $\frac{3}{4}$ of the total per hour will give you.

- ***Now, answer each of the problems in the group above.***

Answers: $\frac{1}{6}$ pound; $\frac{21}{32}$ girls; $\frac{2}{25}$ ton each trip; $\frac{4}{15}$ of a ton; 3 hours; $\frac{5}{4}$ miles

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Find $\frac{7}{8} \div \frac{3}{4}$. ***Answer:*** $\boldsymbol{\frac{7}{6}}$

2. How many $\frac{3}{4}$-cup servings are in $\frac{9}{2}$ cups of yogurt? ***Answer: 6 cups***

3. Jim has $\frac{3}{4}$ yard of string, which he wishes to divide into pieces, each $\frac{1}{8}$ yard long. How many pieces will he have?

   a. 3 b. 4 c. 6 d. 8

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   Answer: c) 6

4. Students in Mrs. Johnson's class were asked to tell why $\frac{4}{9} \div \frac{2}{3} = \frac{2}{3}$. Whose reason is best?

   a. Kelly said, "Because $\frac{1}{2} \times \frac{4}{9} = \frac{2}{3}$."

   b. Keri said, "Because $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$."

   c. Kim said, "Because $\frac{1}{3} \times 9 = 3$."

   d. Kevin said, "The answer is wrong; it should be $\frac{8}{27}$."

Answer: b is correct

5. Use numbers from the given list to make true sentences. (You may use a number more than once.)
   1, 3, 4, 6, 8, 9, 12, 10

   a. $\frac{\square}{8} \div \frac{\square}{6} = \frac{3}{4}$ ***Answer:*** $\boldsymbol{\frac{1}{8} \div \frac{1}{6} = \frac{3}{4}}$

   b. $\frac{\square}{\square} \div \frac{2}{\square} = \frac{5}{3}$ ***Answer:*** $\boldsymbol{\frac{10}{3} \div \frac{2}{1} = \frac{5}{3}}$

   c. $\frac{\square}{4} \div \frac{2}{\square} = \frac{9}{\square}$ ***Answer:*** $\boldsymbol{\frac{3}{4} \div \frac{2}{3} = \frac{9}{8}}$

Student Activity solutions

Vocabulary

identity element of multiplication:

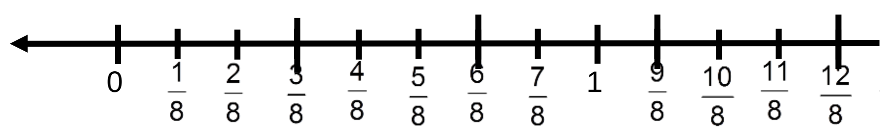
number 1; if you multiply a number by the identity element, the result is that original number

In this activity, you will divide a fraction by a fraction by finding common denominators and by using the relationship between multiplication and division.

1. Sami says the answer to $\frac{7}{8} \div \frac{3}{8}$ is $2\frac{1}{8}$. Joan says it is $2\frac{1}{3}$. Who is correct and why?

Answer: Joan is correct. You are counting groups of $\frac{3}{8}$, and you have two groups of $\frac{3}{8}$ with only a partial group left over. That partial group is $\frac{1}{3}$ of the three copies of $\frac{1}{8}$ that you need to make another whole, so the fraction remaining is $\frac{1}{3}$ not $\frac{1}{8}$.

2. How many $\frac{3}{8}$-foot long small ribbons can you cut from a 2-foot long ribbon? Complete the number line to show your reasoning.



Answer: 2 divided by $\frac{3}{8} = 5\frac{1}{3}$ so 5 small ribbons.

3. Use the file to find the missing value in each equation. Explain your reasoning.

a) $\frac{3}{4}$ of $\frac{1}{2}$ is what? b) $\frac{5}{2}$ is $\frac{3}{2}$ of what? c) $\frac{2}{3}$ of what is 3?

Answers: a) $\frac{3}{8}$ b) $\frac{5}{3}$ c) $\frac{6}{5}$

4. How many $\frac{3}{4}$-cup servings of rice can you get from a container filled with $3\frac{1}{3}$ cups of rice? Explain your reasoning.

Answer: $4\frac{4}{9}$***; I got my answer by dividing*** $\frac{10}{3}$ ***by*** $\frac{3}{4}$ ***to get*** $4\frac{4}{9}$ ***servings.***