Building Concepts: Fraction Multiplication

Lesson Overview

This TI-Nspire™ lesson uses interactive unit squares to explore the concept of fraction multiplication. Multiplication of fractions can be developed using the area of one of the rectangular tiles in a unit square.

The product of two fractions is the product of the numerators and the product of the denominators.

Learning Goals

Students should understand and be able to explain each of the following:

1. The product of two fractions is the product of the numerator and the product of the denominator; in general, $\left(\frac{a}{b}\right) \times \left(\frac{c}{d}\right) = \frac{ac}{bd}$;
2. Because multiplication is commutative, $\frac{ac}{bd}$ can result from either $\frac{a}{b} \times \frac{c}{d}$ or $\frac{c}{b} \times \frac{a}{d}$, i.e., $\frac{3}{5} \times \frac{2}{7} = \frac{2}{5} \times \frac{3}{7}$;
3. Multiplying any number by a fraction less than one produces a product less than the number; multiplying any number by a fraction greater than one produces a product greater than the number;
4. The product of two fractions can often be expressed by an equivalent fraction where the numerator and denominator have been divided by a common factor.

Prerequisite Knowledge

Fraction Multiplication is the tenth lesson in a series of lessons that explore fractions. This lesson builds on the earlier work of finding the area by multiplying side lengths as well as concepts explored in the previous lessons. Students should have prior experience with *Fractions and Unit Squares, Creating Equivalent Fractions* and *Mixed Numbers.* Prior to working on this lesson students should understand:

- the concept of area.
- that tiling means using one or more shapes (the tiles) to cover an area without any gaps or overlaps.
- that improper fractions can be converted into whole numbers and fractions (mixed numbers) and that mixed numbers can be converted into improper fractions.
- that the product of two numbers is not affected by the order in which they are multiplied.

Vocabulary

- **Commutative Property of Multiplication:** a rule that states that the product of two factors is not affected by the order in which they are multiplied
- **reciprocal:** a fraction whose numerator and denominator have been switched

Lesson Pacing

This lesson contains multiple parts and can take 50–90 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Fraction Multiplication_Student.pdf
- Fraction Multiplication_Student.doc
- Fraction Multiplication.tns
- Fraction Multiplication_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity Sheet: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

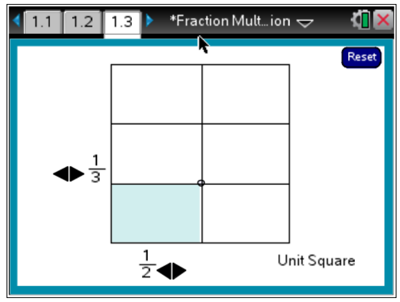
Mathematical Background

This TI-Nspire™ lesson uses interactive unit squares to explore the concept of fraction multiplication. Multiplication of fractions can be developed using the area of one of the rectangular tiles in a unit square. Consider the base of a unit square as a horizontal number line partitioned into $\frac{1}{n}$ equal parts and the left side a vertical number line partitioned into $\frac{1}{m}$ equal parts. This creates a unit square tiled with ***mn*** rectangles, each with dimensions $\frac{1}{n}$ and $\frac{1}{m}$. From earlier lessons, students should remember that the area of a rectangle is the product of the base and height, which in this case are $\frac{1}{n}$ and $\frac{1}{m}$, so the area of any rectangle is $\left(\frac{1}{n}\right)\left(\frac{1}{m}\right)$. But the unit square has been partitioned into ***mn*** rectangles, so the area of any one rectangle is $\frac{1}{mn}$. Thus, the product of $\left(\frac{1}{n}\right)\left(\frac{1}{m}\right)=\frac{1}{mn}$. Investigating the fraction of area covered by shading several rectangles in the unit square in the same way leads to the generalization that the product of two fractions $\frac{a}{n}$ and $\frac{b}{m}$ is the fraction $\frac{(ab)}{mn}$ formed by the product of the numerators, ***ab***, and the product of the denominators, ***mn***. Rearranging the shaded portions can support thinking about equivalent fractions.

Once the process for finding the product of two unit fractions has been developed, the mathematical properties of multiplication could be used to develop the general statement: $\left(\frac{a}{b}\right)\left(\frac{c}{d}\right)=\left(a\left(\frac{1}{b}\right)\right)\left(c\left(\frac{1}{d}\right)\right)$. Rearranging and regrouping leads to $(ac)\left(\left(\frac{1}{b}\right)\left(\frac{1}{d}\right)\right)=(ac)\left(\frac{1}{bd}\right)=\frac{ac}{bd}$. Facility with fraction multiplication should include the ability to recognize that multiplying fractions such as $\frac{2}{3}$ and $\frac{5}{7}$ is equivalent to finding the product of 2 and $\frac{5}{21}$; 5 and $\frac{2}{21}$; $\frac{5}{3}$ and $\frac{2}{7}$.

Part 1, Page 1.3

Focus: Students will use unit squares to develop the concept of fraction multiplication.



In this activity a unit square is used to investigate multiplying a fraction by a fraction. Page 1.3 shows a unit square, where the horizontal and vertical sides of the square can be thought of as perpendicular number lines. Each point on these number lines represents a fraction. Horizontal and vertical lines drawn from those points tile the unit square with congruent rectangles. The area of one of the rectangles is a fraction of the unit square.

The denominators of the fractions (from 1 to 12) are changed using the arrows, which determine the number of congruent rectangles in the unit square. Moving the dot will change the numerators of the fractions (from 0 to 12) and display the shaded area for that portion of the unit square. To reset the page, select **Reset** in the upper right corner. **Note:** Page 1.4 is an optional, second page for this activity. It shows the product of the two fractions identified on page 1.3.

TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

Teacher Tip: Give students time to repeat the activity before asking them focused questions. This will help them internalize the concept of fraction multiplication. Help students to connect the product of the denominators with the number of rectangles in the unit square and the product of the numerators with the number of shaded rectangles.

Class Discussion (continued)

Have students…	Look for/Listen for…
The unit square on page 1.3 has been partitioned into rectangles.	
• ***What is the area of one of the rectangles? Explain how you know this is the area. (You might consider whether the rectangles are congruent in your explanation.)***	Possible answer: The area of one of the rectangles is $\frac{1}{6}$ square units because the unit square has been partitioned into 6 congruent smaller rectangles. You know they are congruent because you could reflect each rectangle over a horizontal or vertical line and note that the length of each side is preserved in the transformation, so one fits on top of the other.
• ***How did you learn to find the area of a rectangle in your earlier work with shapes?***	Possible answer: We found the area of a rectangle by multiplying the base times the height. We also counted the unit squares inside the rectangle.
• ***Remembering that the square is a unit square with each side of length 1, what are the dimensions of the shaded rectangle? Of any of the small rectangles?***	Answer: The base is $\frac{1}{2}$ and the height is $\frac{1}{3}$.
• ***Explain why it makes sense to make the statement:*** $\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$***.***	Possible answer: You know that the area of the rectangle is $\frac{1}{6}$ of the area of the unit square because 1 of the 6 congruent rectangles within the unit square is shaded. You also know that to find area, you multiply the base, $\frac{1}{2}$, and the height, $\frac{1}{3}$. So the two different ways of thinking about the same thing should give you the same result.
• ***What do you think the product of*** $\frac{1}{2}\times\frac{1}{3}$ ***will be?***	Answer: It should give you the same answer as $\frac{1}{3}$ times $\frac{1}{2}$, or $\frac{1}{6}$.

Class Discussion (continued)

- ***The arrows on the left change the denominator of the first fraction; those on the bottom change the denominator of the second fraction. Use the arrows on the left and bottom to create*** $\frac{1}{2} \times \frac{1}{3}$***.***

  How does the picture support your answer to the question above?

  Answer: The unit square is made of 6 congruent rectangles, so each one has area $\frac{1}{6}$ and the rectangle area is the product of the base, $\frac{1}{3}$, and the height , $\frac{1}{2}$. You also know that changing the order in a multiplication problem does not change the result (the commutative property of multiplication).

- ***If you are going to multiply*** $\frac{1}{5}$ ***by*** $\frac{1}{3}$***, how many congruent rectangles do you think will cover the unit square?***

  Answer: 15.

- ***What do you think the area of each rectangle will be?***

  Answer: $\frac{1}{15}$ square unit.

- ***Change the fractions in the file to*** $\frac{1}{5}$ ***and*** $\frac{1}{3}$***. How does the picture support your answers to the two questions above?***

  Answer: There are 15 smaller congruent rectangles and the dimensions of each rectangle are $\frac{1}{5}$ by $\frac{1}{3}$.

Moving the dot vertically will change the numerator of the fraction on the left; moving the dot horizontally will change the numerator of the fraction on the bottom. Change the fractions to $\frac{2}{5}$ ***and*** $\frac{2}{3}$***.***

- ***What fraction of the rectangles is shaded?***

  Answer: $\frac{4}{15}$.

- ***Explain what the product of*** $\frac{2}{5}$ ***and*** $\frac{2}{3}$ ***will be and how you know.***

  Answer: The product will be $\frac{4}{15}$ because that is how you find the total area of the shaded rectangle and 4 of the 15 smaller congruent rectangles are shaded.

Class Discussion (continued)

Have students…

- ***Another way to describe partitioning the unit square is to say the unit square is tiled by smaller congruent rectangles. How many congruent rectangles will tile the unit square if the denominators of the two fractions that are multiplied together are the following? Explain your reasoning in each case.***

  a. 5 and 7 **b.** 4 and 3 **c.** 2 and 11

Look for/Listen for…

Answer: One of the numbers will tell how many rectangles are along the base of the unit square and the other how many are along the vertical side of the unit square (or the other way around), so the number of smaller rectangles will be the product of the numbers in each case.

a. 35 b. 12 c. 22.

Find each and be ready to explain your answers.

- $\frac{1}{9} \times \frac{1}{7}$

  Answer: $\frac{1}{63}$.

- $\frac{2}{9} \times \frac{5}{7}$

  Answer: $\frac{10}{63}$.

- $\frac{0}{9} \times \frac{5}{7}$

  Answer: 0.

✔ ***Explain how you can argue from the unit square that $\frac{6}{10} \times \frac{1}{6}$ is the same as $\frac{1}{10}$.***

(Question #1 on the Student Activity sheet.)

Possible answer: If you moved the 6 shaded rectangles to the bottom row, you would fill up the entire row. There are 10 rows so you have shaded $\frac{1}{10}$ of the rows or $\frac{1}{10}$ of the unit square.

Class Discussion (continued)

Use the TNS file to help determine whether each statement is true. Explain your reasoning in each case.

- ***Tami claims that*** $\frac{3}{4}\times\frac{2}{3}$ ***is*** $\frac{1}{2}$.

  Answer: True. Set the fractions to $\frac{3}{4}\times\frac{2}{3}$. If you move the two shaded rectangles in the top row to fill in the left column on the two bottom rows, you have shaded $\frac{1}{2}$ of the entire unit square, which is $\frac{6}{12}$ reduced.

- ***Holm argues that*** $\frac{4}{5}\times\frac{7}{8}$ ***is*** $\frac{11}{40}$.

  Answer: False. 28 of the smaller congruent rectangles are shaded, not 11. He added the two numbers instead of multiplying them.

- ***Roy says that*** $\frac{4}{5}\times\frac{2}{3}$ ***is the same as*** $\frac{2}{5}\times\frac{4}{3}$.

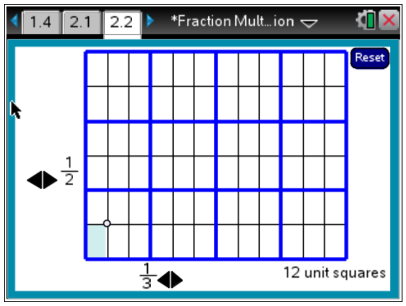
  Answer: True. In both cases 8 out of 15 rectangles are shaded.

- **Sally says that** $\frac{3}{7}\times\frac{5}{8}$ **is** $\frac{24}{35}$.

  Answer: False because the number of rectangles tiling the unit square is 56 and the number of shaded rectangles would be 15. To multiply two fractions, you find the product of the numerators and the product of the denominators.

Part 2, Page 2.2

Focus: Students will multiply a fraction by a fraction to yield products that include fractions greater than 1.



Page 2.2 displays 12 unit squares, which allow students to investigate the product of fractions greater than 1 (improper fractions). The arrows and dot behave the same way as on page 1.3. The denominators in the fractions are represented by the congruent rectangles within each of the 12 unit squares. If the product of the two fractions is greater than one, the shaded portion will fill a unit square and extend into an adjoining unit square. To reset the page, select **Reset** in the upper right corner.

> **Teacher Tip:** Be sure students understand that the whole number 2 would be represented by shading two of the unit squares and by the fraction $\frac{2}{1}$.
> This page can be used to help students think about mixed numbers by rearranging the rectangles to fill as many unit squares as possible.

> **Teacher Tip:** Page 2.3 is an optional, second page for this activity. It shows the product of the two fractions.

Be sure students understand how the interaction with the file supports the mathematics. Asking them how the file is connected to their thinking about multiplying fractions can lead to a productive discussion. Some of the problems confront some typical student misconceptions; in particular problem 14 would be a good problem to emphasize the difference between multiplying and adding fractions.

Class Discussion

Have students…	Look for/Listen for…
Page 2.2 displays 12 congruent unit squares	
• ***What is the area of each of the small rectangles that tile the unit squares?***	Answer: $\frac{1}{6}$ square units.
✓ ***Do you think the product of the fractions $\frac{5}{4}\times\frac{2}{3}$ will be more or less than 1?*** ***Explain your reasoning.*** (Question #2 on the Student Activity sheet.)	Possible answer: Less than 1 because there will be 12 congruent rectangles but only 10 of them will be shaded.

Class Discussion (continued)

- ***Use the file to find the product. How can you tell from the unit squares whether you were right or wrong in the question above?***

  Possible answer: If you move the shaded rectangles around, you cannot fill up one unit square.

- ***Explain why the product of $\frac{5}{3}\times\frac{7}{4}$ is $2\frac{11}{12}$.***

  Possible answer: $\frac{35}{12}$ is the product. If you divide 12 into 35 you will get $2\frac{11}{12}$. You could also think about moving the shaded rectangles so that you have filled 2 of the unit squares and $\frac{11}{12}$ of a third one.

- ***Write the product of $\frac{7}{6}\times\frac{5}{2}$ as a mixed number and as an improper fraction.***

  Answer: It will be the same as the problem in the answer above: $\frac{35}{12}$ and $2\frac{11}{12}$.

For each of the following problems: estimate whether the product will be more or less than 1, then find the product. When possible, write the solutions as mixed numbers.

- $\frac{7}{3}\times\frac{4}{5}$

  Answer: More than 1 because the first factor is more than 2 and you are taking $\frac{4}{5}$ of that; $\frac{28}{15}$; $\frac{28}{15}=1\frac{13}{15}$.

- $\frac{4}{3}\times\frac{7}{3}$

  Answer: More than 1 because both numbers are larger than 1; $\frac{28}{9}=3\frac{1}{9}$.

- $3\times\frac{4}{5}$

  Answer: More than 1 because it is more than $\frac{1}{2}$ times 3; $\frac{12}{5}=2\frac{2}{5}$.

- $\frac{3}{5}\times 4$

  Answer: More than 1 because the answer is the same as part c using the commutative property of multiplication, $2\frac{2}{5}$.

Class Discussion (continued)

- $\frac{4}{5} \times \frac{2}{3}$

Answer: Less than 1 because both fractions are less than 1; $\frac{8}{15}$.

- Explain the difference between

  a. $\frac{7}{8} \times \frac{6}{5}$ b. $\frac{6}{8} \times \frac{7}{5}$

  c. $\frac{7}{5} \times \frac{3}{4}$ d. $\frac{5}{9} \times \frac{9}{2}$

Possible answer: The answer to part a is $\frac{42}{40}$ or $\frac{21}{20}$. The answer to part b is the same because of the commutative property of multiplication. The answer to part c is the same because $\frac{3}{4}$ is equivalent to $\frac{6}{8}$. The answer to part d is $\frac{5}{2}$ because you can change the order of the 5 and 9 in the numerators and then one of the fractions is $\frac{9}{9}$, or 1.

Use the file to help answer each of the following and explain your reasoning in each case.

✓ ***Explain why*** $3 \times \frac{1}{4}$ ***is the same as*** $1 \times \frac{3}{4}$***.*** (Question #3 on the Student Activity sheet.)

Answer: In $3 \times \frac{1}{4}$, $\frac{1}{4}$ of the congruent rectangles in three of the unit squares are shaded. That would be 3 of the total of 12 rectangles in those unit squares or $\frac{3}{12}$. But if you moved the shaded rectangles from the top two unit squares to the bottom one, you would have one unit square with 3 of 4 rectangles shaded, which is the same as $1 \times \frac{3}{4}$.

- ***Find a number so the product of the number and*** $\frac{3}{1}$ **is *exactly* 2.**

Answer: $\frac{2}{3}$.

- ***Find two fractions not equivalent to whole numbers whose product is a whole number.***

Possible answer: $\frac{2}{5}$ times $\frac{5}{2}$ is 1; $\frac{3}{4}$ times $\frac{8}{3}$ is 2.

Class Discussion (continued)

Use the file to describe two different ways to find the product of $\frac{7}{3}$ and $3\frac{5}{8}$.	Possible answer: You could use both as improper fractions and find the product of $\frac{7}{3}$ and $\frac{29}{8}$. Or you could take the product of $\frac{7}{3}$ and 3, which is 7 and add the product of $\frac{7}{3}$ and $\frac{5}{8}$, which is $\frac{35}{24}$. The sum would be $8\frac{11}{24}$
• ***Suppose a recipe called for $3\frac{5}{8}$ ounces of milk and you wanted to make 2 batches. How much milk would you need?***	Answer: You could use improper fractions, so you would find the product of $\frac{29}{8}$ and 2, which would be $\frac{58}{8}$ or $\frac{29}{4}$. You would need $7\frac{1}{4}$ ounces of milk.
• ***A baby crawls $3\frac{1}{3}$ feet in one minute. If the baby crawls at the same pace, how far would he crawl in 2 minutes? How far in $\frac{3}{4}$ of a minute?***	Answer: In 2 minutes, the baby would crawl $6\frac{2}{3}$ feet; in $\frac{3}{4}$ of a minute, the baby would crawl $\frac{10}{3} \times \frac{3}{4} = \frac{30}{12} = \frac{5}{2} = 2\frac{1}{2}$ feet.
• ***If two fractions have unlike denominators, you must find common denominators before you can add them. Explain why you do not need to find common denominators to multiply fractions.***	Possible answer: Adding two fractions was done by putting two line segments end to end on the number line. You could not count the number of unit fractions unless they were the same in both segments. But finding the product of two fractions is like finding area and you have a base and a height for a rectangle. These do not have to have the same denominators--rectangles can have different dimensions. Adding is about length and multiplying is related to area.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. What is the product of $\frac{4}{7}\times\frac{3}{2}$?

   a. $\frac{7}{9}$ b. $\frac{7}{14}$ c. $\frac{12}{14}$ d. $\frac{12}{9}$

   Answer: c.

2. $\frac{4}{3}\times 12=$ ***Answer: 16***.

3. $\left(2\frac{1}{5}\right)\times(3)=$ ***Answer:*** $\boldsymbol{\frac{33}{5}}$ ***or*** $\boldsymbol{6\frac{3}{5}}$.

4. Given the numbers 2 5 7 9 12 24 (you may use a number more than once) insert four values into the boxes that will produce a product

   a. less than 1 $\left(\frac{\square}{\square}\right)\times\left(\frac{\square}{\square}\right)$ ***Possible answer: any values that make both fractions proper fractions (less than 1).***

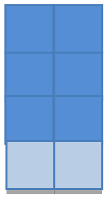
   b. greater than 1 $\left(\frac{\square}{\square}\right)\times\left(\frac{\square}{\square}\right)$ ***Possible answer: any values that make both fractions improper fractions (greater than 1).***

   c. that is a whole number $\left(\frac{\square}{\square}\right)\times\left(\frac{\square}{\square}\right)$ ***Possible answer: The fractions should reduce to a whole number, for example, the product of*** $\boldsymbol{\frac{9}{12}}$ ***and*** $\boldsymbol{\frac{2}{24}}$.

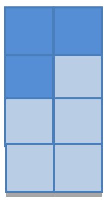
5. Suppose you want to make five and a half batches of a recipe that calls for $\frac{3}{4}$ cup of milk. How much milk should you use? ***Answer:*** $\boldsymbol{4\frac{1}{8}}$ ***cups***.

6. A farmer plants $\frac{3}{4}$ of the field with soybeans.

   a. Shade the field to show the fraction of the field that is planted with soybeans. ***Possible answer: Shade 6 of the 8 squares.***

   b. The next year the farmer decided to plant one half as many soybeans. Shade the field to show the fraction of the field planted with soybeans that year. ***Possible answer: Shade any three of the small squares.***

Adapted from Farmer's Fields (PARRC release 3rd grade item)

Student Activity solutions

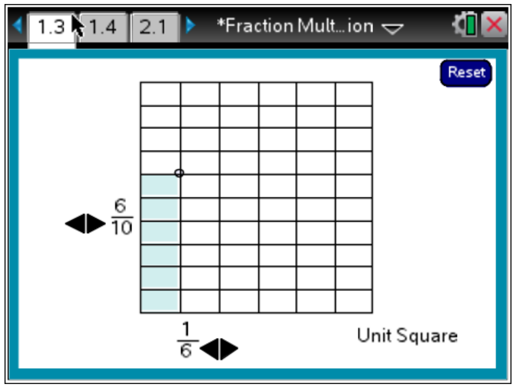
Vocabulary

Commutative Property of Multiplication: a rule that states that the product of two factors is not affected by the order in which they are multiplied

reciprocal: a fraction whose numerator and denominator have been switched

In this activity, you find the product of a fraction and a fraction.

1. Explain how you can argue from the unit square below that $\frac{6}{10} \times \frac{1}{6}$ is the same as $\frac{1}{10}$.



Possible answer: If you moved the 6 shaded rectangles to the bottom row, you would fill up the entire row. There are 10 rows so you have shaded $\frac{1}{10}$ of the rows or $\frac{1}{10}$ of the unit square.

2. Do you think the product of the fractions $\frac{5}{4} \times \frac{2}{3}$ will be more or less than 1? Explain your reasoning.

Possible answer: Less than 1 because there will be 12 congruent rectangles but only 10 of them will be shaded.

3. Explain why $3\times\frac{1}{4}$ is the same as $1\times\frac{3}{4}$.

Possible ***answer: In $3\times\frac{1}{4}$, $\frac{1}{4}$ of the congruent rectangles in three of the unit squares are shaded. That would be 3 of the total of 12 rectangles in those unit squares or $\frac{3}{12}$. But if you moved the shaded rectangles from the top two unit squares to the bottom one, you would have one unit square with 3 of 4 rectangles shaded, which is the same as $1\times\frac{3}{4}$.***

4. Evan and Keisha multiplied $\frac{5}{2}$ and $2\frac{3}{5}$. They each used a different method but got the same solution. Describe the different ways that Evan and Keisha could have used to find the solution.

Possible answer: Evan could have converted the second fraction to an improper fraction and then multiplied both improper fractions to get $\frac{65}{10}$ or $6\frac{1}{2}$. Keisha could have found the product of $\frac{5}{2}$ and 2, which is 5 and then added the product of $\frac{5}{2}$ and $\frac{3}{5}$, which is $\frac{15}{10}$. The sum would be $6\frac{5}{10}$ or $6\frac{1}{2}$.