Building Concepts: Multiplying Fractions by a Whole Number

TEACHER NOTES

Lesson Overview

This TI-Nspire™ lesson can be used to develop multiplication of a fraction by a whole number. The number $\frac{a}{b}$ is ***a*** copies of $\frac{1}{b}$. Multiplying $\frac{a}{b}$ by the whole number ***c***, produces ***c*** copies of $\frac{a}{b}$ joined end to end on the number line.

The product of ***c*** and the fraction $\frac{a}{b}$ would be *c* copies of $\frac{a}{b}$ which is equivalent to ***ca*** copies of $\frac{1}{b}$.

Learning Goals

Students should understand and be able to explain each of the following:

1. The product of a whole number ***n*** and a fraction $\frac{a}{b}$ is $\frac{(na)}{b}$;
2. The fraction $\frac{(na)}{b}$ can be decomposed into the following products:
   - (na) and $\frac{1}{b}$;
   - ***n*** and $\frac{a}{b}$; and
   - ***a*** and $\frac{n}{b}$;
3. If a quantity comes in fraction amounts such as $\frac{a}{b}$, then ***n*** of those amounts will yield $\frac{(na)}{b}$;
4. The whole number factor ***n*** in the product of ***n*** and $\frac{a}{b}$ does not change the denominator except in cases where ***n*** and ***b*** have a common factor.

Prerequisite Knowledge

Multiplying Fractions by a Whole Number is the ninth lesson in a series of lessons that explore fractions. Students should have had experience with the lessons *What is a Fraction? Equivalent Fractions,* and *Mixed Numbers.* Students should be familiar with the following vocabulary: *unit fraction, factors, improper fraction*, and *mixed number*. Prior to working on this lesson students should understand:

- the concept of whole number multiplication
- the concept of equivalent fractions
- that fractions greater than 1 can be represented as improper fractions or mixed numbers.

Vocabulary

- **Commutative Property of Multiplication:** a rule that states that the product of two factors is not affected by the order in which they are multiplied

Lesson Pacing

This lesson should take 50 minutes to complete with students, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- Multiplying Fractions by a Whole Number_Student.pdf
- Multiplying Fractions by a Whole Number_Student.doc
- Multiplying Fractions by a Whole Number.tns
- Multiplying Fractions by a Whole Number_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

✓ **Student Activity Sheet**: The questions that have a check-mark also appear on the Student Activity Sheet. Have students record their answers on their student activity sheet as you go through the lesson as a class exercise. The student activity sheet is optional and may also be completed in smaller student groups, depending on the technology available in the classroom. A (.doc) version of the Teacher Notes has been provided and can be used to further customize the Student Activity sheet by choosing additional and/or different questions for students.

Bulls-eye Question: Questions marked with the bulls-eye icon indicate key questions a student should be able to answer by the conclusion of the activity. These questions are included in the Teacher Notes and the Student Activity Sheet. The bulls-eye question on the Student Activity sheet is a variation of the discussion question included in the Teacher Notes.

Mathematical Background

This TI-Nspire™ lesson can be used to develop multiplication of a fraction by a whole number. The number $\frac{a}{b}$ is ***a*** copies of $\frac{1}{b}$. Multiplying $\frac{a}{b}$ by the whole number ***c***, produces ***c*** copies of $\frac{a}{b}$ joined end to end on the number line. The lesson can be used to illustrate that the product of ***c*** and the fraction $\frac{a}{b}$ is equivalent to $\frac{ca}{b}$, in other words ***ca*** copies of $\frac{1}{b}$.

Another approach is to consider the concept of a quantity used a given number of times. For example, $\frac{a}{b}$ can be considered in essence the "unit" and ***c*** is the number of copies of that "unit" in ***c*** times $\frac{a}{b}$. The activity allows students to locate a quantity, such as $\frac{2}{3}$, on a number line; for example, an item is sold in packages weighing $\frac{2}{3}$ of a pound. Students can then visualize the number of pounds in 5 packages. Mathematically, $c\left(\frac{a}{b}\right)=(c)(a)\left(\frac{1}{b}\right)=(ca)\left(\frac{1}{b}\right)=\frac{ca}{b}$ Students should eventually recognize that $\frac{(ca)}{b}$ is equal to $c\left(\frac{a}{b}\right)$ and to $a\left(\frac{c}{b}\right)$.

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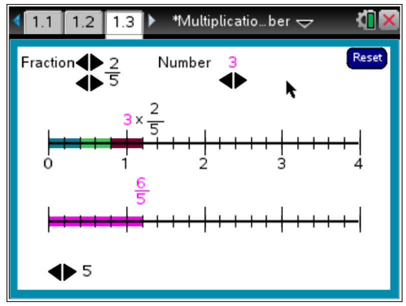
TEACHER NOTES

Part 1, Page 1.3

Focus: Students will use number lines to develop the concept of fraction multiplication.

The horizontal arrows in the upper left change the numerator and denominator of the fraction. The arrows in the middle of the page above the number lines determines the whole number ***n*** in the product $n \times (b)$, that is, the desired number of copies of the fraction. Students should notice that each copy is represented by a different color on the number line.



TI-Nspire Technology Tips

Students may find it easier to use the tab key to toggle between objects and then use the arrow keys to move or change their selections.

To reset the page, select **Reset** in the upper right corner.

Teacher Tip: Have students explain how the visualization on the top number line is related to the value of the product. If students are familiar with mixed numbers, they should note that three copies of $\frac{2}{5}$ can be two copies of $\frac{2}{5}$ and half of the third copy of $\frac{2}{5}$ to make one whole $\left(\frac{5}{5}\right)$, with $\frac{1}{5}$ more so the answer is $1+\frac{1}{5}$ or $\frac{6}{5}$.

The arrow on the bottom left of the page controls the number of equal partitions in each unit on the bottom number line. When the number of partitions is compatible with the number of partitions in the top number line, the value of the fraction associated with the product $n \times \left(\frac{a}{b}\right)$ appears. Note this includes equivalent fractions as well as the value $\frac{(na)}{b}$. Students can use the bottom number line to visualize the result as an improper fraction and as a mixed number.

Teacher Tip: Have students work with a partner to multiply fractions. Have one student write a multiplication composed of a fraction and a whole number and have the partner color partitions on a number line to show the product. Then have students work together using the interactive number line to solve the equation and find an equivalent fraction for the product. Guide students in a discussion about their exploration of fraction multiplication.

Class Discussion

Have students…	**Look for/Listen for…**
Use the arrows on the upper left to create the fraction $\frac{3}{4}$.	
✓ ***If you have three copies of $\frac{3}{4}$, where do you think that value will lie on the number line?*** (Question #1 on the Student Activity sheet.)	Answer: Three sets of $\frac{3}{4}$ will probably be between 2 and 3. The product of $3\left(\frac{3}{4}\right)$ is $2\frac{1}{4}$, which is $2\frac{1}{4}$.
• ***Use the arrow at the top middle to set the number to 3 and check your conjecture for part 1a. What fraction does three copies of $\frac{3}{4}$ represent?***	Answer: 2 and $\frac{1}{4}$; $2\frac{1}{4}$.
• ***How is your answer to part 1b related to copies of the unit fraction $\frac{1}{4}$?***	Answer: It is 9 copies of $\frac{1}{4}$.
• ***Use the arrow on the bottom to change the number of equal partitions in the units on the bottom number line. Describe what happens as you change the number. How does this relate to your answer for the second question?***	Answer: Changing the arrow on the bottom changes the number of partitions for each unit on the bottom number line. Two and $\frac{1}{4}$ is the same as $\frac{9}{4}$.
Use the TNS file to find the fraction values for the following. In each case explain why your answer is reasonable.	
• ***4 copies of $\frac{2}{3}$ =***	Answer: $\frac{8}{3}$; Sample explanation: Multiplying the number of copies and the numerator results in 8, the numerator in the product.
• ***2 copies of $\frac{5}{3}$ =***	Answer: $\frac{10}{3}$; Sample explanation: Multiplying the number of copies and the numerator results in 10, the numerator in the product.

Class Discussion (continued)

- **3 copies of $\frac{3}{2}$ =**

  Answer: $\frac{9}{2}$; Sample explanation: Multiplying the number of copies and the numerator results in 9, the numerator in the product.

- ***2 copies of $\frac{3}{4}$. Is there only one answer? Explain.***

  Possible answer: There could be many answers because one fraction can have many names. Two might be: $\frac{6}{4}$ and $\frac{3}{2}$.

- ***Make a conjecture about how you might find 5 copies of $\frac{3}{7}$. Then use the file to check your conjecture.***

  Answer: multiply the whole number and the numerator to get $\frac{15}{7}$.

The fraction $\frac{4}{5}$ can be written as $4\times\frac{1}{5}$. How could you find each of the following using the unit fraction $\frac{1}{5}$?

- $3\left(\frac{4}{5}\right)=$

  Answer: $3\left(4\times\frac{1}{5}\right)=(3)(4)\left(\frac{1}{5}\right)=\frac{12}{5}=2\frac{2}{5}$.

- $5\left(\frac{4}{5}\right)=$

  Answer: $5\left(4\times\frac{1}{5}\right)=(5)(4)\left(\frac{1}{5}\right)=5\left(\frac{1}{5}\right)(4)=\frac{20}{5}=4$.

- $7\left(\frac{4}{5}\right)=$

  Answer: $7\left(4\times\frac{1}{5}\right)=(7)(4)\left(\frac{1}{5}\right)=\frac{28}{5}=5\frac{3}{5}$.

- $0\left(\frac{4}{5}\right)=$

  Answer: 0.

> **Teacher Tip:** Review the Commutative Property of Multiplication with students. As students write their solutions remind them that that order in which they write the factors does not affect the product. Demonstrate the concept using one of parts above.

Class Discussion (continued)

Having 5 copies of $\frac{3}{7}$ can be thought about as a multiplication problem: $5\times\frac{3}{7}$. Find each of the following. Check your answer using the file.

- $5\times\frac{4}{7}=$

  Answer: $\frac{20}{7}$.

- $4\times\frac{5}{7}=$

  Answer: $\frac{20}{7}$.

- $3\times\frac{7}{6}=$

  Answer: $\frac{21}{6}$ or $\frac{7}{2}$.

Have students…

✓ ***Sally claims that $3\times\frac{5}{6}$ is $\frac{15}{18}$. Is she correct? Explain why or why not.***

(Question #2 on the Student Activity sheet.)

Look for/Listen for…

Possible answer: She is not correct. $\frac{5}{6}$ is 5 copies of $\frac{1}{6}$ and 3 copies of $\frac{5}{6}$ does not change the unit fraction $\frac{1}{6}$ you are using. It just changes the number of copies of that unit fraction. The answer is $\frac{15}{6}$ or $\frac{5}{2}$. If you multiply both the numerator and denominator by 3, you are multiplying by $\frac{3}{3}$, which is 1 and you would not change the value.

How do 3 copies of $\frac{5}{4}$ compare to 5 copies of $\frac{3}{4}$? Explain.

Possible answer: They are the same, $\frac{15}{4}$. (5 copies of $\frac{1}{4}$) is $(3)(5)\left(\frac{1}{4}\right)$ and the order of multiplication does not make a difference so it could be $(5)(3)\left(\frac{1}{4}\right)$ which is 5 copies of $\frac{3}{4}$.

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Class Discussion (continued)

- ***Write $\frac{12}{5}$ as a product of a whole number and a fraction in as many ways as possible. Use the file to check your thinking***

Answer: $1\left(\frac{12}{5}\right)=2\left(\frac{6}{5}\right)=3\left(\frac{4}{5}\right)=6\left(\frac{2}{5}\right)=12\left(\frac{1}{5}\right)$.

Answer each question and explain your reasoning in each case. You may want to check your thinking with the file.

- ***If you figure $\frac{2}{3}$ of a pound of meat per person, how much meat would you need for seven people? Explain your reasoning.***

Answer: $(7)(2)\left(\frac{1}{3}\right)=\frac{14}{3}$ or $4\frac{2}{3}$ pounds.

- ***A six-ounce serving of milk is $\frac{3}{8}$ of a pint. How many pints do you need for 10 six-ounce servings?***

Answer: $10\left(\frac{3}{8}\right)=\frac{30}{8}=\frac{15}{4}$ or $3\frac{3}{4}$ pints.

- ***Each desk is $\frac{7}{3}$ feet long; how long a wall would you need to line up 10 desks next to each other and have the wall equal the total length of the desks? How might your approach change if you thought of $\frac{7}{3}$ feet as 2 and $\frac{1}{3}$ foot?***

Possible answer: $10\left(\frac{7}{3}\right)=\frac{70}{3}$ feet or $23\frac{1}{3}$ feet. If you used 2 and $\frac{1}{3}$ foot you could multiply each part by 10 so you would have 20 and $\frac{10}{3}$, which would be 20 and $3\frac{1}{3}$ or $23\frac{1}{3}$ feet.

✓ ***Find a fraction so that the product of 6 and the fraction is larger than 6. Find another fraction so the product is smaller than 6. Make a general rule about the size of a product if one of the factors is 6.*** (Question #3 on the Student Activity sheet.)

Possible answer: $6\left(\frac{3}{2}\right)=\frac{18}{2}=9$, which is larger than 6. $6\left(\frac{2}{3}\right)=\frac{12}{3}=4$, which is smaller than 6. As a general rule, to get a product larger than 6, multiply by a fraction greater than 1; to get a product smaller than 6, multiply by a fraction less than 1.

Class Discussion

Have students…	Look for/Listen for…
Decide whether each is true or false and explain your reasoning in each case.	
• ***To multiply a fraction by 5 you multiply the numerator and denominator of the fraction by 5.***	Answer: False. You only multiply the numerator; you are not changing the unit fraction.
• ***The product of 7 and*** $\frac{5}{12}$ ***is the same as the product of 5 and*** $\frac{7}{12}$***.***	Answer: True. You either have 7(5) copies of $\frac{1}{12}$ or 5(7) copies of $\frac{1}{12}$, which are both $\frac{35}{12}$.
• $2\times\frac{8}{2}$ ***can be written as the product of two whole numbers.***	Answer: True because $\frac{16}{12}$ is 8, which can be written as 2(4) or as 1(8).
• ***The result of multiplying a fraction by a whole number greater than or equal to 1 is always greater than or equal to the original fraction.***	Answer: True because you are increasing the copies of the fraction by the whole number.
• ***Use the file to describe two different ways to find*** $2\times 3\frac{5}{8}$***.***	Answer: You can think of 3 and $\frac{5}{8}$ as $\frac{29}{8}$ and $2\left(\frac{29}{8}\right)$ would be $\frac{58}{8}$ or $\frac{29}{4}=7$ and $\frac{1}{4}$. Or you can take 2(3) and $2\left(\frac{5}{8}\right)$ to get 6 and $\frac{10}{8}$, which would be 6 and $\frac{5}{4}$ or 1 and $\frac{1}{4}$, so the result would be 7 and $\frac{1}{4}$.
• ***Each member of a three-person relay team runs*** $3\frac{3}{8}$ ***miles. What is the total distance run by the team?***	Answer: 9 and $\frac{9}{8}$ or 10 and $\frac{1}{8}$ miles. It could be written $\frac{81}{8}$ miles.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. Insert values into the blanks to make a true statement (a number may be used more than once).

   3, 5, 6, 8, 2, 10, 12

   a. $\square\left(\frac{\square}{\square}\right)=\frac{\square}{\square}$ ***Possible answer:*** $\boldsymbol{5\left(\frac{3}{10}\right)=\frac{3}{2}}$

   b. $\left(\frac{12}{5}\right)=\square\frac{\square}{\square}$ ***Possible answer:*** $\boldsymbol{6\left(\frac{2}{5}\right)}$

2. ***10 x (3/4) =***

   a. $\frac{30}{40}$ b. $\frac{30}{4}$ c. $\frac{15}{4}$ d. $\frac{3}{40}$

   Answer: b.

3. If $1\frac{2}{3}$ cups of flour are needed for a batch of cookies, how many cups of flour will be needed for 3 batches?

   adapted from NAEP grade 4, 1992

   a. $4\frac{2}{3}$ b. 5 c. $3\frac{2}{3}$ d. $4\frac{1}{3}$

   Answer: b.

4. Sami walks $2\frac{7}{8}$ miles an hour. How far could she walk in 2 hours?

   Answer: $\boldsymbol{\frac{46}{8}}$ ***or*** $\boldsymbol{\frac{23}{4}}$ ***or 5 and*** $\boldsymbol{\frac{3}{4}}$ ***miles***.

5. Tracy said, “I can multiply 6 by another number and get an answer that is smaller than 6.” Pat said, “No, you can't. Multiplying 6 by another number always makes the answer 6 or larger.” Who is correct? Give a reason for your answer.

NAEP 1992 grades 8, 12

Possible answer: Tracy because you can multiply 6 by any fraction that is less than 1 to get a number less than 6. For example, multiplying 6 by $\frac{3}{4}$ results in $\frac{9}{2}$, which is smaller than 6.

Student Activity solutions

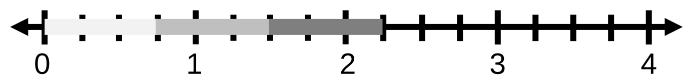
Vocabulary

Commutative Property of Multiplication:

a rule that states that the product of two factors is not affected by the order in which they are multiplied

In this activity, use number lines to multiply whole numbers with fractions.

1. If you have three copies of $\frac{3}{4}$, where do you think that value will lie on the number line? Explain your reasoning. Use the number line below to show your answer.



Answer: Three sets of $\frac{3}{4}$ will probably be between 2 and 3.

The product of $3\left(\frac{3}{4}\right)$ is $\frac{9}{4}$, which is $2\frac{1}{4}$.

2. Sally claims that $3\times\frac{5}{6}$ is $\frac{15}{18}$. Is she correct? Explain why or why not.

Answer: She is not correct. Possible explanation: $\frac{5}{6}$ is 5 copies of $\frac{1}{6}$ and 3 copies of $\frac{5}{6}$ does not change the unit fraction $\frac{1}{6}$ you are using. It just changes the number of copies of that unit fraction. The answer is $\frac{15}{6}$ or $\frac{5}{2}$. If you multiply both the numerator and denominator by 3, you are multiplying by $\frac{3}{3}$, which is 1 and you would not change the value.

3. Find a fraction so that the product of 6 and the fraction is larger than 6. Find another fraction so the product is smaller than 6. Make a general rule about the size of a product if one of the factors is 6.

Possible answer: $6\left(\frac{3}{2}\right)=\frac{18}{2}=9$ ***, which is larger than 6.*** $6\left(\frac{2}{3}\right)=\frac{12}{3}=4$ ***, which is smaller than 6. A general rule would be to get a product larger than 6, multiply by a fraction greater than 1; to get a product smaller than 6, multiply by a fraction less than 1.***

4. Joshua says that 4 copies of $\frac{7}{6}$ is less than 7 copies of $\frac{4}{6}$. Is Joshua correct? Explain your reasoning.

Answer: Joshua is not correct. They are both the same. 4 (7 copies of $\frac{1}{6}$ ***) is*** $(4)(7)\left(\frac{1}{6}\right)$***. Since the order of multiplication does not affect the product,*** $(7)(4)\left(\frac{1}{6}\right)$ ***would result in the same answer*** $\frac{28}{6}$.