# Building Concepts: Solving Proportions

TEACHER NOTES

## Lesson Overview

This TI-Nspire™ lesson uses students' understanding of proportional relationships to solve problems that involve two variables. Many problems can be framed in terms of the unit rate; still this approach is often obscured by the traditional method of setting up proportions. This lesson provides students with another method for visualizing and solving problems using proportional relationships.

The unit rate is the *y*-coordinate of the point $(1, \frac{5}{2})$; the product of the unit rate and the *x*-coordinate is equal to the *y*-coordinate, specifying the equation of the line containing the ray representing the proportional relationship.

### Learning Goals

1. Distinguish situations that involve proportional relationships from those that do not;
2. use a variety of strategies for solving problems involving proportions, including finding a unit rate;
3. attend to the units involved in working with proportional relationships.

### Prerequisite Knowledge

*Solving Proportions* is the twelfth lesson in a series of lessons that explore the concepts of ratios and proportional relationships. The lesson builds on students' prior knowledge of graphing proportional relationships. Prior to working on this lesson, students should have completed *Connecting Ratios to Graphs*, *Connecting Ratios to Equations,* and *Proportional Relationships*. Students should:

- understand how to represent proportional relationships between quantities using tables, graphs, diagrams, and equations;
- be able to determine whether two quantities are in a proportional relationship;
- be able to identify the dependent and independent variables for a given context and understand how they are interpreted in a graph.

### Vocabulary

- **proportional relationship**: a collection of equivalent ratios.
- **constant of proportionality**: For the ratio A:B, the constant of proportionality, *k*, in the equation $y = kx$ is equal to the value $\frac{B}{A}$.

### Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

## Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Solving Proportions_Student.pdf
- Solving Proportions_Student.doc
- Solving Proportions.tns
- Solving Proportions_Teacher Notes
- To download the TI-Nspire lesson (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

## Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

**Class Discussion:** Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS lesson as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

**Student Activity:** Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

**Additional Discussion:** These questions are provided for additional student practice, and to faciliate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

## Mathematical Background

This TI-Nspire™ lesson uses students' understanding of proportional relationships to solve problems. Proportions are tools for solving a variety of problems with two variables involved in a proportional relationship. Students need to examine situations carefully to determine if they describe a proportional relationship. For example, if Joe is 9 and Rae is 7, how old will Rae be when Joe is 15? This problem cannot be solved with the proportion $\frac{9}{7}=\frac{15}{x}$ because it is not the case that for every 9 years that Joe ages, Rae ages 7 years. Instead, when Joe has aged another 6 years, Rae will as well, and so she will be 13 when Josh is 15.

Many problems can be framed in terms of proportional relationships and the constant of proportionality or unit rate. This approach is often obscured by the traditional method of setting up proportions. A common error in setting up proportions is placing numbers in incorrect locations. This is especially easy to do when the order in which quantities are stated in the problem is switched within the problem statement. For this reason students should pay particular attention to the units involved in setting up the problem. A similar difficulty might occur because students confuse a point defining the proportion with the corresponding coefficient of $x$ in the equation; i.e., (2, 5) defines a proportion but the equation is $y=\frac{5}{2}x$. They might need to revisit the entire reasoning chain several times: the ratio 2:5 defines a collection of equivalent ratios; 2:5 is associated with the point (2, 5) that along with (0, 0) determines the ray representing all the equivalent ratios, which is the proportional relationship. The unit rate is the $y$-coordinate of the point $(1, \frac{5}{2})$; the product of the unit rate and the $x$-coordinate is equal to the $y$-coordinate, specifying the equation of the line containing the ray representing the proportional relationship.

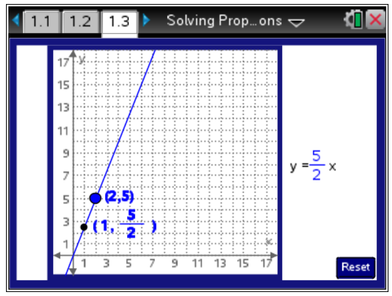
 

## Part 1, Page 1.3

Focus: What different strategies have you learned for solving problems involving proportions? Are some easier to use than others?

On page 1.3, the black point can be moved along the line by grabbing and dragging or by using the left/right arrows on the handheld or computer. Dragging the blue point will change the position of the line.

Have students move the black point along the line to identify additional points on the graph that are part of the proportional relationship represented.




**TI-Nspire Technology Tips**

The [tab] key will toggle between the two points. Using the right/left arrows will move the black point along the line.

To reset the document press **Reset** or [ctrl] [del] to reset.

The following questions engage students in reasoning about problems involving proportions. They use unit rates, equations, graphs, and revisit ratio tables and double number lines. The importance of units in helping set up a problem and in thinking about the solution is stressed in several places.

### Class Discussion

| Have students… | Look for/Listen for… |
|---|---|
| • ***Identify three points in addition to (2, 5) that would be in the proportional relationship represented on the graph. Give at least two reasons for why you think your answer is correct*.** | Possible answer: (4, 10); (6, 15); (8, 20). The points are on the line because I can read them off the graph. They are associated with ratios that are all equivalent to 2:5 because a collection of equivalent ratios lies on a straight line through the origin. They are also solutions to the equation of the line, $y = \frac{5}{2}x$. |
| • ***What is the unit rate and how is it connected to the graph and to the equation?*** | Answer: The unit rate is $\frac{5}{2}$. It comes from the point (1, $\frac{5}{2}$), and it is the coefficient of $x$ in the equation, which is the rate of change or constant of proportionality. |

## Student Activity Questions—Activity 1

1. **George bought 8 pounds of bananas for $5.**

   a. **Use the TNS page to make a graph showing the cost for different amounts of bananas. Use the graph to help find the price per pound of the bananas George bought.**

   Answer: Graph a ray through the point (8, 5) and the origin. The unit rate will give the price per pound, which is $\frac{5}{8}$ of a dollar.

   b. **How can you find the unit rate without using the graph?**

   Answer: Find the ratio equivalent to 8:5 with the first value 1 by dividing both values by 8 to get 1:$\frac{5}{8}$, which is associated with the ordered pair (1, $\frac{5}{8}$).

   c. **How much would 9 pounds of bananas cost? Explain how you can find the answer using the graph and how you can find the answer using the equation.**

   Answer: You can estimate the answer from the points on the graph. Using the unit rate in the equation, you can let *x* be 9 and solve the equation $y = \frac{5}{8} \times 9 = \frac{45}{8}$ or about $5.63.

   d. **Explain whether the units in the equation make sense in the context of the problem.**

   Answer: If *c* represents the cost in dollars and p the number of pounds, the equation would be $c\$ = \frac{\$5}{8 \text{ pounds}} \times p$ pounds. In the expression on the right, pounds in the numerator and denominator reduce leaving only $.

2. **A caterer plans for 8 pounds of beef for every 10 guests.**

   a. **Does this situation represent a proportional relationship? Why or why not?**

   Answer: Yes because it makes sense to have 0 pounds of beef for 0 guests, so the line will go through the origin and be determined by the ordered pair (10, 8), where the number of guests is the independent variable.

   b. **What is the ratio of the amount of beef for every 10 guests? How many pounds will there be per guest?**

   Answer: The ratio 10:8 is equivalent to 1:$\frac{8}{10}$ or 1:$\frac{4}{5}$ Each guest will get $\frac{4}{5}$ pound.

   c. **Write an equation that represents the proportion in the problem. Explain how the units can help you make sense of the problem.**

   Answer: If *y* is the amount of beef in pounds and *x* is the number of guests, $y \text{ pounds} = \frac{8 \text{ pounds}}{10 \text{ guests}} \bullet x \text{ guests}$. The unit "guests" reduces from the right side and leaves $y \text{ pounds} = \frac{4}{5} x \text{ pounds}$.

**Student Activity Questions—Activity 1 (continued)**

d. **Graph the equation on the TNS page and use the graph to find how many pounds of beef should be prepared for 14 guests. How could you get your answer using the unit rate without using the graph?**

Answer: $\frac{56}{5}$ or $11\frac{1}{5}$ pounds. You could multiply the amount per guest times the number of guests $\frac{4}{5}(14)$.

3. **Lila wanted to make a chart so she would know how much meat she should cook for up to 10 guests. Explain how Lila can make her chart and then create one for her based on the caterer's plan of 8 pounds of beef for every 10 guests. You may want to fill out the chart and then check your answer using the TNS lesson.**

Answer: She can use the unit rate of $\frac{4}{5}$ pound per person and fill out a table by adding $\frac{4}{5}$ pound each time she adds a person. Or she could read the ordered pairs from the line representing the proportion.

| Number of Guests | Pounds of Meat |
|---|---|
| 1 | $\frac{4}{5}$ |
| 2 | $\frac{8}{5}$ or $1\frac{3}{5}$ |
| 3 | $\frac{12}{5}$ or $2\frac{2}{5}$ |
| 4 | $\frac{16}{5}$ or $3\frac{1}{5}$ |
| 5 | $\frac{20}{5}$ or 4 |
| 6 | $\frac{24}{5}$ or $4\frac{4}{5}$ |
| 7 | $\frac{28}{5}$ or $5\frac{3}{5}$ |
| 8 | $\frac{32}{5}$ or $6\frac{2}{5}$ |
| 9 | $\frac{36}{5}$ or $7\frac{1}{5}$ |
| 10 | $\frac{40}{5}$ or 8 |

**Student Activity Questions—Activity 1 (continued)**

4. **Think about the following problems from your work in question 3.**

   **A caterer plans for 8 pounds of beef for every 10 guests. How much beef do you need for 14 guests?**

   **George bought 8 pounds of bananas for $5. How much would it cost for 9 pounds of bananas?**

   **Which of them could you have solved using a ratio table? A double number line? Explain why at least one of the strategies will or will not work.**

   Possible answer: Both problems could have been solved using a double number line or a ratio table. For example, using a ratio table for the caterer:

| | Divide both by 4 | Add the first 2 columns |
|---|---|---|
| 8 lbs | 2 | 10 |
| 10 people | $\frac{10}{4} = 2.5$ | 12.5 |

5. **Jon had a blueprint for a house in which 2 centimeters represented 5 feet. He was trying to figure out the actual length of one of the rooms that was 7 centimeters on the blueprint.**

   a. **Show two different strategies you could use to solve his problem. Find the unit rate for each strategy. Check your thinking using the TNS lesson.**

      Possible answer: He could set up the proportion using the ratio 2 centimeters to 5 feet where the unit rate would be $\frac{5}{2}$ feet to 1 centimeter, or he could use a double number line.

   b. **Write the equation for each strategy and explain how the units would work.**

      Answer: The first strategy would fit the equation $d \text{ feet} = \frac{5/2 \text{ feet}}{1 \text{ centimeter}} \times 7 \text{ centimeters}$ and the centimeters would reduce. Students can also use the double number line, with feet on one number line and centimeters on the other, to show the connection between the two quantities.

   c. **Explain which strategy you would choose to use and why.**

      Answers will vary. Possible answer: I would use the first strategy because on a blueprint you probably want to convert a lot of the measurements and you can use the same process every time, just changing the measurement from the blueprint. For the second strategy, you would have to create a new problem each time you wanted to convert a new measurement.

6. **Simon ordered 4 bags of soil for his plants. Each small group of plants needs $\frac{3}{4}$ of a bag of soil. How many groups of plants can he fill completely with soil? How much soil does he have left? Solve this problem in as many different ways you can think of. Reflect on the strategies you used and answer the following questions.**

   a. **Which strategy seemed to be easiest?**

      Answers will vary.

## Student Activity Questions—Activity 1 (continued)

**b. What are some of the advantages or disadvantages of the strategies if you tried to use them on any problem?**

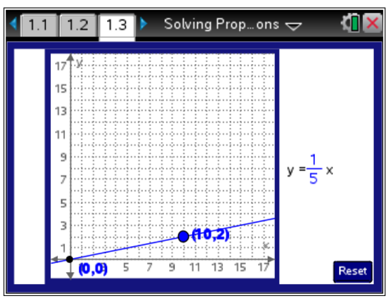
Answer: You can use the soil on 5 groups of plants with $\frac{1}{4}$ bag left over. Students might solve this using a double number line, a table of ratios going by the unit rate, a ratio table, the unit rate, the equation, a graph, or drawing a diagram. The answers to the questions will vary. They might note that making a table with the unit rate might take a long time if the numbers were large, and that the equation would work for any case, no matter how large the numbers. Ratio tables are easy to work with and you can play with the numbers to get the combination you want. A graph shows you how everything is related but might be difficult to use with big numbers.

## Additional Discussion

***There is a proportional relationship between the size of a projected image on a screen and the distance of the screen from the projector. An image that is projected onto a screen 10 feet away is a rectangle with dimensions of 2 feet by 3 feet.*** (Adapted from F-CAT, Florida, 2006, Gr. 9.)

- ***If the projected width for a figure is 2 feet, what is one point in the proportional relationship (distance, projected width)? Use this point to draw the graph of the relationship.***

  Answer: (10, 2) is one possible point.

- ***If the screen is moved to a distance of 15 feet from the projector, what will be the width of the larger image projected onto the screen? Use three different strategies to find your answer.***

  Answer: 1) The new width will be 3 feet because the ratio associated with the point (10, 2) is equivalent to the ratio associated with the point (15, 3). 2) Substitute 15 feet into the equation to get $y = \frac{1}{5}(15) = 3$ feet 3) Read the answer off the graph.

- ***Use the same strategy to find the length of the projected image, which was 3 feet on the first image, when the projector is 15 feet away. Explain what you did to find the answer.***

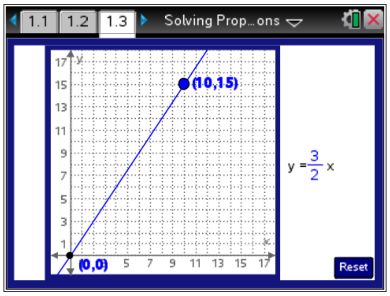
  Possible answer: I made a new line using the point (10, 3) and from that line found the answer of $\frac{9}{2}$ or $4\frac{1}{2}$ feet.

## Additional Discussion (continued)

***Suppose you started the first problem with the question: How will the image of an object 10 feet away from the projector change when it is 15 feet away from the project?***

- ***Find a point in the proportional relationship and use it and the point (0, 0) to draw the graph of the relationship.***

  Possible answer: (10, 15) for (image1, image2)

- ***Explain how you can use your graph to find how the image of a rectangle 2 feet by 3 feet when it is 10 feet from the projector will change when the image is 15 feet from the projector.***

  Possible answer: You can just use the line to find that 2 feet will change to 3 feet from the ordered pair (2, 3) for (image1, image2) and 3 feet will change to $\frac{9}{2}$ or $4\frac{1}{2}$ feet from the ordered pair $(3, \frac{9}{2})$.

- ***Explain the difference in the way the first question and the question at the top of this page were set up and solved.***

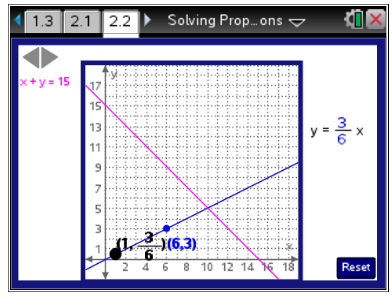
  Answer: In the first question, you needed to set up two different proportions to find the solution, one for the length and one for the width because your original proportion involved one distance and an image dimension. In the second question, you needed to set up only one proportion to find the solution, old distance to new distance, because you could use the same proportion to find both the length and width.

- ***How would using the equation for the proportion have changed the work you needed to do for the first question and the question at the top of this page?***

  Answers will vary: Students might suggest that they just have to substitute the given value into the equation and solve for the missing value.

## Part 2, Page 2.2

Focus: Solving proportional relationships using a graph.




- On page 2.2, the points on the blue line behave the same as the line on page 1.3. The pink line represents a constraint on the sum of the variables.
- Set the equation $x + y = n$ by using the arrow keys or arrow buttons at the top of the page.
- Move the blue point to set the proportion shown on the blue line.
- Move the black point to show the solution to the equation. The point will turn pink when its coordinates satisfy the equation.
- Move the points by dragging the points along graph or using the arrow keys.

> **Teacher Tip:** Toggle between the points and the set of arrows (that set the equation at the top of the page) by pressing the tab key.

In the following questions, students consider ratio problems that involve an additional constraint on the quantities. In particular, the two quantities involved add to a given sum as well as have a proportional relationship. Students look at the problem using a table, a graph and equations.

### Class Discussion

| Have students… | Look for/Listen for… |
|---|---|
| ***Suppose both eighth and seventh graders played on the soccer team.*** | |
| • ***If there were 15 students on the team, how many eighth graders and how many seventh graders might be on the team?*** | Answer: Any combination of two whole numbers that add up to 15. |
| • ***Look at page 2.2. What might the pink line represent in the problem? Use ordered pairs you can read from the graph that are on the line as an example of your thinking.*** | Answer: The pink line could be the number of students in the class showing the sum of the number of seventh graders, *x*, and number of eighth graders, *y* (or it could be the other way around). So the point (14, 1) could be 14 eighth graders and 1 seventh grader, or the point (3, 12) could be 3 eighth graders and 12 seventh graders. |
| • ***The blue line represents the ratio of eighth graders to seventh graders on the team. What is the ratio? Explain your thinking using ordered pairs as an example.*** | Answer: It could be the proportion for the ratio of the eighth graders to the seventh graders. So the point (2, 1) would mean 2 eighth graders for every 1 seventh grader or a ratio of 2:1 or any equivalent ratio. |

**Student Activity Questions—Activity 2**

**Use page 2.2 of the lesson to answer the questions below.**

1. **The blue line represents the ratio of eighth graders to seventh graders on the team.**

   a. **Make a list of possible ordered pairs that would come from the equivalent ratios. If (*x, y*) represents (number of eighth graders, number of seventh graders), how many total students does each ordered pair in your list represent?**

   Answer:

| ***Ordered pairs*** | ***Number eighth graders and number seventh graders*** | ***Total students*** |
|---|---|---|
| ***(2, 1)*** | ***2 grade 8, 1 grade 7*** | ***3*** |
| ***(4, 2)*** | ***4 grade 8, 2 grade 7*** | ***6*** |
| ***(6, 3)*** | ***6 grade 8, 3 grade 7*** | ***9*** |
| ***(8, 4)*** | ***8 grade 8, 4 grade 7*** | ***12*** |
| ***(10, 5)*** | ***10 grade 8, 5 grade 7*** | ***15*** |
| ***(12, 6)*** | ***12 grade 8, 6 grade 7*** | ***18*** |
| ***(14, 7)*** | ***14 grade 8, 7 grade 7*** | ***21*** |
| ***(16, 8)*** | ***16 grade 8, 8 grade 7*** | ***24*** |
| ***(18, 9)*** | ***18 grade 8, 9 grade 7*** | ***27*** |

   b. **Which of the ordered pairs from your list in part a satisfies both requirements: the ratio of eighth graders to seventh graders is 2:1 and the total number of students is 15?**

   Answer: (10, 5)

   c. **On page 2.2 move the black dot to the point where the two lines intersect. What is this point and what does it represent?**

   Answer: The intersection point is (10, 5), which is the ordered pair that makes both equations true. The ratio of eighth graders to seventh graders on the team is 10:5, which is equivalent to 6:3, and there are 15 students, 10 eighth graders and 5 seventh graders.

2. **The ratio of dogs to cats in the animal shelter is 3:5. There are a total of 24 dogs and cats in the shelter. How many cats are in the shelter? Explain how you found your answer.**

   Answer: The ratio 3:5 is equivalent to 9:15, and 9 dogs to 15 cats makes 24 dogs and cats, so there are 15 cats. Strategies will vary; some students may make lists and others try to use the TNS lesson. Some may guess.

## Student Activity Questions—Activity 2 (continued)

**3. If *x* represents the number of dogs and *y* represents the number of cats, which of the following equations describe a condition in the problem for question 2? Explain your thinking.**

**a.** $y = \frac{3}{5}x$ **b.** $y = \frac{5}{3}x$ **c.** $x + y = 24$ **d.** $3x + 5y = 24$ **e.** $5x + 3y = 24$

Answer: b and c. b describes the proportion for a ratio of 3:5, where the constant of proportionality or unit ratio is $\frac{5}{3}$, and c describes the sum of the number of dogs and the number of cats as 24.

**4. Insert the information from question 3 into the TNS page to check your answer to question 2.**

Answer: The point of intersection should be (9, 15), so 15 cats are in the shelter.

## Additional Discussion

**Sand and cement are mixed in a ratio of 3 cubic yards of sand to 2 cubic yards of cement.**

- ***Set up the proportion on the TNS page. The mixture is sold in lots of a whole numbers of cubic yards of sand and of cement. The largest lot is 30 cubic yards. What other size lots are possible? You may want to use the lesson to help your thinking.***

  Answer: 5 cubic yards, 10 cubic yards, 15 cubic yards, 20 cubic yards and 25 cubic yards.

- ***The shop offered a special lot of 15 cubic yards. How many cubic yards of sand and of cement were in the special?***

  Answer: 6 cubic yards of cement and 9 yards of sand.

- ***Sally decided not to use the graph and instead did the following:***
  ***If x = cubic years of sand and y = cubic yards of concrete, $y = \frac{2}{3}x$ and $x + y = 15$.***
  $x + \frac{2}{3}x = 15 \rightarrow \frac{5}{3}x = 15 \rightarrow x = 15 \times \frac{3}{5} \rightarrow x = 9$ ***cubic yards of sand and*** $y = \frac{2}{3} \bullet 9 = 6$ ***cubic yards of cement. Do you agree or disagree with Sally? Explain your thinking.***

  Possible answer: I agree because she uses $\frac{2}{3}$ in her equation to represent the ratio of cubic yards of sand to cement and an equation to represent the total amount of material present. She solved the equations to determine the appropriate amount of sand to cement.

- ***Sally argued that she made one equation out of the two conditions by substituting one into the other, so she only had one variable in the new equation. Then she used what she knows about solving equations with one variable to find an answer. Once she knew one variable, she could find the other. Does this change your answer to the previous question?***

  Possible answer: No, I agree with her reasoning.

**Additional Discussion (continued)**

***Describe how your strategies for solving the following two problems would be different and how they might be alike***.

- ***The Park City School District bought some math books. The district distributed these books to the Brown Middle School and Thompson Middle School using a 4 to 5 ratio. Brown Middle School received 200 books. How many books are there in total?***

  Answer: You can set up the ratio 4:5 and get the proportion $y=\frac{5}{4}x$ where *x* represents the number of books for Brown and *y* represents the number of books for Thompson. You would just use that proportion and/or the graph to figure out the value for *y* when *x* is 200 and then find the sum of *x* and *y*.

- ***The Park City School District bought some math books. The district distributed these books to the Brown Middle School and Thompson Middle School using a 4 to 5 ratio. All together there were 450 books. How many did Brown Middle School receive?***

  Answer: Again you can set up the ratio 4:5 and get the proportion $y=\frac{5}{4}x$ where *x* represents the number of books for Brown and y represents the number of books for Thompson. You know the total is 450, so you really have another condition $x+y=450$. Using the two conditions together you can find the ordered pair that makes them both true and from that get the number of books for Brown.

## Sample Assessment Items

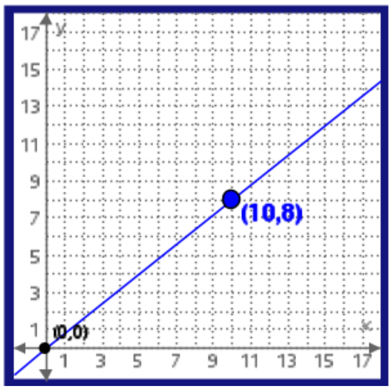
After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS lesson.

1. A bus travels so that the distance traveled is directly proportional to the time spent traveling. If the bus travels 120 km in 5 hours, how many kilometers does it travel in 8 hours?

   a. 168 b. 192 c. 200 d. 245

   ***Answer: b***

2. Which of the following is an equation for the proportion represented in the graph?



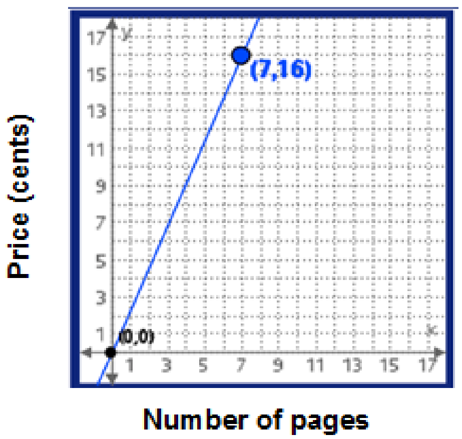

   a. $y = \frac{4}{5}x$ b. $y = 10x + 8$ c. $y = \frac{5}{4}x$ d. $y = 8x + 10$

   ***Answer: a***

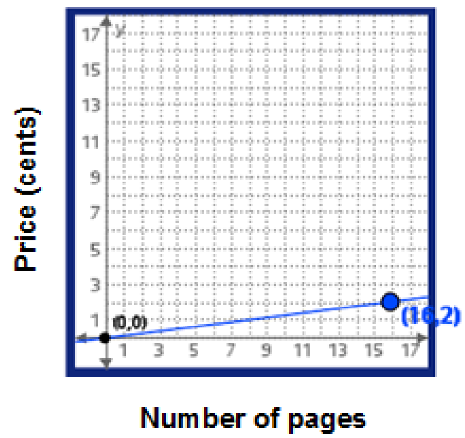
3. A board is cut into two pieces in a ratio of 4:5. If the length of the board is 36 inches, how long is each piece? ***Answer: 16 inches and 20 inches***

4. Which of the graphs below can be used to answer the following question? A copy shop offered a bargain: 7 cents for copying 2 pages, single-sided.

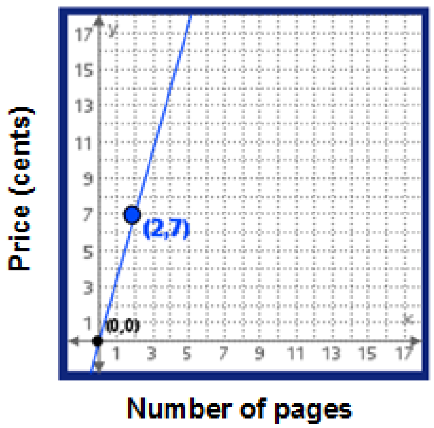
a.



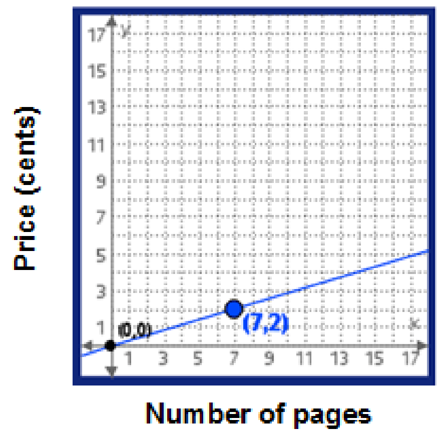

b.




c.




d.



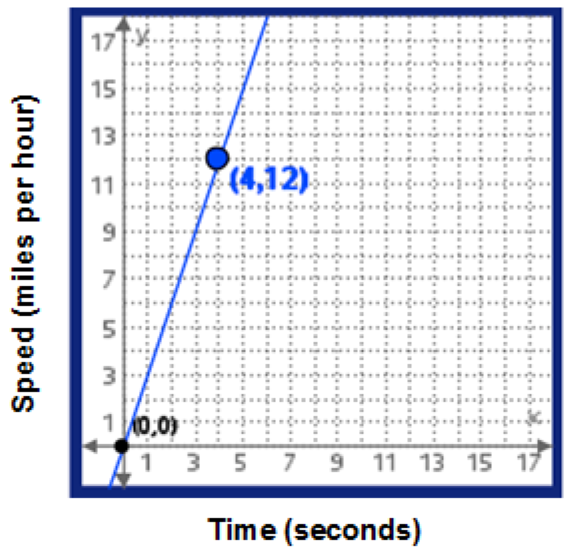

***Answer: c***

5. Two copy shops offered bargains. Which shop has the better deal?
Shop A -7 cents for copying 2 pages, single-sided
Shop B- 10 cents for copying 3 pages, single-sided.

***Answer: the second shop because it is*** $3\frac{1}{3}$ ***cents per page and the first shop is*** $3\frac{1}{2}$ ***cents per page.***

6. An automobile testing organization is verifying the acceleration characteristics of a car. The car will accelerate at a rate of 3 mph per second from 0 miles per hour (mph) to 45 mph. The graph below shows the beginning of the ideal acceleration plot. If the rate of acceleration remains constant, how many seconds will it take the car to reach its final test speed of 45 mph?

(Adapted from FCAT Florida 2006, Grade 9.)




***Answer: 15 seconds***

## Student Activity Solutions

In these activities you will use a variety of strategies for solving problems involving proportions. After completing each activity, discuss and/or present your findings to the rest of the class.

**Activity 1 [Page 1.3]**

1. George bought 8 pounds of bananas for $5.

   a. Use the TNS page to make a graph showing the cost for different amounts of bananas. Use the graph to help find the price per pound of the bananas George bought.

   *Answer: Graph a ray through the point (8, 5) and the origin. The unit rate will give the price per pound, which is* $\frac{5}{8}$ *of a dollar.*

   b. How can you find the unit rate without using the graph?

   *Answer: Find the ratio equivalent to 8:5 with the first value 1 by dividing both values by 8 to get 1:* $\frac{5}{8}$*, which is associated with the ordered pair (1,* $\frac{5}{8}$*).*

   c. How much would 9 pounds of bananas cost? Explain how you can find the answer using the graph and how you can find the answer using the equation.

   *Answer: You can estimate the answer from the points on the graph. Using the unit rate in the equation, you can let x be 9 and solve the equation* $y = \frac{5}{8} \times 9 = \frac{45}{8}$ *or about $5.63.*

   d. Explain whether the units in the equation make sense in the context of the problem.

   *Answer: If c represents the cost in dollars and p the number of pounds, the equation would be* $c\$ = \frac{\$5}{8\ pounds} \times p$ *pounds. In the expression on the right, pounds in the numerator and denominator reduce leaving only $.*

2. A caterer plans for 8 pounds of beef for every 10 guests.

   a. Does this situation represent a proportional relationship? Why or why not?

   *Answer: Yes because it makes sense to have 0 pounds of beef for 0 guests, so the line will go through the origin and be determined by the ordered pair (10, 8), where the number of guests is the independent variable.*

   b. What is the ratio of the amount of beef for every 10 guests? How many pounds will there be per guest?

   *Answer: The ratio 10:8 is equivalent to 1:* $\frac{8}{10}$ *or 1:* $\frac{4}{5}$ *Each guest will get* $\frac{4}{5}$ *pound.*

c. Write an equation that represents the proportion in the problem. Explain how the units can help you make sense of the problem.

*Answer: If y is the amount of beef in pounds and x is the number of guests,*

$y \text{ pounds} = \frac{8 \text{ pounds}}{10 \text{ guests}} \bullet x \text{ guests}$. *The unit "guests" reduces from the right side and leaves*

$y \text{ pounds} = \frac{4}{5} x \text{ pounds}$.

d. Graph the equation on the TNS page and use the graph to find how many pounds of beef should be prepared for 14 guests. How could you get your answer using the unit rate without using the graph?

*Answer:* $\frac{56}{5}$ *or* $11\frac{1}{5}$ *pounds. You could multiply the amount per guest times the number of guests* $\frac{4}{5}(14)$.

3. Lila wanted to make a chart so she would know how much meat she should cook for up to 10 guests. Explain how Lila can make her chart and then create one for her based on the caterer's plan of 8 pounds of beef for every 10 guests. You may want to fill out the chart and then check your answer using the TNS lesson.

*Answer: She can use the unit rate of* $\frac{4}{5}$ *pound per person and fill out a table by adding* $\frac{4}{5}$ *pound each time she adds a person. Or she could read the ordered pairs from the line representing the proportion.*

| Number of Guests | Pounds of Meat |
|---|---|
| *1* | $\frac{4}{5}$ |
| *2* | $\frac{8}{5}$ or $1\frac{3}{5}$ |
| *3* | $\frac{12}{5}$ or $2\frac{2}{5}$ |
| *4* | $\frac{16}{5}$ or $3\frac{1}{5}$ |
| *5* | $\frac{20}{5}$ or 4 |
| *6* | $\frac{24}{5}$ or $4\frac{4}{5}$ |
| *7* | $\frac{28}{5}$ or $5\frac{3}{5}$ |
| *8* | $\frac{32}{5}$ or $6\frac{2}{5}$ |
| *9* | $\frac{36}{5}$ or $7\frac{1}{5}$ |
| *10* | $\frac{40}{5}$ or 8 |

4. Think about the following problems from your work above.

   A caterer plans for 8 pounds of beef for every 10 guests. How much beef do you need for 14 guests?

   George bought 8 pounds of bananas for $5. How much would it cost for 9 pounds of bananas?

   Which of them could you have solved using a ratio table? A double number line? Explain why at least one of the strategies will or will not work.

   *Possible answer: Both problems could have been solved using a double number line or a ratio table. For example, using a ratio table for the caterer:*

| | *Divide both by 4* | *Add the first 2 columns* |
|---|---|---|
| *8 lbs* | *2* | *10* |
| *10 people* | $\frac{10}{4} = 2.5$ | *12.5* |

5. Jon had a blueprint for a house in which 2 centimeters represented 5 feet. He was trying to figure out the actual length of one of the rooms that was 7 centimeters on the blueprint.

   a. Show two different strategies you could use to solve his problem. Find the unit rate for each strategy. Check your thinking using the TNS lesson.

   *Possible answer: He could set up the proportion using the ratio 2 centimeters to 5 feet where the unit rate would be* $\frac{5}{2}$ *feet to 1 centimeter, or he could use a double number line.*

   b. Write the equation for each strategy and explain how the units would work.

   Answer: The first strategy would fit the equation, $d\ \textit{feet} = \frac{5/2\ \textit{feet}}{1\ \textit{centimeter}} \times 7\ \textit{centimeters}$ and the centimeters would reduce. Students can also use the double number line, with feet on one number line and centimeters on the other, to show the connection between the two quantities.

   c. Explain which strategy you would choose to use and why.

   *Answers will vary. Possible answer: I would use the first strategy because on a blueprint you probably want to convert a lot of the measurements and you can use the same process every time, just changing the measurement from the blueprint. For the second strategy, you would have to create a new problem each time you wanted to convert a new measurement.*

6. Simon ordered 4 bags of soil for his plants. Each small group of plants needs $\frac{3}{4}$ of a bag of soil. How many groups of plants can he fill completely with soil? How much soil does he have left? Solve this problem in as many different ways you can think of. Reflect on the strategies you used and answer the following questions.

   a. Which strategy seemed to be easiest?

   *Answers will vary.*

   b. What are some of the advantages or disadvantages of the strategies if you tried to use them on any problem?

   *Answer: You can use the soil on 5 groups of plants with* $\frac{1}{4}$ *bag left over. Students might solve this using a double number line, a table of ratios going by the unit rate, a ratio table, the unit rate, the equation, a graph, or drawing a diagram. The answers to the questions will vary. They might note that making a table with the unit rate might take a long time if the numbers were large, and that the equation would work for any case, no matter how large the numbers. Ratio tables are easy to work with and you can play with the numbers to get the combination you want. A graph shows you how everything is related but might be difficult to use with big numbers.*

**Activity 2 [Page 2.2]**

**Use page 2.2 of the lesson to answer the questions below.**

**1.** The blue line represents the ratio of eighth graders to seventh graders on the team.

a. Make a list of possible ordered pairs that would come from the equivalent ratios. If (*x, y*) represents (number of eighth graders, number of seventh graders), how many total students does each ordered pair in your list represent?

*Answer:*

| **Ordered pairs** | **Number eighth graders and number seventh graders** | **Total students** |
|---|---|---|
| *(2, 1)* | *2 grade 8, 1 grade 7* | *3* |
| *(4, 2)* | *4 grade 8, 2 grade 7* | *6* |
| *(6, 3)* | *6 grade 8, 3 grade 7* | *9* |
| *(8, 4)* | *8 grade 8, 4 grade 7* | *12* |
| *(10, 5)* | *10 grade 8, 5 grade 7* | *15* |
| *(12, 6)* | *12 grade 8, 6 grade 7* | *18* |
| *(14, 7)* | *14 grade 8, 7 grade 7* | *21* |
| *(16, 8)* | *16 grade 8, 8 grade 7* | *24* |
| *(18, 9)* | *18 grade 8, 9 grade 7* | *27* |

b. Which of the ordered pairs from your list in part a satisfies both requirements: the ratio of eighth graders to seventh graders is 2:1 and the total number of students is 15?

*Answer: (10, 5)*

c. On page 2.2 move the black dot to the point where the two lines intersect. What is this point and what does it represent?

*Answer: The intersection point is (10, 5), which is the ordered pair that makes both equations true. The ratio of eighth graders to seventh graders on the team is 10:5, which is equivalent to 6:3, and there are 15 students, 10 eighth graders and 5 seventh graders.*

2. The ratio of dogs to cats in the animal shelter is 3:5. There are 24 dogs and cats in the shelter. How many cats are in the shelter? Explain how you found your answer.

*Answer: The ratio 3:5 is equivalent to 9:15, and 9 dogs to 15 cats makes 24 dogs and cats, so there are 15 cats. Strategies will vary; some students may make lists and others try to use the TNS lesson. Some may guess.*

3. If $x$ represents the number of dogs and $y$ represents the number of cats, which of the following equations describe a condition in the problem for question 2? Explain your thinking.

   a. $y = \frac{3}{5}x$ b. $y = \frac{5}{3}x$ c. $x + y = 24$ d. $3x + 5y = 24$ e. $5x + 3y = 24$

   *Answer: b and c. b describes the proportion for a ratio of 3:5, where the constant of proportionality or unit ratio is* $\frac{5}{3}$*, and c describes the sum of the number of dogs and the number of cats as 24.*

4. Insert the information from question 3 into the TNS page to check your answer to question 2.

   *Answer: The point of intersection should be (9, 15), so 15 cats are in the shelter.*