Building Concepts: Mean as Fair Share

TEACHER NOTES

Lesson Overview

This lesson introduces students to mean as a way to describe the center of a set of data. Often called the average, the mean can also be visualized as "leveling out" the data in the sense of "fair share".

The concept of "fair share" can be developed in two different ways: 1) those with the most give something to those with the least until everyone has the same amount or 2) total all the contributions and divide the total equally among the contributors.

Learning Goals

1. Interpret "fair share" as taking from those with more and giving to those with less;
2. interpret "fair share" as totaling all of the data and sharing the total equally, that is dividing the sum of the data values by the number of data values, which leads to the standard definition of mean;
3. associate the term *mean* with the idea of "fair share."

Prerequisite Knowledge

Mean as Fair Share is the fourth lesson in a series of lessons that investigates the statistical process. In this lesson, students will use graphs to explore "fair share." Prior to working on this lesson, students should:

- understand how to construct and read bar graphs;
- understand how to multiply and divide numbers;
- understand how to add and subtract fractions.

Vocabulary

- **mean**: the sum of all the data values in a set of data divided by the number of data values.

Lesson Pacing

This lesson contains multiple parts and can likely be completed in 2-3 class periods, though you may choose to extend, as needed.

Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software

- Mean as Fair Share_Student.pdf
- Mean as Fair Share_Student.doc
- Mean as Fair Share.tns
- Mean as Fair Share_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

Class Discussion: Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

Student Activity: Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

Deeper Dive: These questions are provided to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

Mathematical Background

The arithmetic mean is another way to describe a center for a set of data. The mean can be visualized as "leveling out" the data in the sense of "fair share." For example, if the total cost of lunch for six students is $48 and they decide to pay equal shares of the cost, then each student's share is $8. This notion of "fair share" can be developed in two different ways: 1) those with the most give something to those with the least until everyone has the same amount or 2) total all the contributions and divide the total equally among the contributors.

The representation of shares is typically done with blocks or some model involving physical objects that can be associated with bar graphs, and the blocks are moved from one "bar" to the other to obtain equal shares. Students should bring experience working with bar graphs from earlier grades. Bar graphs are associated with categorical data—data that describe the number of observations in each category. Categories are typically classifications: colors, sports, grade levels, gender. The categories can be displayed in any order; for example, from least to most in terms of frequency of outcomes in each category, alphabetically, or chronologically. Bar graphs represent counts as a height or length of each bar while dot plots represent a count on a number line without a category or label attached. In shifting from a bar graph to a dot plot, you may lose categorical identification, but you gain information about the distribution. Bar graphs can be rearranged at will; data associated with a number line cannot be rearranged. Part of this lesson is designed to help students make the transition from bar graphs to representing data in a dot plot on a number line.

The mean is often called the average. Students should recognize the mean as a convenient summary statistic that is used extensively in the world around them, such as the average score on an exam, mean temperature for the day, average height and weight of a person of their age, and so on.

Part 1, Page 1.3

Focus: Students explore "fair share" as taking from those with more and giving to those with less.

Page 1.3 is designed to support the strategy of "fair share" by taking from the one with the most and giving it to the one with the least until everyone has the same amount.

Left and right arrows will display from 2 to 6 dogs.

Go gives random numbers of bags of dog food to each dog.

Selecting a bag will highlight it, allowing the bag to be dragged to another dog or moved using the up and down arrow keys.

Split Bag divides a bag into the specified size parts. The right and left arrows control the size of the parts.

Combine adds the fractional parts of bags.

Give students time to explore the file before asking them a focused set of questions. Encourage them to use the TNS activity to explain or demonstrate their reasoning.

TI-Nspire Technology Tips

tab selects a bag of food.

enter splits bags and combines when splitting isn't possible.

ctrl del resets page.

Class Discussion

***Select Go*.**

- ***Does each dog have the same number of bags of dog food? If not, drag bags from the dog with the most bags of dog food to the dog with the fewest bags. You may have to split a bag by using the arrows that appear in the upper right. Explain how you were able to give each dog the same number of bags of food.***

  Answers will vary. Have students demonstrate using the TNS activity.

- ***How many bags of dog food does each dog have if they share equally?***

  Answers will vary. For 45 bags of dog food, each dog has $22\frac{1}{2}$ bags of dog food.

- ***The number of bags of dog food for each dog changes every time. When will you have to split a bag in order for the two dogs to have the same amount of dog food?***

  Answer: If the number of bags is odd, one bag will have to be split into two parts.

Class Discussion (continued)

***Reset the page. Use the arrow to create 4 dogs. Select* Go.**

- ***How many bags of dog food does the dog with the most dog food have? The least?***

  Answers will vary. In one example with 30 bags of dog food, one dog had 10, and one only had 5.

- ***Move bags from one dog to another until they all have the same amount. How many bags will each dog have if they share equally?***

  Answers will vary. For 30 bags of dog food, each dog will have 7.5 bags.

***Reset. Select 4 dogs and click* Go**.

- ***Will each dog get a whole number of bags of dog food? Why or why not?***

  Answers will vary depending on the number of bags. One example might be 12 bags shared among 4 dogs, which will give each dog 3 bags of dog food. 13 bags when shared among the 4 dogs will give each dog 3 whole bags of dog food and $\frac{1}{4}$ of the 13^{th} bag for $3\frac{1}{4}$ bags per dog. It depends on whether the number is a multiple of 4.

- ***For what numbers of bags will each dog get a whole number of bags of dog food?***

  Answer: If the number of bags is a multiple of 4, then each dog will get a whole number of bags of dog food. If not, the bags will have to be split.

- ***What possible fraction of bags will come up when there are 4 dogs? Fill in the table, and then explain how you can tell by looking at the number of bags.***

Extra Bags	Split	Fraction per Dog

Answer: There might be 1, 2, or 3 extra bags depending on the remainders when dividing by 4.

Extra Bags	*Split*	*Fraction per Dog*
1	***fourths***	$\frac{1}{4}$
2	***halves or fourths***	$\frac{1}{2}$ ***or*** $\frac{2}{4}$
3	***fourths***	$\frac{3}{4}$ ***or*** $\frac{1}{2}+\frac{1}{4}$
0	***no split***	***0***

Class Discussion (continued)

***Reset the page. Choose 6 dogs and then select* Go**.

- ***Predict the number of whole bags of dog food each dog will have. Check your answer using the TNS activity.***
- Answers will vary depending on the number of bags of dog food.
- ***Describe your strategy for sharing the bags equally. Do you think it is efficient? Explain your reasoning.***

  Answers will vary. Some might say no, they just drag until the rows even out; others might say they start with taking from the dog with the most bags and giving it to the dog with the least number of bags.

Student Activity Question—Activity 1

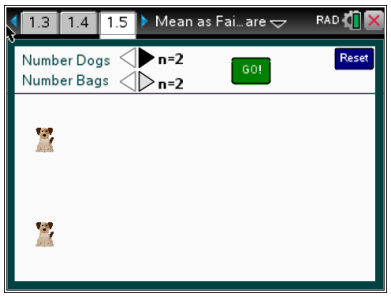
1. What possible fraction of bags will come up when there are 6 dogs? Make a table showing the number of extra bags, how they are split, and what fraction goes to each dog. Fill in the table, and then explain how you can tell by looking at the number of bags.

Answer: The remainders will be 1, 2, 3, 4, or 5 bags. The easiest thing to do is just divide each extra bag into sixths and give out the sixths until each dog has the same number of sixths. Some of the sixths will combine; for example, two $\frac{1}{6}$s will make $\frac{1}{3}$.

Extra Bags	Split	Fraction per Dog
1	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$ or $\frac{1}{3}$	$\frac{2}{6}$ or $\frac{1}{3}$
3	$\frac{1}{6}$ or $\frac{1}{2}$	$\frac{3}{6}$ or $\frac{1}{2}$
4	$\frac{1}{6}$	$\frac{4}{6}$ or $\frac{2}{3}$
5	$\frac{1}{6}$	$\frac{5}{6}$
0	*No split*	*0*

Part 2, Page 1.5

Focus: Students investigate the standard definition of *mean* by totaling the number of bags of dog food and sharing the amount equally between the number of dogs.



Page 1.5 lays the foundation for the standard approach to *mean*: combine all of the contributions and then redistribute them among the contributors by dividing the total by the number of contributors.

The arrows can be used to choose from 2 to 6 dogs and a total number of bags of dog food.

Go randomly gives the bags to the dogs.

Total arranges all of the bags in a column at the right of the screen.

Fair Share sends one bag from the total to each dog.

Split then Share divides a bag into parts according to the number of dogs and distributes the partial bags to the dogs.

TI-Nspire Technology Tips

tab selects a bag of food.

enter toggles through Go, Total, Fair Share, and Split then Share.

ctrl del resets page.

Class Discussion

Go to page 1.5. Select 3 dogs and 6 bags of dog food.

- ***How many bags of dog food will each dog have if they share equally?***

  Answer: Two bags.

- ***What are the possible arrangements of the bags of dog food?***

  Answer: One dog could have all 6 bags, which could be represented as 6,0,0; then the other outcomes could be 5,0,1; 4,2,0; 4,1,1; 3,3,0; 3,2,1; 2,2,2. (Note that the order is not important in listing these outcomes; i.e., 6,0,0 is the same as 0,6,0 and 0,0,6.)

- ***Select* Total*. Describe what happened.***

  Answer: Each dog's bags moved to the right.

- ***Select* Fair Share *and describe what happened.***

  Answer: 3 bags on the right moved to the dogs and 1 bag went to each dog.

- ***How many times do you think you will have to select* Fair Share *in order to give out the bags of dog food equally to the dogs? Explain your reasoning. Select* Fair Share *to check your answer.***

  Answer: Once more because there are only 3 bags left.

***Reset the page. Choose 5 dogs and 8 bags of dog food. Select* Go.**

- ***Describe the distribution of the bags of dog food to the five dogs.***

  Answers will vary. For example, one dog could have 4 bags and the other 4 dogs each have one bag.

Class Discussion (continued)

- ***Select* Total *and* Fair Share*. How many bags are left to split?***

  Answer: 3

- ***Select* Split then Share*. Describe what happens.***

  Answer: For each selection, a bag divided into fifths and $\frac{1}{5}$ of a bag went to each dog.

- ***What is the number of bags of dog food per dog when all the dogs have the same amount?***

  Answer: $1\frac{3}{5}$ bags

The number of bags of dog food each dog has when they share equally is called the mean. Reset the page. Select 4 dogs and 10 bags of dog food.

- ***Describe what you think will happen when you select* Go*, Total, and then* Fair Share*. Check your conjecture using the TNS activity.***

  Answers will vary.

- ***What is the mean number of bags of dog food per dog?***

  Answer: $2\frac{2}{4}$ or $2\frac{1}{2}$ bags

- ***Suppose you had 6 dogs and 42 bags of dog food. How many times do you think you would have to select* Fair Share*? Explain your thinking, and then use the TNS activity to check your conjecture.***

  Answer: 7 times, because there are 7 sets of 6 in 42 and each time you select Fair Share you give away a set of 6 bags from the total amount of 42 bags.

Student Activity Questions—Activity 2

1. **Alyssa had 2 dogs with 10 bags of dog food, 1 dog with 8 bags, 2 dogs with 5 bags of food, and 1 dog with no bags of food. Simone had 3 dogs with 4 bags of dog food, 1 dog with 12 bags of food, and 2 dogs with 7 bags of food. Alyssa claimed they should both get the same mean. Simone argued that they had very different problems so the means would not be the same. Who is right and why? Use the TNS activity to support your thinking.**

   Answer: Alyssa is correct because they both have 6 dogs and 38 bags of dog food, so they will both end up with a mean of $6\frac{1}{3}$ bags.

Student Activity Questions—Activity 2 (continued)

2. Explain the difference between finding the mean on page 1.3 and finding the mean on page 1.5.

Answer: To find the mean on page 1.3, you took bags from some dogs that had a lot of bags and gave them to dogs with hardly any bags until every dog had the same amount. To find the mean on page 1.5, you put all of the bags into a common pile and then shared them out one at a time for each dog until all of the bags were gone.

3. Find the mean number of bags of dog food for each of the following. Use either page in the TNS activity to help your thinking.

a. 5 dogs, 18 bags of dog food

Answer: The mean is $3\frac{3}{5}$ bags.

b. 4 dogs, 17 bags of dog food

Answer: The mean is $4\frac{1}{4}$ bags.

c. One dog has 6 bags of dog food, a second has 3 bags of dog food, a third has 2 bags of dog food and a fourth has 9 bags of dog food.

Answer: The mean is 5 bags.

d. Two dogs each having 11 bags of dog food, one dog has 7 bags of dog food, three dogs each have 9 bags of dog food.

Answer: The mean is $\frac{56}{6}$ or $\frac{28}{3} = 9\frac{1}{3}$ bags.

Part 3, Page 1.7

Focus: Students continue to explore the standard definition of *mean*.

Page 1.7 works in the same way as page 1.3, with the addition of a number line that displays a dot plot associated with the number of bags per dog. This page supports the transition from representing data in bar graphs to representing the data on a number line. In the original representation each bar is a dot (i.e., a category), and the number of bags of dog food determines the length of the bar. Each dot is represented by a dot positioned on the number line to indicate the number of bags of food that dog has.

Class Discussion

Have students…	Look for/Listen for…
***Select* Go**.	
• ***Describe the distribution of the dog food. Explain what the dot plot on the bottom of the screen represents.***	Answers will vary. For example, for a total of 41 bags, 1 dog had 22 bags and the other 19. Each dot represents a dog; one dot is located on the number line at 19 and the other at 22.
• ***Move the bags so each dog has an equal amount of dog food. How much will each dog have?***	Answers will vary. For the example above, the mean will be 20.5 bags.
• ***How did the dot plot change when you moved the bags?***	Answer: The dots stacked up on 20.5 on the number line.
Reset the page. Choose 5 dogs and select Go.	
• ***Sketch the dot plot and describe what it means.***	Answers will vary. Sample answer: There are 16 bags of dog food with one dog having 2 bags, 2 dogs having 3 bags each, and 2 dogs having 4 bags each.
• ***Move the bags to give each dog the same number of bags of dog food. Describe what happens to the dot plot as you move the bags***.	Answer: The dots get closer together on the dot plot and begin to pile up on one number. When you get to the “fair share” or mean, the dots are all on the same number.
• ***Explain what the stack of dots on the number line represents.***	Answer: Each dot represents a dog, so there are 5 dots because there are 5 dogs. The dots are all at the same place on the number line because the dogs all have the same share of the dog food. The number on which they are stacked is the share of dog food each dog has.

Class Discussion (continued)

Reset. ***Choose 6 dogs and select*** **Go**.

- ***Record the dogs and the bags of dog food in the table.***

Dog	Number of Bags of Dog Food
Dog 1	
Dog 2	
Dog 3	
Dog 4	
Dog 5	
Dog 6	

Answers will vary. One example might be:

Dog	*Number of Bags of Dog Food*
Dog 1	*4*
Dog 2	*2*
Dog 3	*2*
Dog 4	*7*
Dog 5	*10*
Dog 6	*5*

- ***Find the mean number of bags of dog food. Explain how you might use the table to help you find the mean.***

  Answers will vary. For the example, there are 30 bags of dog food all together, and the mean would be $\frac{30}{6}$ or 5 bags of dog food.

Student Activity Questions—Activity 3

1. **Which of the following strategies make sense for finding the mean? Explain why or why not in each case.**

   a. **Divide the number of bags of dog food by the number of dogs.**

   b. **Take one bag and divide it equally among the dogs. Multiply the fraction share each dog received by the total number of bags.**

   c. **Find the largest multiple of the number of dogs that will go into the number of bags of dog food. Subtract that number from the number of bags and figure out what fractions to split the number of bags in the difference so each dog has the same fraction.**

   Answer: All of the strategies will work because they all involve getting the total and dividing it evenly among the dogs.

2. **Which of the correct strategies in the question above seems to be the most efficient? Explain your thinking.**

   Answers may vary. Strategy a) would seem to be the most efficient because you only add and divide. In b) you could be working with lots of fractions, and then would have to add the fractions to find the mean. Strategy c) is almost like a) but finding multiples is more complicated than simply dividing.

Deeper Dive – Page 1.5

Suppose you have 3 dogs and 5 bags of dog food.

- ***How many different ways can the 5 bags be distributed to the dogs?***

  Answer: five ways assuming that the order does not matter, 0,0,5 is the same as 5,0,0- A) 0, 0, 5; B) 0, 1, 4; C) 0, 2, 3; D) 1, 1, 3; E) 1, 2, 2

- ***Find the mean for each of the arrangements you described above. Explain your reasoning. Use the TNS activity to check your thinking.***

  Answer: All of the means will be $1\frac{2}{3}$ because in every case the total will be 5 bags. So each dog will get 1 whole bag and then each of the leftover 2 bags will be split into thirds, so each dog will get $\frac{2}{3}$ bag for a total of $1\frac{2}{3}$ bags. How the bags are originally shared does not matter.

Deeper Dive – Page 1.7

Make up three different problems to use with page 1.7 and give them to a classmate. For each, they should describe the distribution of the bags of dog food and predict the point at which the dots will stack on the number line. You should check their answers using the TNS activity.

Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. What is the mean of the numbers: 4, 8, 3, 2, 5, 8, 12?

   a. 4

   b. 5

   c. 6

   d. 7

   e. 8

   Answer: c) 6

2.

Baskets	Students
7	Tom
8	Jenny
2	Patty
3	Pete
5	Lucy

   The table above shows the number of baskets each member of the starting team made in a practice drill. What is the mean number of baskets made by the starting players on the team? ***Answer: 5 baskets***

3. Bettina and four of her friends wanted to buy something to eat. One of them had $3.50. The others had $4.00, $2.75, $5.50, and $3.75. Did they have enough to buy each of them a smoothie that cost $3.25? Explain why or why not.

   Answer: Yes because if they pooled their money, they would have $19.50. Each one could spend $3.90, which is more than the cost of the smoothie.

4. Tina, Lou, Franz, and Simon entered a lottery to win small plots of land to use for a garden. Tina won 2 plots, Lou won 3 plots, Franz won 1 plot, and Simon won 5 plots. They decided to share their plots so they could plant more vegetables. Find the number of plots each student had to plant after they had shared.

   Answer: 11 total plots divided by 4 would be $2\frac{3}{4}$ ***plots per person.***

Student Activity Solutions

In these activities you will work together to explore mean as "fair share." After completing each activity, discuss and/or present your findings to the rest of the class.

Activity 1 [Page 1.3]

1. What possible fraction of bags will come up when there are 6 dogs? Make a table similar to the table in the Class Discussion. Fill in the table, and then explain how you can tell by looking at the number of bags.

   Answer: The remainders will be 1, 2, 3, 4, or 5 bags. The easiest thing to do is just divide each extra bag into sixths and give out the sixths until each dog has the same number of sixths. Some of the sixths will combine; for example, two $\frac{1}{6}$s will make $\frac{1}{3}$.

Extra Bags	*Split*	*Fraction per Dog*
1	$\frac{1}{6}$	$\frac{1}{6}$
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4	$\frac{1}{6}$	$\frac{4}{6}$ or $\frac{2}{3}$
5	$\frac{1}{6}$	$\frac{5}{6}$
0	*No split*	*0*

Activity 2 [Page 1.4]

1. Alyssa had 2 dogs with 10 bags of dog food, 1 dog with 8 bags, 2 dogs with 5 bags of food, and 1 dog with no bags of food. Simone had 3 dogs with 4 bags of dog food, 1 dog with 12 bags of food, and 2 dogs with 7 bags of food. Alyssa claimed they should both get the same mean. Simone argued that they had very different problems so the means would not be the same. Who is right and why? Use the TNS activity to support your thinking.

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   Answer: To find the mean on page 1.3, you took bags from some dogs that had a lot of bags and gave them to dogs with hardly any bags until every dog had the same amount. To find the mean on page 1.5, you put all of the bags into a common pile and then shared them out one at a time for each dog until all of the bags were gone.

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      Answer: The mean is $3\frac{3}{5}$ *bags.*

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   c. One dog has 6 bags of dog food, a second has 3 bags of dog food, a third has 2 bags of dog food and a fourth has 9 bags of dog food

      Answer: The mean is 5 bags.

   d. Two dogs each having 11 bags of dog food, one dog has 7 bags of dog food, three dogs each have 9 bags of dog food

      Answer: The mean is $\frac{56}{6}$ *or* $\frac{28}{3} = 9\frac{1}{3}$ *bags.*

Activity 3 [Page 1.7]

1. Which of the following strategies make sense for finding the mean? Explain why or why not in each case.

   a. Divide the number of bags of dog food by the number of dogs.

   b. Take one bag and divide it equally among the dogs. Multiply the fraction share each dog received by the total number of bags.

   c. Find the largest multiple of the number of dogs that will go into the number of bags of dog food. Subtract that number from the number of bags and figure out what fractions to split the number of bags in the difference so each dog has the same fraction.

   Answer: All of the strategies will work because they all involve getting the total and dividing it evenly among the dogs.

2. Which of the correct strategies in the question above seems to be the most efficient? Explain your thinking.

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