# Building Concepts: What is Probability?

## Lesson Overview

**Mathematical Focus:** How do you calculate theoretical and experimental probabilities?

Students use a simulation of card draws to develop a relative-frequency view of probability. They will explore the relationship between the theoretical probability of an outcome calculated from knowing the sample space and the empirical probability of an outcome calculated by conducting an experiment and recording the long-run relative frequency of the outcome. Students draw chips from a bag to approximate the probability of outcome based observing its long-run relative frequency.

The greater the probability the more likely an event is to occur.

### Learning Goals

1. Recognize that probability is a measure of the chance that an event will occur;
2. understand that the probability of an event is between 0 and 1, and be able to interpret a probability of 0 and of 1;
3. understand what the long run relative frequency of an event means;
4. recognize that short term relative frequencies can have a lot of variation.

### Prerequisite Knowledge

*What is Probability?* is the twelfth lesson in a series of lessons that explore statistics and probability. This lesson is the first lesson on probability. Prior to working on this lesson students should understand:

- how to construct and read a table;
- how to construct and read a graph;
- the concepts of fractions and decimals.

### Vocabulary

- **probability:** the chance that something will happen; how likely it is that an event will happen
- **outcome:** a possible result of a probability experiment
- **sample space:** the collection of all possible individual outcomes
- **probability model:** a graphic that shows the probability for each possible, unique outcome for a situation, so that the total probability over all such outcomes is one

### Lesson Pacing

This lesson should take 50–90 minutes to complete with students, though you may choose to extend, as needed.

## Lesson Materials

- Compatible TI Technologies:

  TI-Nspire CX Handhelds, TI-Nspire Apps for iPad®, TI-Nspire Software
- What is Probability_Student.pdf
- What is Probability_Student.doc
- What is Probability.tns
- What is Probability_Teacher Notes
- To download the TI-Nspire activity (TNS file) and Student Activity sheet, go to http://education.ti.com/go/buildingconcepts.

## Class Instruction Key

The following question types are included throughout the lesson to assist you in guiding students in their exploration of the concept:

**Class Discussion:** Use these questions to help students communicate their understanding of the lesson. Encourage students to refer to the TNS activity as they explain their reasoning. Have students listen to your instructions. Look for student answers to reflect an understanding of the concept. Listen for opportunities to address understanding or misconceptions in student answers.

**Student Activity:** Have students break into small groups and work together to find answers to the student activity questions. Observe students as they work and guide them in addressing the learning goals of each lesson. Have students record their answers on their student activity sheet. Once students have finished, have groups discuss and/or present their findings. The student activity sheet can also be completed as a larger group activity, depending on the technology available in the classroom.

**Deeper Dive:** These questions are provided for additional student practice, and to facilitate a deeper understanding and exploration of the content. Encourage students to explain what they are doing and to share their reasoning.

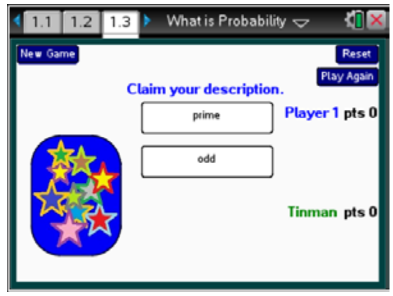
## Mathematical Background

In Grade 7, students develop their understanding of probability from two perspectives: *theoretical* (based on the structure of a process and its outcomes) or *empirical* (based on observed data generated by the process). Tossing two fair coins has four possible outcomes that form the sample space for the experiment tossing two fair coins. A theoretical probability model of this process can be created by assigning equal probabilities to each outcome, (HH, HT, TH, TT) assuming that an outcome of heads is just as likely as an outcome of tails. A relative frequency view would involve recording repeated observations of the outcomes of tossing two coins, where you count the proportion of each outcome in the chance process. Students relate probability to the long-run (more than five or ten trials) relative frequency of a chance event, using coins, number cubes, cards, spinners, bead bags, and so on.

A *probability model* provides a probability for each possible unique outcome for a chance process so that the total probability over all such outcomes is one. The collection of all possible individual outcomes is called the *sample space* for the model.

## Part 1, Page 1.3

Focus: Students will recognize that the number of opportunities to win a chance game (the probability) can help them choose a winning outcome.




On page 1.3, students play a card game against Tinman. They select a description then draw cards one at a time. If the number on the card matches the selected description the player earns one point. Play continues until the student or Tinman has 10 points, or until 20 cards have been drawn.

**Draw Card** shows a card with a number from 1 to 10.

**Fast Play** speeds up the draws (available after 11 draws).

**Play Again** allows the user to play the same game again.

**New Game** generates a new set of descriptions. (Note the two sets of descriptions on page 1.3 will be the same every time.)

**TI-Nspire Technology Tips**

menu accesses page options.

tab cycles through on page options.

enter draws cards or activates on page option.

1 or 2 chooses a description.

ctrl del resets the page.

### Class Discussion

The following questions engage students in a game in which students recognize a strategy for winning is to select the option that has the greater chance of occurring. This leads to a definition of *probability*.

**Teacher Tip:** Students might be allowed to play the game and try to figure out a winning strategy or they can work through the questions. Listen for words such as *chance*, *likely to win*, etc. which are informal ideas related to probability that will be explained later in the lesson.

***The descriptions on page 1.3 refer to the numbers 1 to 10. Choose one of the descriptions. Then draw a card to see who wins the point, you, Tinman or neither. Continue to draw cards until one of you has 10 points or you have drawn 20 cards. (Note that after drawing 11 cards, you have the option of Fast Play).***

- ***Who is winning and by how much?*** Answers will vary.
- ***At least one of the cards did not give either you or Tinman a point. Explain why not.*** Answer: 4, 6, 8, and 10 are neither odd nor prime.
- ***Did any cards give both of you points? If so, which ones and why?*** Answer: 3, 5, and 7 are all odd and prime so we both got points

**Class Discussion (continued)**

| Have students… | Look for/Listen for… |
|---|---|
| ***Choose*** **Play Again*.*** | |
| • ***Did you change the description you chose? Why or why not.*** | Answers will vary. Some students might recognize they should choose the option that has more possible ways of occurring. |
| • ***Did you win?*** | Answers will vary. The probabilities are close together so the outcome might go either way. |
| • ***Which description do you think is most likely to be the winning description? Explain your reasoning.*** | Answers will vary. Some students might recognize there are 5 possible odd numbers between 1 and 10 and 4 possible prime numbers, so it is more likely that you will get an odd number. |
| ***Select*** **New Game** | |
| • ***Which description do you want to claim? Why?*** | Answers will vary. Some may figure there are more numbers greater than 3 than there are multiples of 4, so they will choose numbers greater than 3. |
| • ***Play the game. Who won?*** | Answers will vary. |
| • ***Play the game again and try to choose the winning description. Explain your thinking.*** | Answers will vary. Students should be thinking about which description has the most possible ways of occurring. |
| ***In the first game the two claims were: the number is odd or the number is prime.*** | |
| • ***How many of the numbers from 1 to 10 are odd? Prime?*** | Answer: 5 of the numbers are odd, and 4 are prime (students might need reminding that 1 is not a prime number). |
| • ***Which claim would you expect to win? Why?*** | Answer: There are more chances to get an odd number than a prime so I would expect odd to win. |
| ***In the second game the two claims were: the number is a multiple of 4, and the number is greater than or equal to 3.*** | |
| • ***How many of the numbers from 1 to 10 are a multiple of 4? Greater than or equal to 3?*** | Answer: Only two numbers (4 and 8) are multiples of 4. Eight numbers from 1 to 10 are greater than or equal to 3. |

**Class Discussion (continued)**

- ***Which claim would you expect to win? Why?*** Answer: There are more numbers greater than or equal to 3 than multiples of 4, so I would expect that the description greater than or equal to 3 will win.

**Student Activity Questions—Activity 1**

**The *probability* of an event is the number of successes divided by the total number of possible outcomes (assuming the probability of each outcome is equally likely).**

1. **For the numbers from 1 to 10, describe the outcomes that satisfy each condition and then give the probabilities.**

   a. **the number is odd**

   Answer: {1, 3, 5, 7, 9}; $\frac{5}{10}=\frac{1}{2}$

   b. **the number is prime**

   Answer: {2, 3, 5, 7}; $\frac{4}{10}=\frac{2}{5}$

   c. **the number is a multiple of 4**

   Answer: {4, 8}; $\frac{2}{10}=\frac{1}{5}$

   d. **the number is greater than or equal to 3**

   Answer: {3, 4, 5, 6, 7, 8, 9, 10}; $\frac{8}{10}=\frac{4}{5}$

2. **Reset page 1.3. Sometimes the probability of an outcome is calculated experimentally by recording the number of times the outcome occurs in a large number of repetitions of the experiment and dividing the result by the total number of repetitions. This is called the *relative frequency* of that outcome.**

   **Play the game again. This time when either you or Tinman gets 10 points, go to Fast Play. Play until you have drawn 100 cards.**

   a. **Calculate the probability of each outcome using the relative frequencies.**

   Answers will vary. One possibility might be $\frac{38}{100}$ for prime numbers and 47 out of 100 for odd numbers.

## Student Activity Questions—Activity 1 (continued)

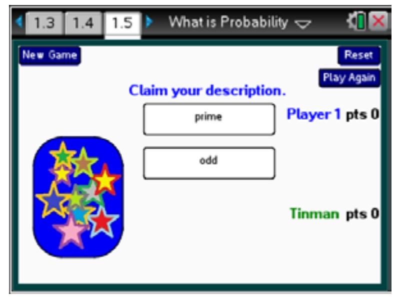
**b. How close is the probability calculated from the relative frequency for the outcome "prime" to the theoretical probability you found in question 1?**

Answers will vary. For the example, out of 100 trials, prime numbers occurred $\frac{38}{100} = 0.38$. The theoretical probability is $\frac{4}{10}$ or 0.40. The two are close.

## Part 2, Page 1.5

Focus: Students continue the exploration of how they can use probability to help them choose a winning outcome.

Page 1.5 has the same functionality as page 1.3, but the descriptions are randomly generated.




## Class Discussion

| Have students… | Look for/Listen for… |
|---|---|
| ***Choose* New Game.** | |
| • ***Which of the outcomes has the greater probability of being chosen? Justify your reasoning.*** | Answers will vary. Note that each student could get a different set of descriptions. Students should begin to choose the option that has the best probability. |
| • ***Play the game. Who won?*** | Answers will vary. |
| • ***Find the probability based on the relative frequency. How does it compare to the theoretical probability?*** | Answers will vary. |
| ***Play* New Game *until you have won at least 10 different games.*** | |
| • ***Describe your strategies for winning.*** | Answers will vary. Students might mention finding the probability for each option and choosing the one with the greatest probability. |
| • ***Did your strategy ever fail? If so, describe when.*** | Answers will vary. If the probabilities of the two outcomes are close or the same, the results might vary between them even over the long run. |

## Student Activity Questions—Activity 2

1. **Decide whether each of the following is true or false. Give an example from the game, using numbers from 1 to 10, to support your reasoning. The probability of an outcome can be**

   a. **1**

   Answer (examples will vary). True: One of the descriptions was a number less than or equal to 10, which was true for all of the numbers from 1 to 10.

   b. **0**

   Answer (examples will vary). True: One of the descriptions was a number can be greater than 11, which cannot happen when you only have cards with the numbers 1 to 10 on them.

   c. **more than 1**

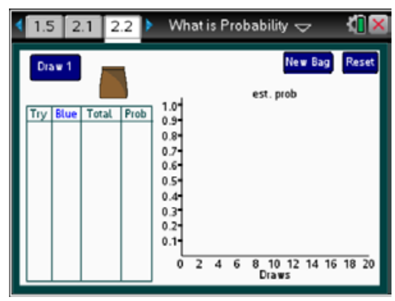
   Answer (examples will vary). False: You cannot have more successes than possible outcomes; you cannot have 11 successful outcomes if there are only 10 possible outcomes.

   d. **0.5**

   Answer (examples will vary). True: One of the descriptions was the probability that a number is even.

## Part 3, Pages 2.2

Focus: Students will recognize that the probability of an outcome can be estimated from the long-run relative frequencies.




**Draw 1** draws a chip from the bag and plots the cumulative proportion of blue chips.

**Draw 10** plots the results of drawing a chip ten times. (Available after the first 10 chips are drawn. Note: this is not drawing 10 chips at one time.)

**New Bag** changes to a new bag that has a different proportion of blue chips.

**Reset** resets the page with the original bag.

## Class Discussion

**Teacher Tip:** In the following questions students investigate the probability of an outcome when nothing is known about the population as they draw a chip from a bag of chips some of which are blue and others white. Be sure students understand how to interpret the table and how the entries in the table are related to the graph, plotting the relative frequencies, the number of blue chips so far divided by the total number of draws. Students will use the outcomes from repeatedly drawing a chip to estimate the probability of getting a blue chip. The results from different simulations will vary due to chance but should be relatively close.

## Class Discussion (continued)

| Have students… | Look for/Listen for… |
|---|---|
| ***The bag contains blue and white chips. Draw a sample from the bag.*** | |
| • ***Explain what the first line in the table tells you. How does this relate to the graph?*** | Answer: One example might be 1, no, 0, 0, which would indicate that in Draw 1, the chip was not blue, the total number of blue chips was 0 and the relative frequency of a blue chip after 1 draw was 0. The point (1, 0) would be plotted on the graph, which indicates that after Draw 1, the relative frequency of a blue chip was 0. Another example might be 1, yes, 1, 1, which would indicate that in Draw 1, the chip was blue, the total number of blue chips was 1 and the relative frequency of a blue chip after 1 draw was 1. The point (1, 1) was plotted on the graph representing the relative frequency of a blue chip after 1 draw is 1. |
| • ***Draw three times. Explain what the entries in the row for* Try 3 *tell you.*** | Answers will vary. One example might be Try 3, Yes, 2, 0.67. This would indicate that in the third draw from the bag, the chip was blue; this was the second blue out of three draws and the relative frequency of a blue chip so far was $\frac{2}{3}$ or 0.67. |
| • ***Draw ten times. Make a sketch of the graph. Based on what you see so far, what do you think the probability of drawing a blue chip from the bag is?*** | Answers will vary. Some students may think it will be close to the last result; others might not be sure how the trend will settle after more draws. |
| • ***Describe the graph of the first ten draws and indicate what it means about the probability of getting a blue chip.*** | Answers will vary. Most descriptions should include a lot of variability from the early draws. For example, the relative frequency might go from 0 after 1 draw to 0.5 after 2 draws to 0.67 after 3 draws to 0.5 after 6 draws and to about 0.3 after 10 draws. Be sure students recognize the cumulative nature of the relative frequencies as the number of draws increase. |
| ***Select* Draw 10*.*** | |
| • ***Explain the results in the table and on the graph.*** | Answer: The table and graph show the results after you draw 10 more chips 1 at a time and replace them back in the bag each time. |

**Class Discussion (continued)**

| | |
|---|---|
| • ***Continue to*** **Draw 10*****. Describe the graph as you increase the number of times you draw a chip from the bag.*** | Answers will vary, but the probability should settle in close to 0.30. |
| • ***Reset.*** **Draw 10*****. How does the graph compare to the graph of the first ten draws you sketched in the earlier question?*** | Answers will vary. |
| • ***Continue to*** **Draw 10*****. Estimate the probability of a blue chip and compare it to your estimate in the question above.*** | Answers will vary but the result again should be close to 0.3 |
| ***Suppose you were to reset the page. Describe what the graph will look like if*** | |
| • ***the first three draws are blue chips.*** | Answer: The dots will form a horizontal line connecting (1, 1), (2, 1) and (3, 1). |
| • ***the first draw is a blue chip, and the second and third chips are not blue.*** | Answer: The dots will go from (1, 1) down to (2, 0.5) and (3, 0.33). |

**Student Activity Questions—Activity 3**

1. **Select New Bag**.

   a. **Select *Draw 1* and *Draw 10* several times. When do you think you can estimate the probability of a blue chip in a bag?**

   Answers will vary. Some may think around 30 draws, while others will be around 80 or 90 draws.

   b. **Continue to *Draw 10* to check your answer to the question above. How close did your estimate come?**

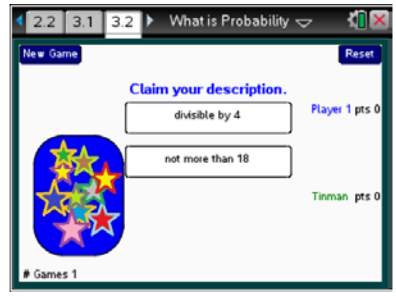
   Answers will vary.

   c. **Scroll back through the column for the number of tries in the table and see how different the relative frequency for the 10$^{th}$ draw was from the relative frequency for the 200$^{th}$ draw. Make a conjecture and check it out by selecting more new bags.**

   Answers will vary.

## Part 4, Pages 3.2

Focus: Students build upon their understanding that the number of opportunities to win a chance game (the probability) can help them choose a winning outcome.




Page 3.2 extends the game from page 1.3.

Players begin at Level 1, where the descriptions refer to the set of numbers from 1 to 25. (Note that some of the descriptions are designed to provide experience with reasoning about expressions and mathematical terms.)

Students unlock Level 2 by playing at least 3 Level 1 games and earning 30 points. The descriptors in Level 2 descriptors are for the numbers 1 to 50. At this level students have to win at least 4 games and 40 points to become a “champion”.

### Deeper Dive — Page 3.2

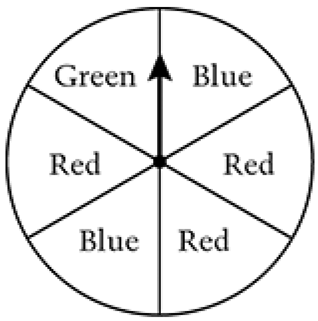
| | |
|---|---|
| • ***Make up two descriptions for some of the numbers from 1 to 30, one description you know has a high chance of occurring when you draw a card and one that has a low change of occurring.*** | Answers will vary. A high probability description might be less than or equal to 25; a low probability description might be multiple of 25. |
| • ***Describe how you might use the TNS activity to investigate the probability that two cards in a row show consecutive numbers.*** | Answer: Choose any descriptor and start drawing cards. Record on paper every time two consecutive numbers show up. |

## Sample Assessment Items

After completing the lesson, students should be able to answer the following types of questions. If students understand the concepts involved in the lesson, they should be able to answer the following questions without using the TNS activity.

1. The spinner below is divided into 6 congruent sectors. If the arrow is spun once, what is the probability that the arrow will land on either Red or Blue?

   a. $\frac{5}{6}$ b. $\frac{2}{3}$ c. $\frac{1}{2}$ d. $\frac{1}{5}$ e. $\frac{1}{6}$




NAEP 2013 grade 8

***Answer: a)*** $\boldsymbol{\frac{5}{6}}$

2. Marty has 6 red pencils, 4 green pencils, and 5 blue pencils. If he picks out one pencil without looking, what is the probability that the pencil he picks will be green?

   a. 1 out of 3

   b. 1 out of 4

   c. 1 out of 15

   d. 4 out of 15

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***Answer: d) 4 out of 15***

3. Identify each as true or false

   a. The probability of an outcome can be 2/1.

   b. The probability of an outcome can be 0.

   c. If an outcome is sure to happen, the probability of the outcome is 1.

***Answers: a) False, b) True, c) True***

4. (1, 1) (2, 1) (3, 2) (1, 2) (2, 2) (3, 2) (1, 3) (2, 3) (3, 3)

   A pair of numbers will be chosen at random from the list above. What is the probability that the first number in the pair will be less than the second number in the pair?

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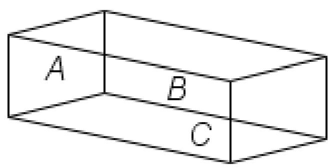
   ***Answer:*** $\frac{3}{9}$ ***or*** $\frac{1}{3}$

5. Tomas tossed a rectangular block 50 times.




The table shows the ways it landed

| **Block landed** | **# of times** |
|---|---|
| On small side A | \|\|\|\| |
| On long side B | \|\|\|\|\| \|\|\|\|\| \|\|\| |
| On top or bottom side C | \|\|\|\|\| \|\|\|\|\| \|\|\|\|\| \|\|\|\|\| \|\|\|\|\| \|\|\|\|\| \|\|\| |

Estimate the probability that the block will land

a. on the top or bottom side.

   ***Answer:*** $\frac{33}{50}$

b. on small side A or long side B.

   ***Answer:*** $\frac{17}{50}$

c. on the small side A or the top or bottom side C.

   ***Answer:*** $\frac{37}{50}$

## Student Activity Solutions

In these activities you will calculate the probability of events. After completing the activities, discuss and/or present your findings to the rest of the class.

**Activity 1 [Page 1.3]**

1. The *probability* of an event is the number of successes divided by the total number of possible outcomes (assuming the probability of each outcome is equally likely).

   For the numbers from 1 to 10, describe the outcomes that satisfy each condition and then give the probabilities.

   a. the number is odd

   *Answer: {1, 3, 5, 7, 9};* $\frac{5}{10}=\frac{1}{2}$

   b. the number is prime

   *Answer: {2, 3, 5, 7};* $\frac{4}{10}$

   c. the number is a multiple of 4

   *Answer:* $\{4, 8\}$*;* $\frac{2}{10}=\frac{1}{5}$

   d. the number is greater than or equal to 3

   *Answer: {3, 4, 5, 6, 7, 8, 9};* $\frac{8}{10}$

2. Reset page 1.3. Sometimes the probability of an outcome is calculated experimentally by recording the number of times the outcome occurs in a large number of repetitions of the experiment and dividing the result by the total number of repetitions. This is called the *relative frequency* of that outcome.

   Play the game again. This time when either you or Tinman gets 10 points, go to Fast Play. Play until you have drawn 100 cards.

   a. Calculate the probability of each outcome using the relative frequencies.

   *Answers will vary. One possibility might be* $\frac{38}{100}$ *for prime numbers and 47 out of 100 for odd numbers.*

   b. How close is the probability calculated from the relative frequency for the outcome "prime" to the theoretical probability you found in question 1?

   *Answers will vary. For the example, out of 100 trials, prime numbers occurred* $\frac{38}{100}=0.38$*. The theoretical probability is* $\frac{4}{10}$ *or 0.40. The two are close.*

## Activity 2 [Page 1.5]

1. Decide whether each of the following is true or false. Give an example from the game, using numbers from 1 to 10, to support your reasoning. The probability of an outcome can be

   a. 1

   *Answer (examples will vary). True: One of the descriptions was a number less than or equal to 10, which was true for all of the numbers from 1 to 10.*

   b. 0

   *Answer (examples will vary). True: One of the descriptions was a number can be greater than 11, which cannot happen when you only have cards with the numbers 1 to 10 on them.*

   c. more than 1

   *Answer (examples will vary). False: You cannot have more successes than possible outcomes; you cannot have 11 successful outcomes if there are only 10 possible outcomes.*

   d. 0.5

   *Answer (examples will vary). True: One of the descriptions was the probability that a number is even.*

## Activity 3 [Page 2.2]

1. Select **New Bag**.

   a. Select **Draw 1** and **Draw 10** several times. When do you think you can estimate the probability of a blue chip in a bag?

   *Answers will vary. Some may think around 30 draws, while others will be around 80 or 90 draws.*

   b. Continue to **Draw 10** to check your answer to the question above. How close did your estimate come?

   *Answers will vary.*

   c. Scroll back through the column for the number of tries in the table and see how different the relative frequency for the 10$^{th}$ draw was from the relative frequency for the 200$^{th}$ draw. Make a conjecture and check it out by selecting more new bags.

   *Answers will vary.*