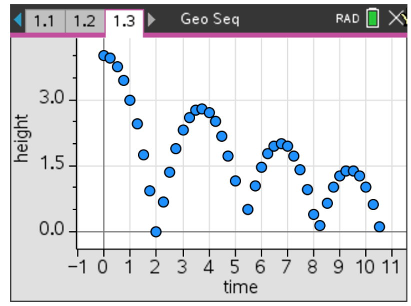



## Math Objectives

- Students will explore geometric sequences and series.
- Students will consider the effect of the value for the common ratio and first term using sliders on the TI-Nspire CX II.
- Students will graphically analyze the geometric series using graphs.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

## Vocabulary

- Geometric Sequence
- Geometric Series
- Common Ratio
- Converge
- Diverge

## About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations SL/HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Core Content Topic 1 Algebra:

**1.3a** Geometric sequences and series

**1.3b** Use of the formula for the $n^{th}$ term and the sum of the first n terms of the sequence

**1.3d** Applications

- As a result, students will:

  Apply this information to real world situations

## TI-Nspire™ Navigator™

- Transfer a File.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to demonstrate.
- Use Teacher Edition computer software to review student documents.
- Use Quick Poll to assess students' understanding

## Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

**Tech Tips:**

- This activity includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at [http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials](http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials)

**Lesson Files:**

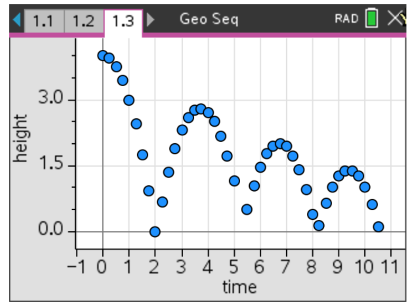
*Student Activity*

Exploring_Geometric_Sequences_Student-Nspire.pdf

Exploring_Geometric_Sequences_Student-Nspire.doc

Exploring_Geometric_Sequences.tns

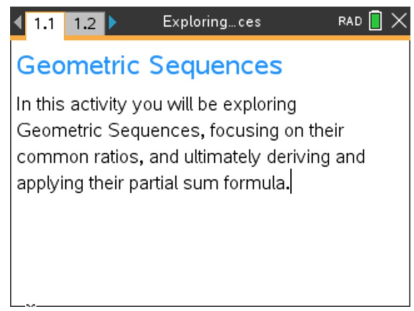
**Example of Geometric Sequence**




The height that a ball rebounds to after repeated bounces is an example of a geometric sequence. The top of the ball appears to be about 4.0, 2.8, 2.0, and 1.4 units. If the ratios of consecutive terms of a sequence are the same, then it is a geometric sequence. The common ratio *r* for these values is about 0.7.

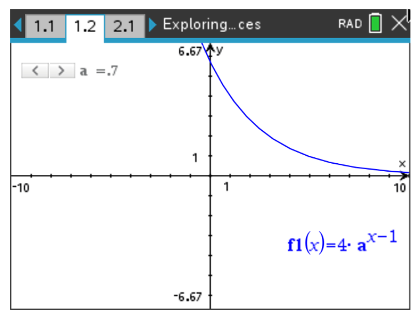
**Demonstration:** $r \approx \frac{2.8}{4.0} = 0.7 \approx \frac{2.0}{2.8} \approx 0.71 \approx \frac{1.4}{2.0} = 0.7$

## Problem 1 – Changing the Common Ratio




Explore what happens when the common ratio changes.

Open the *Exploring_Geometric_Sequences.tns* file. Move to **page 1.2**. On this graph is the function $\boldsymbol{f1(x) = 4 \cdot a^{x-1}}$. In the top left corner you will see a slider for the common ratio, $\boldsymbol{a}$. By pressing on the left and right arrows of the slider, you will change the value of $a$ on the interval $\boldsymbol{-0.1 \leq a \leq 1.1}$, with a step of $\boldsymbol{0.1}$. Use the slider and the resulting function to answer the following questions.




**1.** Discuss why you think the *r*-value is called the common ratio.

**Possible discussion points:** The ratios of consecutive terms are the same; they have that in common.

**2.** With a classmate observe what happens when you change the common ratio from positive to negative. Explain why this happens.

**Possible discussion points:** The smooth continuous graph disappears. The *y* values of the graph oscillate between positive and negative values.

**3.** If each output from this function was a term in a geometric sequence, describe what would happen if you added all the terms of this sequence. Explain what common ratio conditions that would be needed so that the sum will diverge, (get larger, and not converge to some number). Use the table functions on the handheld as an aid.

**Possible discussion points:** The sum would converge to some finite value. If *r* is greater than or equal to 1, the sum will diverge.

4. When the common ratio is larger than 1, explain what happens to the graph and values of *y*.

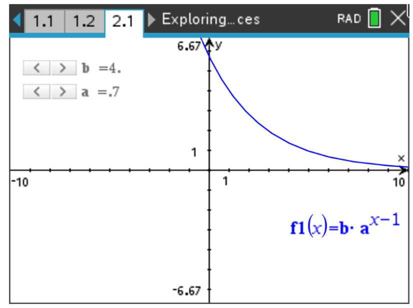
**Possible discussion points:** The graph increases from left to right making the values of *y* larger.

5. Discuss with a classmate the *r*-values that could model the heights of a ball bounce. Share your results with the class.

**Possible discussion points:** The *r*-value would be between 0 and 1 because the height of the ball becomes smaller with each bounce.

### Problem 2 – Changing the Initial Value and the Common Ratio

Move to **page 2.1**. On this graph is the function $\boldsymbol{f1(x) = b \cdot a^{x-1}}$. In the top left corner you will see one slider for the common ratio, $\boldsymbol{a}$ and one slider for the initial value, $\boldsymbol{b}$. By pressing on the left and right arrows of each slider, you will change the value of $\boldsymbol{a}$ on the interval $\boldsymbol{-0.1 \le a \le 1.1}$, with a step of $\boldsymbol{0.1}$, and the value of $\boldsymbol{b}$ on the interval $\boldsymbol{1 \le b \le 5}$, with a step of $\boldsymbol{0.1}$. Use the sliders and the resulting function to answer the following question.




**6.** Explain your observations of what happens when $\boldsymbol{b}$ changes. Describe, in the context of a real world problem, what $\boldsymbol{b}$ is also known as.

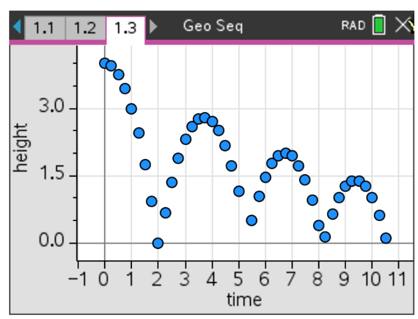
**Possible discussion points:** The graph becomes more steep. $\boldsymbol{b}$ is the initial value.

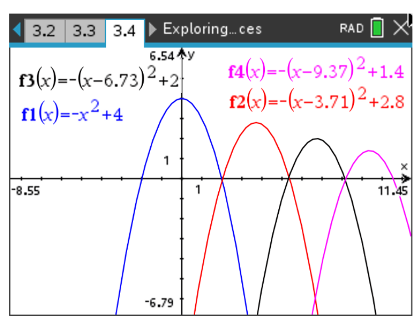
**7.** Discuss with a classmate which variable seems to have a more profound effect on the sequence. Share your results with the class.

**Possible discussion points:** This question can spawn some constructive discussion. Students may have differing opinions.

### Further Discussion

If time permits, move to pages 3.1 – 3.4. Discuss how the bouncing ball data was generated, making connections to the modeling of real world quadratic data, quadratic transformations, and gravity. Finally, discuss how the bouncing ball connects to geometric sequences.




**Possible discussion points:** This is a great opportunity to use motion sensors or high speed digital cameras to collect and model data of bouncing balls. You can then discuss that the parabolas are basically have the same “a” values but are only translated, and then you can discuss the “a” value of each parabola and how close it is to the value of gravity.

**Extension – Deriving and Applying the Partial Sum Formula**

The sum of a finite geometric series can be useful for calculating funds in your bank account, the depreciation of a car, or the population growth of a city.

e.g. $S_6 = 4 + 8 + 16 + 32 + 64 + 128$

In this example, the common ratio is 2, the first term is 4, and there are 6 terms.

The general formula:

$$\boldsymbol{S_n = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n}$$

Because $a_n = r \cdot a_{n-1}$, substituting gives

$$S_n = a_1 + r \cdot a_1 + r^2 \cdot a_1 + r^3 \cdot a_1 + \cdots + r^{n-2} \cdot a_1 + r^{n-1} \cdot a_1$$

$$r \cdot S_n = r \cdot a_1 + r^2 \cdot a_1 + r^2 \cdot a_1 + \cdots + r^{n-1} \cdot a_1 + r^n \cdot a_1$$

Subtract the previous two lines.

$$S_n - r \cdot S_n = a_1 - r^n \cdot a_1$$

$$S_n(1 - r) = a_1(1 - r^n)$$

So, $S_n = a_1 \cdot \frac{1-r^n}{1-r}$

Use the formula to find the sum of the following finite geometric series.

**8.** Find $S_5$ for $a_n = 6\left(\frac{1}{3}\right)^{n-1}$.

**Solution:** $S_5 = \frac{6\left(1-\left(\frac{1}{3}\right)^5\right)}{1-\frac{1}{3}} = \frac{242}{27} \approx 8.96$

**9.** $\frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \frac{1}{7^5} + \frac{1}{7^6} =$

**Solution:** $S_6 = \frac{\frac{1}{7}\left(1-\left(\frac{1}{7}\right)^6\right)}{1-\frac{1}{7}} = \frac{19608}{117649} \approx 0.167$

**10.** Find $S_{25}$ for $a_n = 2(1.01)^{n-1}$.

**Solution:** $S_{25} = \frac{2(1-1.01^{25})}{1-1.01} \approx 56.5$

**11.** $64-32+16-8+4-2+1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\frac{1}{64}-\frac{1}{128}+\frac{1}{256}=$

**Solution:** $S_{15}=\frac{64\left(1-\left(-\frac{1}{2}\right)^{15}\right)}{1-\left(-\frac{1}{2}\right)}=\frac{10923}{256}\approx 42.7$

> **TI-Nspire Navigator Opportunity:** ***Quick Poll (Open Response)***
> (to review before the Further IB Extension)
> Solve the exponential equation using logarithms, round to three significant figures:
> $$6^y = 219$$

**Further IB Extension**

Mac was trying out a new cheesecake recipe. Once completed, he will be serving it to his family. Loving the art of math, he decides to cut the slices using the cheesecake's volume. Each slice will represent a term in a geometric sequence, with the smallest being cut first.

The second smallest slice has a volume of $80\ cm^3$. The fourth smallest slice has a volume of $1280\ cm^3$.

(a) Find the common ratio. [2 marks]

**Solution:** $u_n = u_1 \cdot r^{n-1}$

$u_2 = u_1 \cdot r^{2-1}$ and $u_4 = u_1 \cdot r^{4-1}$

$80 = u_1 \cdot r$ and $1280 = u_1 \cdot r^3$

$\frac{1280 = u_1 \cdot r^3}{80 = u_1 \cdot r}$

$r^2 = 16$

$r = \pm 4$, since the volumes are all positive,

$r = 4$

(b) Find the volume of the smallest slice. [2 marks]

**Solution:** $u_n = u_1 \cdot r^{n-1}$

$80 = u_1 \cdot 4^{2-1}$

$80 = 4 \cdot u_1$

$u_1 = 20\ cm^3$

(c) The cheesecake has a total volume of $27{,}300\ cm^3$, find how many family members get to try Mac's delicious cheesecake.

[2 marks]

**Solution:** $S_n = \frac{u_1(1-r^n)}{1-r}$

$27{,}300 = \frac{20(1-4^n)}{1-4}$

$27{,}300 = \frac{20(1-4^n)}{-3}$

$-81{,}900 = 20(1-4^n)$

$-4095 = 1-4^n$

$-4096 = -4^n$

$4096 = 4^n$

$\log_4 4096 = n$

$n = 6$

**Teacher Tip:** Please know that throughout this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review Geometric Sequences and Series, but also to generate discussion.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*