



Math Objectives

- Students will use sinusoidal regression to determine equations to model various sets and use the equations to make inferences.
- Students will have discussions about their results and apply them to real world situations.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Sinusoidal
- Regression
- Radian Measure

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 3 Geometry and Trigonometry **3.7**:
 - (a) The circular functions $\sin x$, $\cos x$, and $\tan x$; amplitude, their periodic nature, and their graphs
 - (b) Composite functions of the form $f(x) = a \sin(b(x + c)) + d$
 - (c) Transformations
 - (d) Real-life contexts

As a result, students will:

- Apply this information to real world situations.

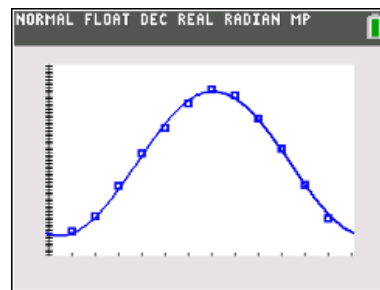
Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

Lesson Files:

Student Activity
 Find The Sine_Student-84CE.pdf
 Find The Sine_Student-84CE.doc
 KANSTEMP.8xp



In this activity, you will use the data in the file *KANSTEMP.8xp*, which contains the temperatures and hours of sunlight in Kansas City, and the heights of tides in the Bay of Fundy. You will find the equations of Sine curves that model the given data and answer several questions about what you have found.

****Note:** Make sure that your handheld is in Radian mode.

Teacher Tip: To see the data for this activity, the *KANSTEMP.8xp* file either must be downloaded to your student's handhelds or you must give them the data for each problem and have them answer the questions accordingly. The file is there to make visuals and analyzing easier to understand.

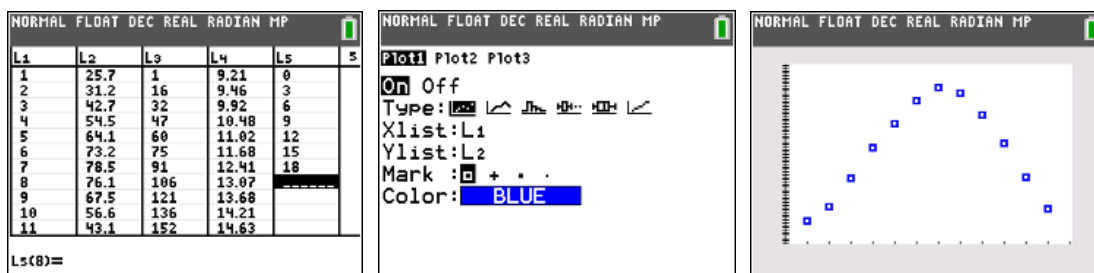
Problem 1 – Temperature

In this problem, you will graph data and find a sinusoidal function.

The temperature in Kansas City fluctuates from cold in the winter to hot in the summer. The average, monthly temperature (°F) will be loaded into **L₁** and **L₂** when you run the program. After your teacher has transferred the file to your handheld, press **prgm** to access the Program menu. Choose the **KANSTEMP** program and press **enter**. This will load the six lists with the data for all three problems.

Press **stat**, **1 Edit...** to see the data in the lists. The number of the month is in **L₁** and the temperature is in **L₂**. Note: The data that appears in **L₃** through **L₆** will be used later in the activity.

To graph the data, press **2nd y =** and **enter** to access **Plot1**. Make sure that the **Plot1** settings are the same as shown below. Press **zoom** and select **9:ZoomStat**. You will get a graph similar to the one below.



- (a) Find the sine equation that models the data. Press **stat**, arrow over to **CALC**, and select **C:SinReg**. Fill in the information with the following:
- XList: L YList: L₂ Store EQ: Y₁ **enter** to calculate

Rounding to the nearest hundredth, write down the sine equation in the form

$$y = a \cdot \sin[bx + c] + d.$$



Solution: $f(x) = 26.75 \sin(0.47x - 1.80) + 51.1$

Press **graph** to see the sine regression with the data.

- (b) Discuss with a classmate how well the sine curve of regression models the data. Share your thoughts with the class.

Possible Discussion: The equation is a good fit for the data. The set of data is periodic. It repeats the same cycle every 12 months.

- (c) Find the approximate temperature in Kansas City on March 15. Explain if this is a reliable temperature you found.

Solution: Approximately, between 47 and 48 degrees (47.6°). Since this value is for March, within the given data set (interpolation), it is reliable.

- (d) Using your sine curve of regression, estimate when your temperature will reach 85° in Kansas City and give an explanation of your results.

Solution: Since the average temperatures have a maximum of approximately 77.8° , the temperature will never reach 85° given this data set to be modeled.

Teacher Tip: This may be a good time to discuss the reliability of making predictions within the given data set and outside the given data set. Also, teachers may want to demonstrate how to find these estimates using the regression curve on the handheld.

Problem 2 – Hours of Sunlight

The amount of light a location on the Earth receives from the Sun changes each day depending upon the time of year and latitude of that location. The amount of daily sunshine Kansas City experiences has been recorded in the lists where the calendar day is in **L₃**, and the hours of sunlight is **L₄**.

Create the scatter plot and sine equation that models the data as outlined in **Problem 1**. To create the scatter plot, make sure to change the **XList** to **L₃** and the **YList** to **L₄**.

In early cultures, certain days of the year had significant importance because of the planting cycle. These days were the winter and summer solstices, and the spring and fall equinoxes. The equinoxes are the days with equal amounts of light and dark. The summer solstice has the greatest amount of sunlight, while the winter solstice has the fewest amount of sunlight.



(a) Rounding to the nearest hundredth, write down the sine equation.

Solution: $f(x) = 2.80 \sin(0.02x - 1.38) + 12.00$

(b) Discuss with a classmate how well the sine curve of regression models the data. Share your thoughts with the class.

Possible Discussion: The equation is a good fit for the data. The set of data is periodic. It repeats the same cycle every 365 days.

(c) Find the solstice and equinox dates using your sine curve of regression. Make sure that after you find your regression curve that you turn off **Plot1** so you can see your curve and not the data points.

Winter Solstice _____ Summer Solstice _____

Solution: The summer solstice is on 173.3 calendar days or June 22. The winter solstice is on 357.76 calendar days or December 22.

Fall Equinox _____ Spring Equinox _____

Solution: The vernal (spring) equinox is on 81.64 calendar days or March 22. The autumnal (fall) equinox is on 266.97 calendar days or September 22.

Problem 3 – Tides

The Bay of Fundy has the highest tides in the world. If a tape measure were attached at the water line of a pier, and the water level height were recorded over a period of eighteen hours, data like that in **L₅** and **L₆** would be generated.

Create the scatter plot and sine equation that models the data as outlined in **Problems 1 and 2**. To create the scatter plot, make sure to change the **XList** to **L₅** and the **YList** to **L₆**.

(a) Rounding to the nearest hundredth, write down the sine equation.

Solution: $f(x) = 4.44 \sin(0.52x - 3.11) + 6.22$



- (b) Discuss with a classmate how well the sine curve of regression models the data. Share your thoughts with the class.

Possible Discussion: The equation is a good fit for the data. The set of data is periodic. It repeats the same cycle every 12 hours.

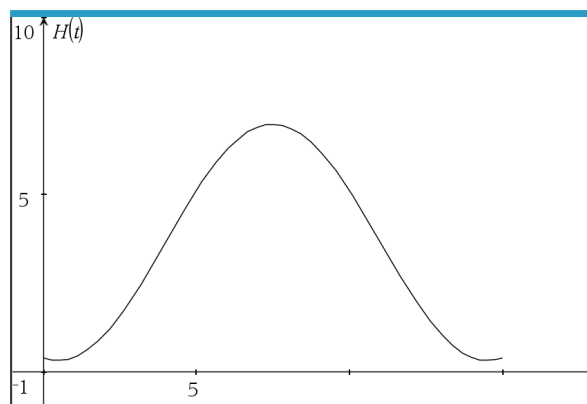
- (c) Using the sine curve of regression for the data, predict the water level when the time is 49 hours after the readings were started. Share one or more ways you can attain this answer by using your TI-84 Plus CE handheld.

Solution: At 49 hours, the water will be at approximately 4.52 feet.

Teacher Tip: It will be useful at some point to check your student’s handheld knowledge when it comes to making these predictions using the regression curve. An example may be reminding them that drawing horizontal lines and finding points of intersection can be helpful.

Further IB Applications

The height of the tides off the coast of Fripp Island, SC is modelled by the function $H(t) = a \sin(b(t - c)) + d$, where t is the number of hours after midnight, and a , b , c , and d are constants, where $a > 0$, $b > 0$, and $c > 0$. The following graph shows the water for 15 hours, starting at midnight.



The first high tide occurs at 7:10 am and the next occurs 12 hours later. Throughout the day, the height of the water fluctuates between 6.9 ft. and 0.4 ft. All heights are given correct to one decimal place.



(a) Show that $b = \frac{\pi}{6}$.

Solution: $\frac{2\pi}{b} = 12 \quad 12b = 2\pi \quad b = \frac{2\pi}{12} \text{ or } \frac{\pi}{6}$

(b) Find the value of a .

Solution: $\frac{\max - \min}{2} = \frac{6.9 - 0.4}{2} = \frac{6.5}{2} = 3.25$

(c) Find the value of d .

Solution: $\frac{\max + \min}{2} = \frac{6.9 + 0.4}{2} = \frac{7.3}{2} = 3.65$

(d) Find the smallest possible value for c .

Solution: Substituting $t = 7:10 \text{ am } (7.167)$ and $H = 6.9 \text{ ft}$ into their equation for H

$$6.9 = 3.25 \sin\left(\frac{\pi}{6}(7.167 - c)\right) + 3.65$$

$$c \approx 4.167$$

(e) Find the height of the tide at 12:00 pm.

Solution: Using the full equation found from parts (a) – (d), plug in $t = 12$ or $t = 0$

$$H = 0.98808 \dots$$

(f) Determine the number of hours, over a 24 hour period, that the tide is over 4 ft.

Solution: Solve the equation $4 = 3.25 \sin\left(\frac{\pi}{6}(x - 4.167)\right) + 3.65$

$$t = 4.37, 9.96 \quad \text{and} \quad t = 16.37, 21.96$$

Between each of those pairs of solutions, the tide is over 4 ft.

$$9.96 - 4.37 = 5.59 \quad \text{and} \quad 21.96 - 16.37 = 5.59$$

$$5.59 + 5.59 = 11.18 \text{ hours the tide is over 4 ft in a 24 hour period}$$

Teacher Tip: Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review sinusoidal regression and interpretation, but also to generate discussion.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*