Too Many Choices!

ID: 11762

Time required
40 minutes

Activity Overview

In this activity, students will investigate the fundamental counting principle, permutations, and combinations. They will find the pattern in each situation and apply it to make predictions. After a teacher-led discussion on the formulas, students will apply them to several problems.

Topic: Probability

- *Counting methods*
- *Permutations*
- *Combinations*

Teacher Preparation and Notes

- *This activity should be teacher-led. It allows for some student discovery and some inquiry questioning by the teacher.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "11762" in the keyword search box.***

Associated Materials

- *TooManyChoices_Student.doc*

Suggested Related Activities

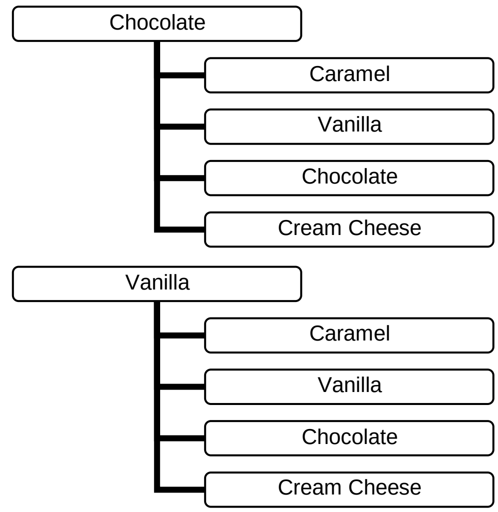
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Combinations (TI-Nspire technology) — 8433*
- *Permutations (TI-Nspire technology) — 8432*
- *What's Your Combination (TI-84 Plus family) — 10126*
- *How Likely Is It? Exploring Probability (TI-Nspire technology) — 9236*
- *Permutations & Combinations (TI-84 Plus family) — 12601*

Problem 1 – Exploring the Fundamental Counting Principle

Students may construct a diagram to determine the different cakes that Jayden can choose from if each cake has one kind of cake flavor and one kind of icing. One possible diagram is shown.

Students are to determine a multiplication sentence that represents the problem. This will help them develop a formula at the end of this part of the activity.



Students will determine how many outfits of one pair of pants and one shirt that Jess has if she owns 3 pairs of pants and 5 shirts. Then they can create the multiplication sentence, pairs of pants × shirts.

Students may find it helpful to draw a diagram using 3 types of pants (e.g., Jeans, Khakis, and Black) and 5 different color shirts (e.g., Pink, Red, White, Green, and Purple) to determine the number of outfits.

Students are given a general problem of Tiana choosing one entrée and one side from *m* entrees and *n* sides. They need to determine the formula for the total number of meals Tiana can choose from (**m × n**).

Discuss with students the **Fundamental Counting Principle**, which says that if one event can happen *m* ways and a second event can happen *n* ways, then together they can happen $m \times n$ ways.

Students can then work through Try These on the worksheet using the Counting Principle.

1. $31 \cdot 30 = 930$ days
2. $10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 = 676{,}000$ different plates

Discussion Questions

- How would the number of ice cream cones change if the parameters were changed? (The two flavors could be the same. Strawberry/Vanilla and Vanilla/Strawberry are considered the same cone.)
  - Would the number of cones be more or less?
  - What mathematical operation must take place for this to happen?
- How would the number of license plates change if the digits could not be repeated?

Problem 2 – Exploring Permutations

Students are to investigate the number of arrows that connect two points in a graph with a given number of points. It is important to know that arrows have direction, i.e., an arrow from point A to point B is not considered the same as an arrow from point B to point A. Students will use the answers from their diagrams to fill in the first four rows of the chart.

Note: It is important for students to understand that they are finding the number of arrows from one point to another no matter how many total points on the page.

When there are 3 points, the paths are:

A to B	B to A
A to C	C to A
B to C	C to B

Encourage students to look for a pattern and determine a formula they think will find the number of arrows for *n* points. They can use that formula to calculate the number of arrows for 6 and 7 points, completing the last two rows of the table.

Number of Points	Number of Arrows
2	2
3	6
4	12
5	20
6	30
7	42

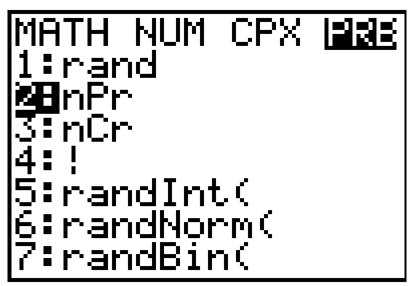
Discuss with students the definition of permutations and the formula.

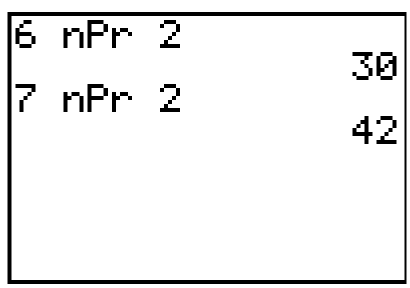
Explain that this arrangement of the paths is an example of a **permutation**, an arrangement of objects in which order matters. In general, the number of permutations is written:

$${}_nP_r = \underbrace{n(n-1)(n-2)\ldots}_{r \text{ factors}},$$

where n is the total number of objects and r is the number to be arranged.

Students will use the ***nPr*** command to check their answers in the table and complete Try These questions.





1. 9 nPr 9 = 362,880 batting orders
2. 6 nPr 3= 120 slates of officers
3. 16 nPr 3 = 3360 ways

Discussion Questions

- How does the formula for permutations follow from the fundamental counting principle?
- Introduce the factorial notation (n!).

Problem 3 – Exploring Combinations

Students will now focus on a version of one of the Try These questions from Problem 2 to consider a three-person committee versus a slate of three officers. The table illustrates the number of ways that each committee of three people could be chosen from a pool of six people (a, b, c, d, e, and f). Column 1 lists all possible ways to select the committee with people a, b, and c. Column 2 lists all possible ways to select a committee with people b, c, and d.

Make sure that students understand that order does not matter for a committee. All the groups in a column represent a single committee.

Students are encouraged to find another set of choices that correspond to one committee of people c, d, and e. They should see that there are still 6 possible combinations. For further investigation, have students determine how many slates there would be for a committee of 4 or 5, etc.

abc	bcd	cde	def
acb	bdc	ced	dfe
bac	cbd	dce	edf
bca	cdb	dec	efd
cab	dbc	ecd	fde
cba	dcb	edc	fed

Students then continue their investigation of the paths from Problem 2, but now they will count edges and ignore direction, e.g., consider an edge from point A to point B as the same as an edge from point B to point A. They should see that the number of edges is half the number of arrows.

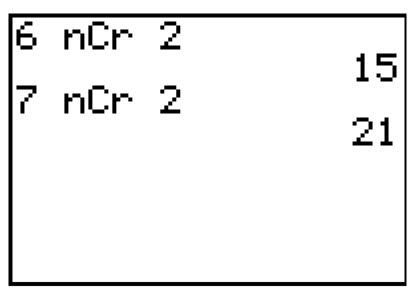
Discuss with students the definition of combinations and the formula to compute the answer.

Number of Points	Number of Arrows	Number of Edges
2	2	1
3	6	3
4	12	6
5	20	10
6	30	15
7	42	21

The committee problem and the edge problem are examples of **combinations,** an arrangement of objects in which order does not matter. In general, this can be written:

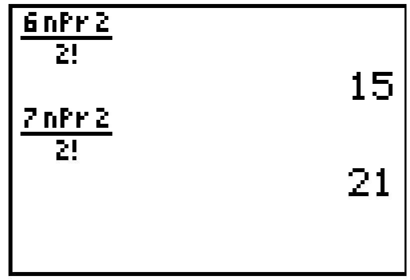
$$ {}_nC_r = \frac{\text{number of permutations}}{r!} $$

The students may use the **MATH > PRB > nCr** command to determine the number of edges for 6 and 7 points.



If using MathPrint™ OS:

When using the formula, students can use the fraction template to compute the answers. They should press [ALPHA] [F1] and select **n/d**. Then they can enter the number of permutations either using the nPr command or the permutations formula. Then they can press [▼] to move to the denominator and enter the appropriate expression.



Discussion Questions

- Why do we divide the number of permutations by ***r*!** to find the number of combinations?
- Can there ever be more combinations than permutations for the same number of elements?
- Can ${}_nP_r = {}_nC_r$ for the same *n* and *r*?

Students are to use the combinations formula and the **nCr** command to answer the Try These problems.

1. 12 nCr 5 = 792 teams
2. 52 nCr 5 = 2,598,960 hands
3. 42 nCr 3 * 57 nCr 3 = 335,904,800 committees