

TI in Focus: AP[®] Calculus

2017 AP[®] Calculus Exam: AB-2

Technology Solutions and Problem Extensions

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Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Solutions using technology
- (4) Problem extensions

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \leq 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4 \ln(t^2 + 2t) \text{ for } 3 < t \leq 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

- (a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?
- (b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.
- (c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.
- (d) How many pounds of bananas are on the display table at time $t = 8$?

$$(a) \int_0^2 f(t) dt = 20.051175$$

20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

$$(b) f'(7) = -8.120 \text{ (or } -8.119 \text{)}$$

After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.

$$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$$

$$(c) g(5) - f(5) = -2.263103 < 0$$

Since $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.

$$2 : \begin{cases} 1 : \text{considers } f(5) \text{ and } g(5) \\ 1 : \text{answer with reason} \end{cases}$$

$$(d) 50 + \int_3^8 g(t) dt - \int_0^8 f(t) dt = 23.347396$$

23.347 pounds of bananas are on the display table at time $t = 8$.

$$3 : \begin{cases} 2 : \text{integrals} \\ 1 : \text{answer} \end{cases}$$

Part (a)

Use a definite integral to accumulate the pounds of bananas removed from the display table during the first 2 hours the store is open.

$$\int_0^2 f(t) dt = 20.051$$

A screenshot of a TI-84 Plus calculator interface. The top status bar shows '1.1', '*Doc', 'RAD', and a red 'X' icon. The main display area shows the function definition $f(t) := 10 + 0.8 \cdot t \cdot \sin\left(\frac{t^3}{100}\right)$ on the left and the word 'Done' on the right. Below this, the definite integral $\int_0^2 f(t) dt$ is shown on the left, and the numerical result '20.0512' is shown on the right. The calculator interface includes a scroll bar on the right side.

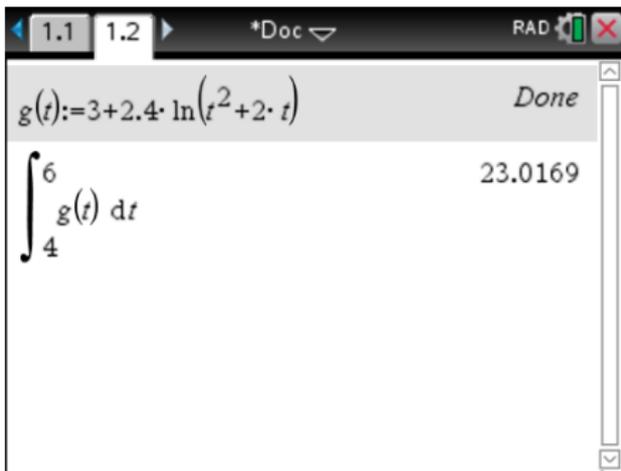
Example 1 Banana Removal

- (a) How many pounds of bananas are added to the display table during the time interval $4 \leq t \leq 6$?
- (b) Find the maximum rate at which bananas are added to the display table for $3 < t \leq 12$.

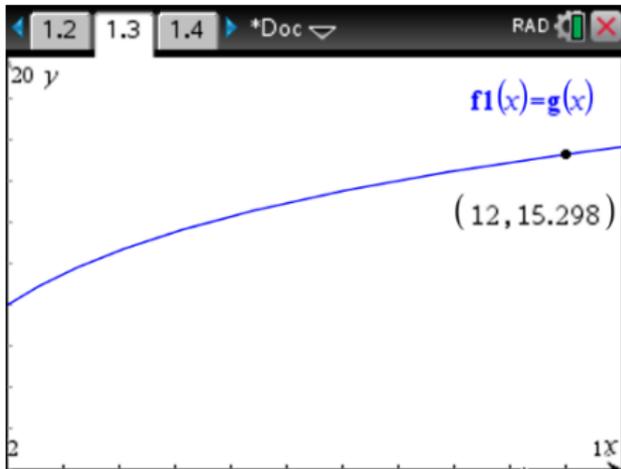
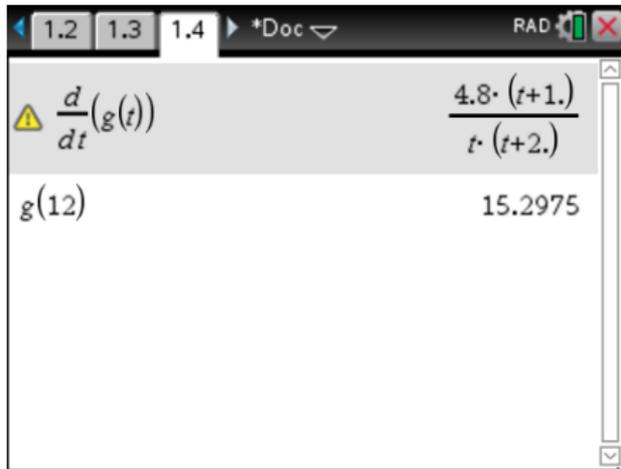
Solution

- (a) Use a definite integral to accumulate the pounds of bananas added to the display table during the time interval $4 \leq t \leq 6$.

$$\int_4^6 g(t) dt = 23.017$$



$$(b) g'(t) = 2.4 \cdot \frac{2t + 2}{t^2 + 2t} = \frac{4.8(t + 1)}{t(t + 2)} > 0 \text{ for } 3 < t \leq 12$$



The maximum rate at which bananas are added to the display table is 15.298 bananas/hr².

Part (b)

Use technology to find $f'(7)$.

$$f'(7) = -8.120$$

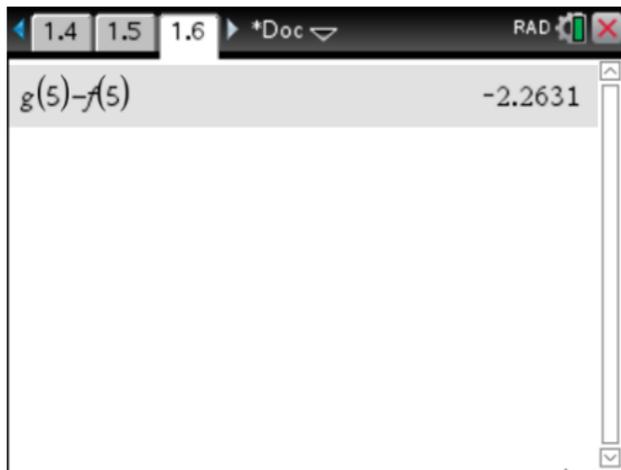


At the moment the store has been open for 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 pounds per hour per hour.

Part(c)

Consider the difference in rates at time $t = 5$.

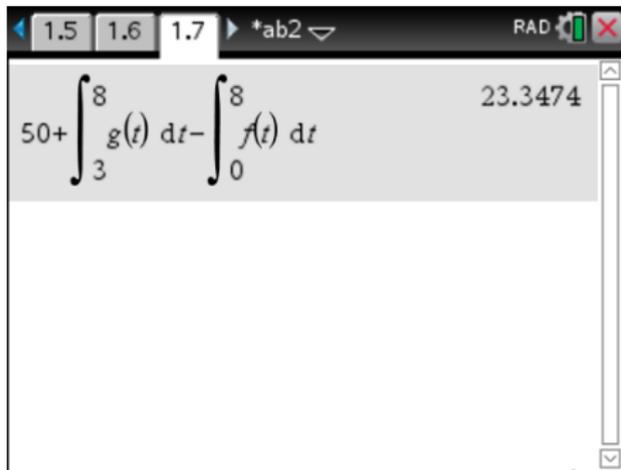
$$g(5) - f(5) = -2.263$$



Since $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.

Part(d)

Consider an expression that represents the total number of pounds of bananas on the display table at time $t = 8$.



A TI-84 Plus calculator screen showing the evaluation of the expression $50 + \int_3^8 g(t) dt - \int_0^8 f(t) dt$. The result displayed is 23.3474. The mode is set to RAD (Radians).

$$50 + \int_3^8 g(t) dt - \int_0^8 f(t) dt = 23.3474$$

23.347 pounds of bananas are on the display table at time $t = 8$.

Example 2 Could We Use a Smaller Table

- (a) Find the minimum pounds of bananas on the display table for $0 < t \leq 12$.
- (b) Suppose the grocery store is open for $0 < t \leq 24$ and assume the rate models are valid on this extended interval. Find the first time the display table will have 75 pounds of bananas.

Solution

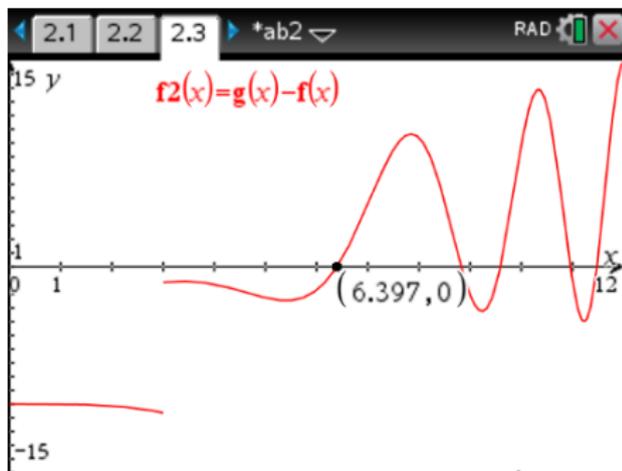
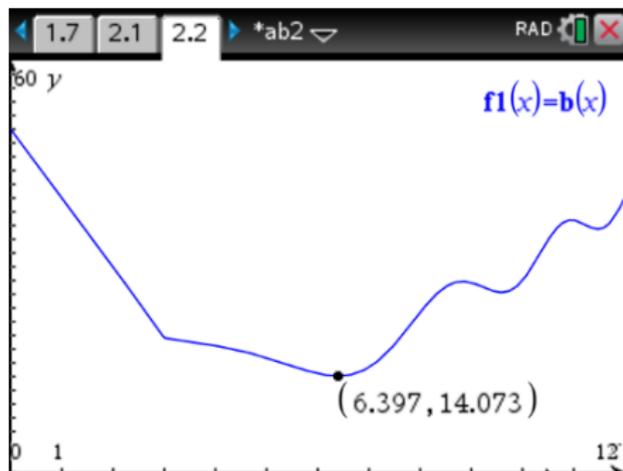
Redefine g so that it includes the interval $0 < t \leq 3$.

$$g(t) = \begin{cases} 0 & 0 < t \leq 3 \\ 3 + 2.4 \ln(t^2 + 2t) & 3 < t \leq 24 \end{cases}$$

Let the function b represent the pounds of bananas on the display table at any time t .

$$b(x) = 50 + \int_0^x g(t) dt - \int_0^x f(t) dt$$

Consider $b'(x) = g(x) - f(x) = 0$

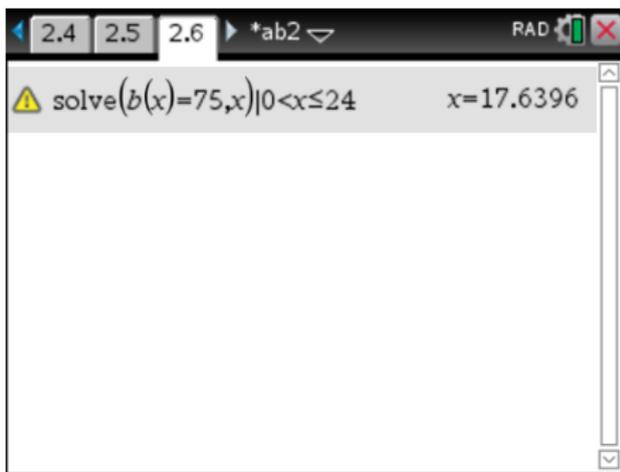
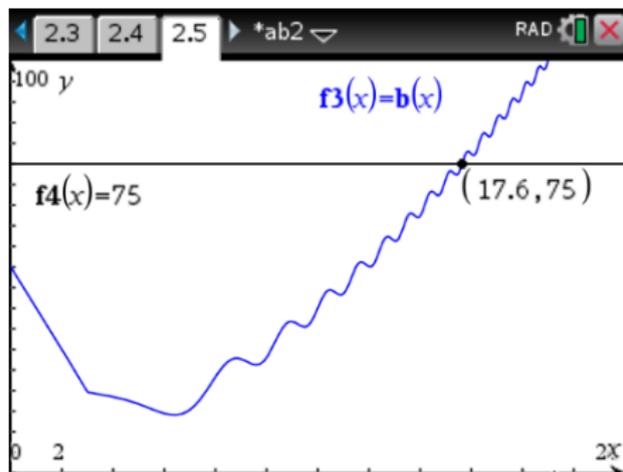


The minimum pounds of bananas on the display table is 14.073.

Note

- For $0 < t \leq 3$ the graph of b looks like a straight line?
- Why is there a break in the graph of $b'(x)$?
- There are other values of x such that $b'(x) = 0$. How can we be sure the absolute minimum does not occur at any of these values?

Consider the graph of b for $0 < x \leq 24$.



The display table will have 75 pounds of bananas at time $t = 17.640$.

More Questions

- (1) Describe the graph of b as x increases without bound?
- (2) Does the graph of b have an oblique, or slant, asymptote?
- (3) Suppose the rates can be modeled by

$$f(t) = 20 + 0.8t \sin\left(\frac{t^{3/2}}{5}\right)$$

$$g(t) = 12 + \frac{20 \ln(t + 2)}{1 + 0.2(t - 10)^2}$$

for $0 < t \leq 24$. Find the absolute minimum and the absolute maximum pounds of bananas on the display table.

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