2017 AP Calculus Exam: AB-5 Scoring Guidelines

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5. Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln\left(t^2 - 2t + 10\right)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

   (a) For $0 \le t \le 8$, when is particle P moving to the left?

   (b) For $0 \le t \le 8$, find all times t during which the two particles travel in the same direction.

   (c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

   (d) Find the position of particle Q the first time it changes direction.

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(a) $x_P'(t) = \dfrac{2t-2}{t^2-2t+10} = \dfrac{2(t-1)}{t^2-2t+10}$

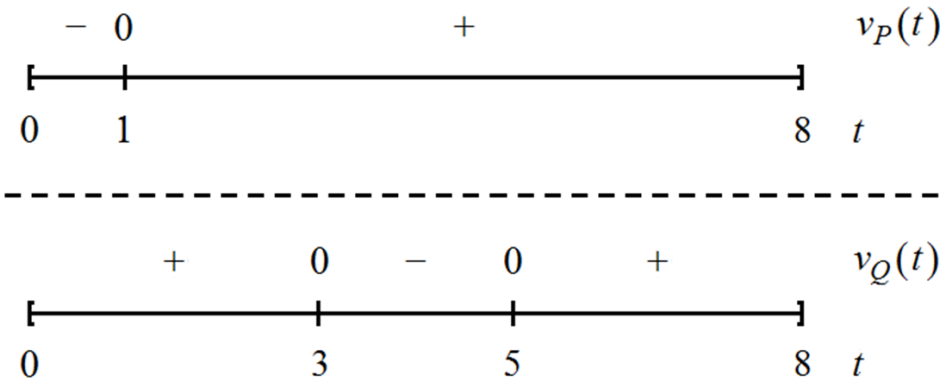
$t^2 - 2t + 10 > 0$ for all t.

$x_P'(t) = 0 \Rightarrow t = 1$
$x_P'(t) < 0$ for $0 \le t < 1$.

Therefore, the particle is moving to the left for $0 \le t < 1$.

$2 : \begin{cases} 1 : x_P'(t) \\ 1 : \text{interval} \end{cases}$

(b) $v_Q(t) = (t-5)(t-3)$
$v_Q(t) = 0 \Rightarrow t = 3, t = 5$



Both particles move in the same direction for $1 < t < 3$ and $5 < t \le 8$ since $v_P(t) = x_P'(t)$ and $v_Q(t)$ have the same sign on these intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{analysis using } v_P(t) \text{ and } v_Q(t) \end{cases}$

Note: $1/2$ if only one interval with analysis

Note: $0/2$ if no analysis

(c) $a_Q(t) = v_Q'(t) = 2t - 8$

$a_Q(2) = 2 \cdot 2 - 8 = -4$

$a_Q(2) < 0$ and $v_Q(2) = 3 > 0$

At time $t = 2$, the speed of the particle is decreasing because velocity and acceleration have opposite signs.

$$2 : \begin{cases} 1 : a_Q(2) \\ 1 : \text{speed decreasing with reason} \end{cases}$$

(d) Particle Q first changes direction at time $t = 3$.

$$x_Q(3) = x_Q(0) + \int_0^3 v_Q(t)\,dt = 5 + \int_0^3 \left(t^2 - 8t + 15\right) dt$$

$$= 5 + \left[\frac{1}{3}t^3 - 4t^2 + 15t\right]_{t=0}^{t=3} = 5 + (9 - 36 + 45) = 23$$

$$3 : \begin{cases} 1 : \text{antiderivative} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$$

Student Performance

(1) Subscripts in the function notation confused some students.

(2) Incorrect interval notation (notational fluency).

(3) Use of the correct function: position, velocity, acceleration.

(4) Part (a): Many successful; some trouble finding the derivative of a composite function involving $\ln$.

(5) Part (b): algebraic attempts; incorrect use of rules; relating the movement of two particles.

(6) Part (c): most found derivative of the velocity function; some used only the sign of the acceleration to determine speed.

(7) Part (d): most could find an antiderivative; some did not use the initial condition to find position.

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Part (a)

1: $x'_P(t)$

Derivatives that earn the first point:

(1) $\dfrac{2t-2}{t^2-2t+10}$ (standard, not factored)

(2) $\dfrac{2(t-1)}{t^2-2t+10}$

(3) $\dfrac{1}{t^2-2t+10}\cdot(2t-2)$

(4) $\dfrac{1}{t^2-2t+10}\cdot 2t-2=\dfrac{2t-2}{t^2-2t+10}$ (presentation error; clarified)

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Part (a)

1: $x'_P(t)$

Derivatives that do not earn the first point:

(1) $\dfrac{1}{t^2 - 2t + 10} \cdot 2t - 2$ (presentation error)

(2) $\displaystyle\int \ln(t^2 - 2t + 10)\, dt = \frac{2t - 2}{t^2 - 2t + 10}$

(3) Any other derivative.

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Part (a)

1: interval

With a correct derivative or the presentation error, the following earn the 2nd point:

(1) $0 \le t < 1;\ [0, 1)$ $\quad 0 < t < 1;\ (0, 1)$ $\quad 0 \le t \le 1;\ [0, 1]$

(2) $t < 1$ $\quad t \le 1$

(3) For t between 0 and 1; from 0 to 1; 0 to 1.

These expression do not earn the 2nd point:

(1) $t = 1$ $\quad (-\infty, 1]$ $\quad (-\infty, 1)$

Note: Using $v_Q(t)$: 0 - 0; bald correct interval: 0 - 0

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Part (b)

1: intervals

(1) Intervals may include or exclude the endpoints.

$1 < t < 3$ and $5 < t \leq 8$ $\quad$ $1 \leq t \leq 3$ and $t \geq 5$

$1 < t \leq 3$ and $5 < t < 8$ $\quad$ $(1, 3) \cup (5, 8]$

from $t = 1$ to $t = 3$ and $t = 5$ to $t = 8$

(2) Not acceptable: $(1, 3) \cup (5, \infty)$

We want our intervals and a correct analysis.

(1)	$(1, 3)$, $(5, 8]$ with sufficient analysis	1 - 1
(2)	$(1, 3)$, $(5, 8]$ with insufficient analysis	1 - 0
(3)	$(1, 3)$, $(5, 8]$ with no analysis	0 - 0

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Part (b)

1: intervals

Special Cases:

(1) $(1,3) \cup (5,\infty)$ with sufficient analysis 1/2

(2) Exactly one correct interval with sufficient analysis. 1/2

Any other intervals 0 - 0

Common Error: $(0,3) \cup (5,8]$

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Part (b)

1: analysis using $v_P(t)$ and $v_Q(t)$

Sufficient analysis with correct intervals must address both particle P and particle Q.

Particle Q:

(1) $V_Q(t)$ used with roots 3 and 5, and

(2) Each interval $(0, 3)$, $(3, 5)$, $(5, 8)$ analyzed with either

- Signs of $v_Q(t)$, or
- A value of $v_Q(t)$ with conclusion about the behavior of particle Q.

Particle P:

(1) $(0, 1)$ in part (a), or

(2) Similar velocity reasoning using $v_P(t)$.

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Part (c)

1: $a_Q(2)$

(1) The work presented must be correct.

(2) The following earn the point:

- $a_q(2) = 2 \cdot 2 - 8 = -4 \qquad a_q(t) = 2t - 8 = -4$
- $4 - 8 \qquad a(2) = -4$
- -4
- $-4 \quad 3$

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Part (c)

1: speed decreasing with reason

The student must consider the signs of both acceleration and velocity.

Eligibility:

(1) The correct value: -4, or

(2) Declared value: $a_Q(2) = k$, or

(3) Declared sign of $a_Q(2)$.

The student must present a consistent statement about the speed using both

(1) Sign of $a_Q(2)$, and

(2) Positive velocity at $t = 2$.

Note: $v_Q(2) = 3$ need not be shown, but if presented, must be correct.

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Part (d)

1: antiderivative

Must be the correct antiderivative (x is OK)

1: uses initial condition

Earns the point:

(1) $x_Q(0) + \int_0^b v_Q(t)\,dt$ where $0 < b \leq 8$

(2) $5 + \int_0^b v_Q(t)\,dt$ where $0 < b \leq 8$

Does not earn the point:

(1) $k + \int_0^b v_Q(t)\,dt$ where $k \neq 5$s

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Part (d)

1: answer

(1) Eligibility: earned the initial condition point.

(2) Linkage errors: cannot earn the answer point.

(3) Three common solutions:

- Definite integral approach: $x_Q(3) = x_Q(0) + \int_0^3 (t^2 - 8t + 15)\,dt$
- Indefinite integral approach: $x_Q(t) = \int (t^2 - 8t + 15)\,dt$
- Position function approach: $x_Q(t) = \frac{1}{3}t^3 - 4t^2 + 15t + 5$