

TI in Focus: AP[®] Calculus

2018 AP[®] Calculus Exam: AB-6

Technology Solutions and Problem Extensions

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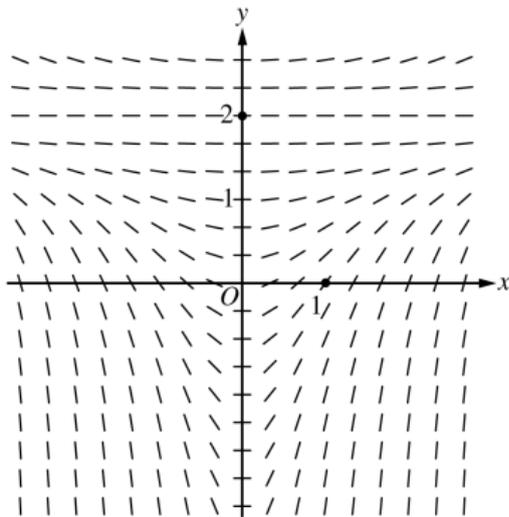
Former AP[®] Calculus Chief Reader

Outline

- (1) Free Response Question
- (2) Scoring Guidelines
- (3) Solutions using technology
- (4) Problem Extensions

6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$.

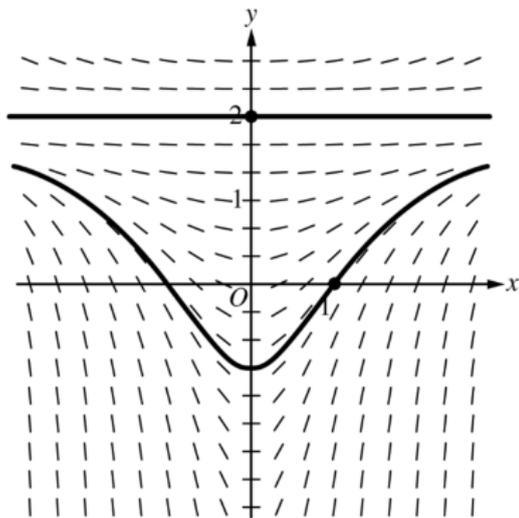
- (a) A slope field for the given differential equation is shown below. Sketch the solution curve that passes through the point $(0, 2)$, and sketch the solution curve that passes through the point $(1, 0)$.



6. Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$.

- (b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = 0$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$. Use your equation to approximate $f(0.7)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(1) = 0$.

(a)



$$2: \begin{cases} 1: \text{solution curve through } (0, 2) \\ 1: \text{solution curve through } (1, 0) \end{cases}$$

Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

$$(b) \left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = \frac{4}{3}$$

An equation for the line tangent to the graph of $y = f(x)$ at

$$x = 1 \text{ is } y = \frac{4}{3}(x - 1).$$

$$f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4$$

$$2: \begin{cases} 1: \text{equation of tangent line} \\ 1: \text{approximation} \end{cases}$$

$$(c) \frac{dy}{dx} = \frac{1}{3}x(y-2)^2$$

$$\int \frac{dy}{(y-2)^2} = \int \frac{1}{3}x \, dx$$

$$\frac{-1}{y-2} = \frac{1}{6}x^2 + C$$

$$\frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3}$$

$$\frac{-1}{y-2} = \frac{1}{6}x^2 + \frac{1}{3} = \frac{x^2 + 2}{6}$$

$$y = 2 - \frac{6}{x^2 + 2}$$

Note: this solution is valid for $-\infty < x < \infty$.

- | | | |
|-----|---|---|
| 5 : | { | 1 : separation of variables |
| | | 2 : antiderivatives |
| | | 1 : constant of integration
and uses initial condition |
| | | 1 : solves for y |
| | | |

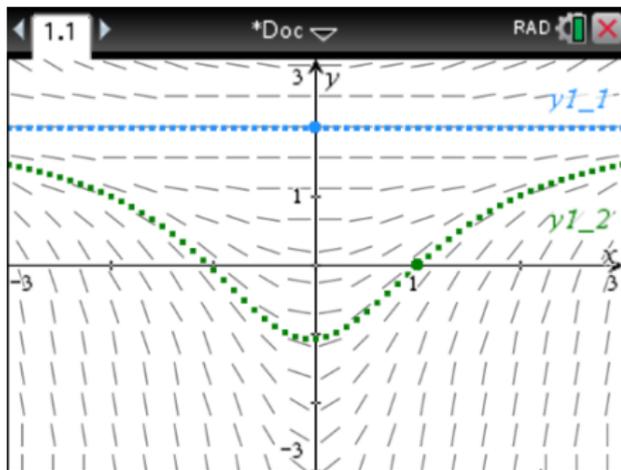
Note: 0/5 if no separation of variables

Note: max 3/5 [1-2-0-0] if no constant of integration

Part (a)

The solution curves must go through the indicated points, follow the given slope field lines, and extend to the boundary of the slope field.

Technology Solution



Example 1 Extended Slope Field

Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$

- (a) Sketch the slope field for this differential equation in the viewing window $[-3, 3] \times [0, 10]$.
- (b) Sketch the solution curve that passes through the point $(0, 5)$.
- (c) Consider the solution curve through the point (a, b) where $b \neq 2$. Explain why this solution curve cannot pass the line $y = 2$.

Part (b)

Find the slope of the tangent line.

$$\left. \frac{dy}{dx} \right|_{(x,y)=(1,0)} = \frac{1}{3} (1)(0 - 2)^2 = \frac{4}{3}$$

Write an equation of the tangent line.

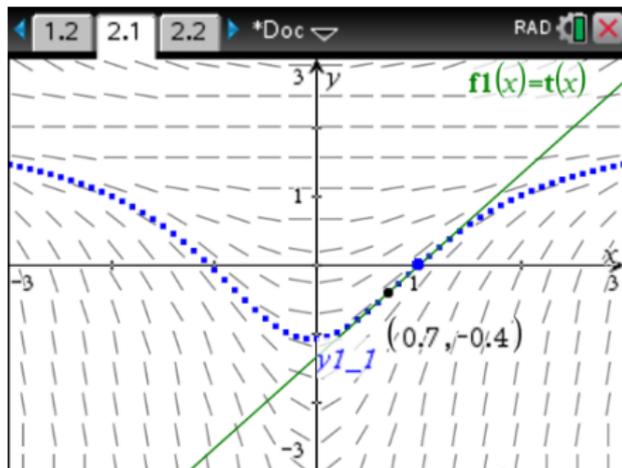
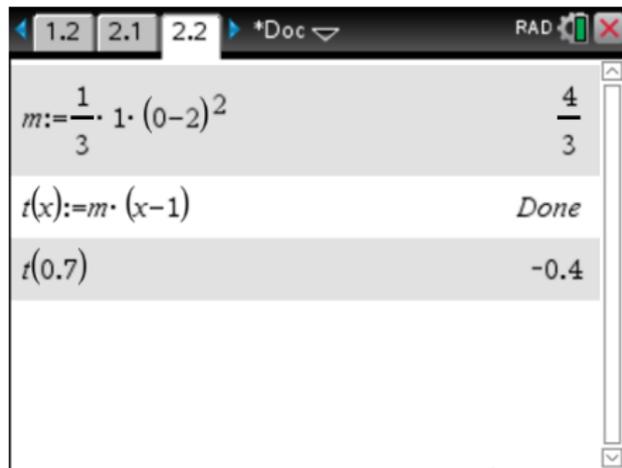
$$y - 0 = \frac{4}{3}(x - 1) \implies y = \frac{4}{3}(x - 1)$$

Use the tangent line equation to approximate $f(0.7)$.

$$f(0.7) \approx \frac{4}{3}(0.7 - 1) = \frac{4}{3} \left(-\frac{3}{10} \right) = -\frac{4}{10} = -0.4$$

Part (b)

Technology Solution



Part (c)

Find the particular solution $y = f(x)$ to the differential equation with initial condition $f(1) = 0$, using the method of separation of variables.

A screenshot of a TI-84 Plus calculator interface. The top status bar shows the mode set to 'RAD' and the angle indicator set to degrees. The calculator is in the 'DE Solver' application, with the equation $y' = \frac{1}{3} \cdot x \cdot (y-2)^2$ and the initial condition $y(1) = 0$ entered. The solution $y = 2 - \frac{6}{x^2 + 2}$ is displayed on the screen.

deSolve($y' = \frac{1}{3} \cdot x \cdot (y-2)^2$ and $y(1) = 0, x, y$)

$$y = 2 - \frac{6}{x^2 + 2}$$

Example 2 Under or Over

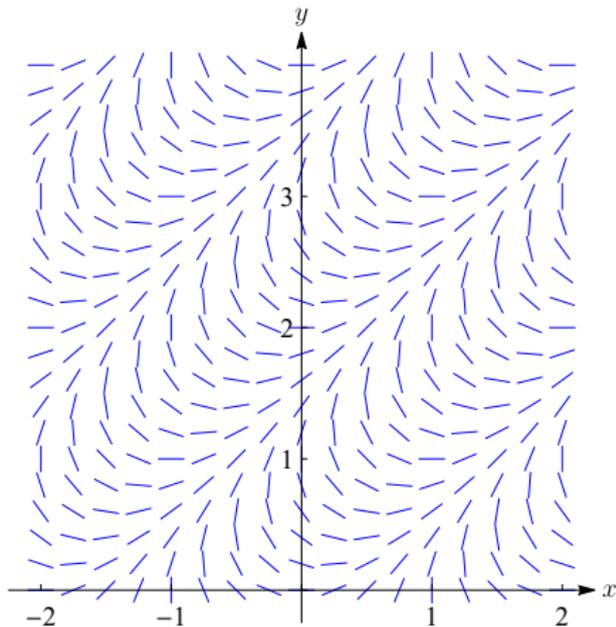
Consider the differential equation $\frac{dy}{dx} = \frac{1}{3}x(y - 2)^2$

- (a) Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(2) = 1$. Write an equation for the tangent line to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(0.5)$.
- (b) Is your approximation in part (a) an overestimate or an underestimate? Justify your answer.
- (c) Find the particular solution to the given differential equation with initial condition $f(2) = 1$.
- (d) Use your particular solution in part (c) to confirm your answer in part (b).

Example 3 Solution Curve Sketches

A slope field for the differential equation $\frac{dy}{dx} = \tan\left(\frac{1}{2}\pi(x - y)\right)$ is shown in the figure.

- (a) Sketch the graph of the solution curve through the given point.
- (i) $(0, 1)$
 - (ii) $(-1, 2)$
 - (iii) $(0, 2.5)$
- (b) Let $y = f(x)$ be the particular solution to the given differential equation with initial condition $f(1) = \frac{3}{2}$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.



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