TI in Focus: AP® Calculus

2019 AP® Calculus Exam: AB-1/BC-1
Scoring Guidelines

Stephen Kokoska
Professor, Bloomsburg University
Former AP® Calculus Chief Reader

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Outline

(1) Free Response Question
(2) Scoring Guidelines
(3) Student performance
(4) Interpretation
(5) Common errors
(6) Specific scoring examples

1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

   (a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ? Give your answer to the nearest whole number.

   (b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$) ?

   (c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

   (d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$) ? Explain your reasoning.

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(a) $\int_0^5 E(t)\,dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{1}{5-0}\int_0^5 L(t)\,dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$E(t) - L(t) = 0 \;\Rightarrow\; t = 6.20356$

$E(t) - L(t) > 0$ for $0 \le t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \le 8$. Therefore, the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

$3 : \begin{cases} 1 : \text{sets } E(t) - L(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

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Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$$A(t) = \int_0^t (E(s) - L(s))\, ds$$

$$A'(t) = E(t) - L(t) = 0 \;\Rightarrow\; t = C = 6.20356$$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore, the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{considers } E'(5) \text{ and } L'(5) \\ 1 : \text{answer with explanation} \end{cases}$

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Student Performance

Part (a)

- Most students knew they had to find $\int_0^5 E(t)\,dt$ using technology.
- Some students did not round to the nearest whole number. Those that did not; most had three decimal places of accuracy.
- Notational errors forgiven: $\int_0^5 E(t)\,dx \quad \int_0^5 E(t)$
- Some students simply found $E(5)$.
- Some interpreted $E(t)$ as an amount rather than a rate.

Part(b)

- Most students knew they had to find $\frac{1}{5}\int_0^5 L(t)\,dt$
- Some decimal presentation errors; 6.06, 6.
- Some students found an average rate of change; others average values: $L(0), \ldots, L(5)$.

Student Performance

Part(c)

- Many students knew they had to determine where $E(t) - L(t)$ changed signs. Many students solved the equation $E(t) = L(t)$ successfully.
- Some students found the maximum value of $E(t)$.
- Some students found the maximum value of $E(t) - L(t)$.
- Some students presented a local argument as justification.

Part (d)

- Most students considered $E'(5)$ and $L'(5)$.
- Some students considered $E(5)$ and $L(5)$.
- Some notational errors forgiven: $\frac{d}{dx}\left[E(t) - L(t)\right]_{x=5}$
- Some students gave incomplete explanations.

Part (a) 1: integral

(1) Must present $\int_0^5 E(t)\,dt$; Missing dt not penalized.

(2) Must be convinced that only $\int_0^5 E(t)\,dt$ is used to produce an answer.

(3) Copy error: 1 - 0

Part (a) 1: answer

(1) Bald answers do not earn the point.

(2) 153 or 154: acceptable answers.

(3) Non integer answers can earn the point:
Must be correct to the number of decimal places presented (up to 3).

(4) Sequential rounding not penalized:

$$\int_0^5 E(t)\,dt = 153.458 = 153.46 = 153.5 = 154$$

(5) Degree mode errors.

Part (b) 1: integral

(1) Must present $\int_0^5 L(t)\,dt$

(2) $1/5$ part of the second point.

(3) No other integrals involved in the answer.

(4) May import $\int_0^5 L(t)\,dt = 30.29519$ from part (a).

Part (b) 1: answer

(1) Average value must be presented to at least three decimal places.

(2) If answer is 6: decimal presentation error.
This does not earn the point.

Part (c) 1: sets $E(t) - L(t) = 0$

(1) Must be a clear attempt to determine where $E(t) - L(t)$ changes signs or where the graphs of $E(t)$ and $L(t)$ intersect.

(2) This can be expressed in words:
The place where it switches from $E(t) > L(t)$ to $E(t) < L(t)$.

(3) The words *functions* and *equations*:
Interpreted as referring to $E(t)$ and $L(t)$.

(4) However, the word *graphs*: without specifying which graphs, is ambiguous.

Part (c) 1: answer

(1) $t = 6.204$ (or 6.203) must be correct to at least three decimal places.

(2) Can earn this point without the first.
But there must be sufficient mathematics that motivates $t = 6.204$.

Example: $t = 6.204$ because that is where $\displaystyle\int_0^t [E(s) - L(s)]\,ds$ is maximum.

Part (c) 1: justification

Sign Test Approach

(1) Must be a convincing global argument.

$E(t) > L(t)$ for all values $0 \leq t < 6.204$.

$E(t) < L(t)$ for all values $6.204 < t \leq 8$.

(2) Endpoints, 0 and 8, do not need to included in the intervals.

(3) Sign chart is not sufficient justification.

Candidates Test Approach

(1) Ignore claims that $A(t)$ is the number of fish in the lake at time t.
Initial number of fish in the lake not known.

(2) Values of $A(t)$ must be correct to the number of decimal places provided.

(3) $A(0) = 0$ does not need to be included in the table of values.

Part (d) 1: considers $E'(5)$ and $L'(5)$

Philosophy: must be convinced there is an attempt to use $E'(5)$ and $L'(5)$.

(1) An attempt to find $E'(5)$ and $L'(5)$ symbolically earns the first point.

(2) If $E'(5)$ and $L'(5)$ are reported without any symbolic work, at least one must be correct.

(3) If only a difference is reported, must be ± 10.7228.

(4) $E'(5)$, $L'(5)$, $E'(5) - L'(5)$: do not need three decimal places.

Part (d) 1: answer with explanation

(1) Must be a connection between $E'(5)$ and $L'(5)$.

(2) Any reported values must be correct.

(3) Values do not need to be reported.

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