

TI in Focus: AP[®] Calculus

2020 Mock AP[®] Calculus Exam

AB-1: Solutions, Concepts, and Scoring Guidelines

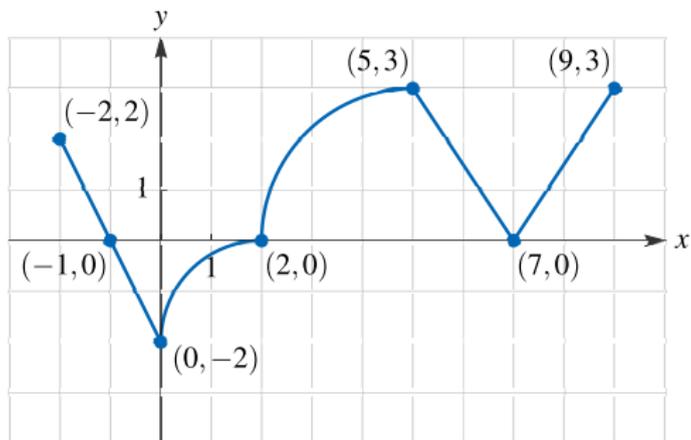
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AB 1

The continuous function f has domain $-2 \leq x \leq 9$. The graph of f , consisting of three line segments and two quarter circles, is shown in the figure.

Graph of f

Let g be the function defined by $g(x) = \int_0^x f(t) dt$ for $-2 \leq x \leq 9$.

- (a) Find the x -coordinate of each critical point of g on the interval $-2 \leq x \leq 9$.

Definition

A **critical number**, or **critical point**, of a function f is a number c in the domain of f such that $f'(c) = 0$ or $f'(c)$ does not exist.

The Fundamental Theorem of Calculus, Part 1

If f is a continuous function on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t) dt \quad a \leq x \leq b$$

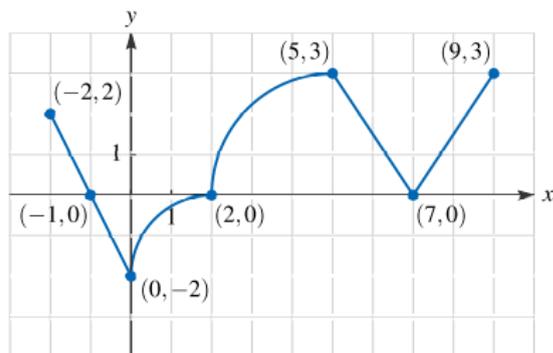
is continuous on $[a, b]$ and differentiable on (a, b) , and $g'(x) = f(x)$.

A Closer Look

1. In words: the derivative of a definite integral with respect to its upper limit is the integrand evaluated at the upper limit.
2. Other notation: $\frac{d}{dx} \left[\int_a^x f(t) dt \right] = f(x)$

Solution

$$g(x) = \int_0^x f(t) dt \Rightarrow g'(x) = f(x)$$



$$g'(x) = f(x) = 0: x = -1, x = 2, x = 7$$

$$g'(x) = f(x) \text{ DNE: none}$$

The critical points of g are $x = -1, 2,$ and 7 .

Scoring Guidelines

$$(a) g'(x) = f(x) = 0 \Rightarrow x = -1, 2, 7$$

1: answer

Scoring Notes

- To earn this point, the student must present all three critical values: -1 , 2 , and 7 .
- No supporting work required; any presented work must be correct.
- The point is not earned if extra numbers in the open interval $(-2, 9)$ or if any critical values are missing.
- Disregard any presentations of the endpoints or any other numbers outside the open interval $(-2, 9)$.
The point is still earned if the three critical points are presented.
- If the critical points are presented as ordered pairs: $(-1, 0)$, $(2, 0)$, and $(7, 0)$: read the x -coordinates and disregard the y -coordinates.

- (b) Classify each critical point from part (a) as the location of a relative minimum, a relative maximum, or neither for g . Justify your answers.

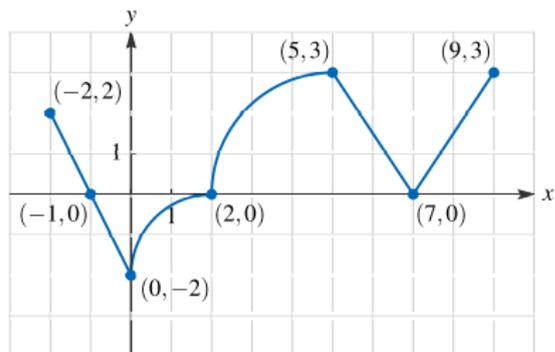
Background

- Fermat's Theorem: If f has a local maximum or minimum at c , then c must be a critical number of f .
- Not every critical number indicates a local maximum or a minimum.
- We need a method to determine whether f has a local maximum, or a local minimum, or neither, at a critical number c .

The First Derivative Test

Suppose that c is a critical number of a continuous function f .

- (a) If f' changes from positive to negative at c , then f has a local maximum at c .
- (b) If f' changes from negative to positive at c , then f has a local minimum at c .
- (c) If f' is positive to both the left and right of c , or negative to both the left and right of c , then f has neither a local maximum nor a local minimum at c .

Solution

$x = -1$: relative maximum; $g'(x) = f(x)$ changes from positive to negative there.

$x = 2$: relative minimum; $g'(x) = f(x)$ changes from negative to positive there.

$x = 7$: neither; $g'(x) = f(x)$ does not change sign there.

Scoring Guidelines

(b) At $x = -1$, g has a relative maximum because $g'(x) = f(x)$ changes from positive to negative there.

At $x = 2$, g has a relative minimum because $g'(x) = f(x)$ changes from negative to positive there.

At $x = 7$, g has neither because $g'(x) = f(x)$ does not change sign there.

3: $\left\{ \begin{array}{l} 1 : x = -1 \text{ relative maximum with} \\ \text{justification} \\ 1 : x = 2 \text{ relative minimum with} \\ \text{justification} \\ 1 : x = 7 \text{ neither with justification} \end{array} \right.$

Scoring Notes

- Each critical point from part (a) must be accompanied by a correct justification to earn the point.
- Justifications based on g' are eligible for all three points.
- If the response is based only on f :
Eligible for all three points if connection presented, $g' = f$, part (a) or part (b).
If no connection, then eligible for at most two of the three points.

Scoring Notes

- Read references to *the graph* as references to the graph of f as given in the statement of the problem.
- A response referencing *the function*, or *it*, or *the slope* is too vague.
This type of response can earn a maximum of one point in this part.
Earns the point if replacing every occurrence of *the function*, or *it*, or *the slope* with g' produces a response that would earn all three points.
- A response that discusses g' at a particular point: $g'(-1)$ changes from positive to negative.
Poor communication: can earn at most two of the three points.
(All three earns two points; two correct classifications earns one point.)
- A sign chart alone is not sufficient to earn any points.
A sign chart must be accompanied by a sufficient written explanation.
- A response that uses a candidate's test is eligible for all three points.
- Global versus local argument: at most two of three points.

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