TI in Focus: AP® Calculus

2022 AP® Calculus Exam: BC2
Scoring Guidelines and Solutions

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Outline

(1) Free Response Question

(2) Point Distribution

(3) Solutions (using technology)

(4) Scoring Notes

(5) Common Errors

(6) Problem Extensions

Part A (BC): Graphing calculator required

Question 2 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

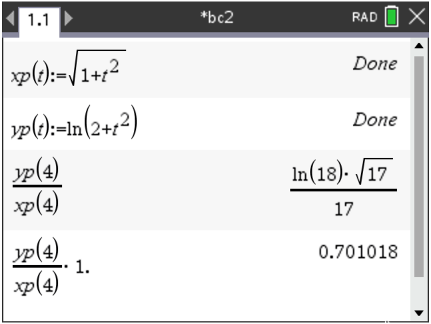
Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A particle moving along a curve in the xy-plane is at position $(x(t), y(t))$ at time $t > 0$. The particle moves in such a way that $\frac{dx}{dt} = \sqrt{1+t^2}$ and $\frac{dy}{dt} = \ln\left(2+t^2\right)$. At time $t = 4$, the particle is at the point $(1, 5)$.

Model Solution	Scoring	
(a) Find the slope of the line tangent to the path of the particle at time $t = 4$.		
$\left.\frac{dy}{dx}\right\|_{t=4} = \frac{y'(4)}{x'(4)} = \frac{\ln 18}{\sqrt{17}} = 0.701018$ The slope of the line tangent to the path of the particle at time $t = 4$ is 0.701.	Answer	**1 point**

Solution

$$\left.\frac{dy}{dx}\right|_{t=4} = \frac{y'(4)}{x'(4)} = \frac{\ln(2+4^2)}{\sqrt{1+4^2}} = \frac{\ln 18}{\sqrt{17}} = 0.701$$



Scoring notes:

- To earn the point, the setup used to perform the calculation must be evident in the response. The following examples earn the point: $\frac{y'(4)}{x'(4)} = 0.701$, $\frac{\ln\left(2 + 4^2\right)}{\sqrt{1 + 4^2}}$, or $\frac{\ln 18}{\sqrt{17}}$.

- Note: A response with an incorrect equation of the form "function = constant", such as $\frac{y'(t)}{x'(t)} = \frac{\ln(18)}{\sqrt{17}}$, will not earn the point. However, such a response will be eligible for any points for similar errors in subsequent parts.

Common Errors

(1) Most students recognized the need to find $\dfrac{y'(4)}{x'(4)}$.

(2) There were some communication errors, especially in equating a function to a specific value.

(3) Some students presented the reciprocal of the correct answer.

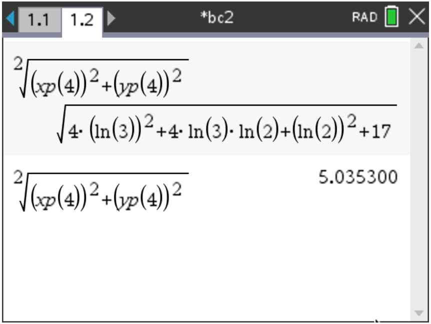
(b) Find the speed of the particle at time $t = 4$, and find the acceleration vector of the particle at time $t = 4$.

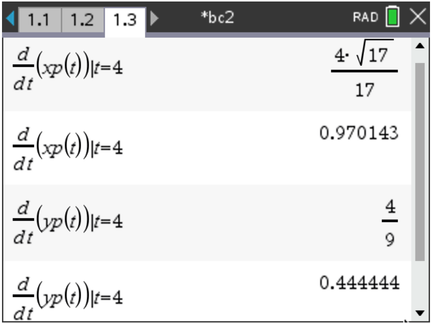
$\sqrt{(x'(4))^2 + (y'(4))^2} = \sqrt{17 + (\ln 18)^2} = 5.035300$ The speed of the particle at time $t = 4$ is 5.035.	Speed	**1 point**
$a(4) = \langle x''(4), y''(4) \rangle = \left\langle \frac{4}{\sqrt{17}}, \frac{4}{9} \right\rangle = \langle 0.970143, 0.444444 \rangle$ The acceleration vector of the particle at time $t = 4$ is $\langle 0.970, 0.444 \rangle$.	First component of acceleration	**1 point**
	Second component of acceleration	**1 point**

Solution

$$\text{Speed} = \sqrt{[x'(4)]^2 + [y'(4)]^2} = \sqrt{17 + (\ln 18)^2} = 5.035300$$

$$a(4) = \langle x''(4), y''(4)\rangle = \left\langle \frac{4}{\sqrt{17}}, \frac{4}{9} \right\rangle = \langle 0.970143, 0.444444\rangle$$





Scoring notes:

- To earn any of these points, the setup used to perform the calculation must be evident in the response. The following examples earn the first point: $\sqrt{(x'(4))^2 + (y'(4))^2} = 5.035$ or $\sqrt{17 + (\ln 18)^2}$ and $\langle x''(4), y''(4) \rangle = \left\langle \frac{4}{\sqrt{17}}, \frac{4}{9} \right\rangle$ would earn both the second and third points.

There must be supporting work. (See last item.)

- The second and third points can be earned independently.

The first and the second component of the acceleration vector can be earned independently.

- If the acceleration vector is not presented as an ordered pair, the x- and y-components must be labeled.

Good communication skills are necessary.

- If the components of the acceleration vector are reversed, the response does not earn either of the last 2 points.

$$a(4) = \langle y''(4), x''(4) \rangle = \langle 0.444, 0.970 \rangle$$

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- A response which correctly calculates expressions for both $x''(t) = \dfrac{t}{\sqrt{1 + t^2}}$ and $y''(t) = \dfrac{2t}{2 + t^2}$, but which fails to evaluate both of these expressions at $t = 4$, earns only 1 of the last 2 points.

- An unsupported acceleration vector earns only 1 of the last 2 points.

$$a(4) = \langle 0.970, 0.444 \rangle$$

Common Errors

(1) Equating the speed function with the value of the function at $t = 4$.

(2) Use of the notation $|v|$ without any indication about how the quantity v was defined.

(3) Parentheses errors that resulted in ambiguous or incorrect expressions.

(4) Many students found the components of the acceleration vector symbolically.
In this case: chain rule errors, power rule errors.

(5) Some responses found the length of the acceleration vector.

(6) Incorrect labels or no labels on vector components.

(c) Find the y-coordinate of the particle's position at time $t = 6$.

$y(6) = y(4) + \int_4^6 \ln\left(2 + t^2\right)\, dt$	Integrand	**1 point**
	Uses $y(4)$	**1 point**
$= 5 + 6.570517 = 11.570517$ The y-coordinate of the particle's position at time $t = 6$ is 11.571 (or 11.570).	Answer	**1 point**

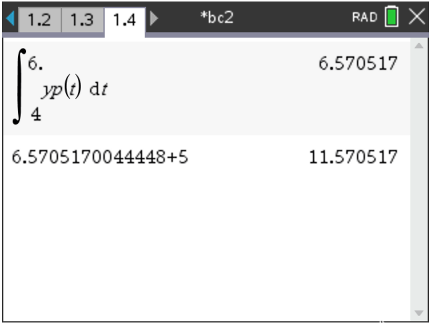
Solution

Given: $y(4) = 5$, Need $y(6)$

$$\int_4^6 y'(t)\,dt = y(6) - y(4) \quad \Rightarrow \quad y(6) = y(4) + \int_4^6 y'(t)\,dt$$

$$y(6) = 5 + 6.570517 = 11.570517$$

The y-coordinate of the particle's position at time $t = 6$ is 11.571.



Scoring notes:

- For the first point, an integrand of $\ln(2 + t^2)$ can appear in either an indefinite integral or an incorrect definite integral.

We want to see the correct integrand.

- A definite integral with incorrect limits is not eligible for the answer point.

A definite integral with correct integrand and incorrect limits cannot resolve to the correct answer. Therefore, this cannot earn the answer point.

- Similarly, an indefinite integral is not eligible for the answer point.

If there are no bounds on the integral, then the response is not eligible for the third point.

- For the second point, the value for $y(4)$ must be added to a definite integral. A response that reports the correct x-coordinate of the particle's position at time $t = 6$ as $x(6) = x(4) + \int_4^6 \sqrt{1 + t^2}\, dt = 11.200$ (or 11.201) instead of the y-coordinate, earns 2 out of the 3 points.

- A response that earns the first point but not the second can earn the third point with an answer of 6.571 (or 6.570).

$$\int_4^6 y'(t)\, dt = 6.571$$

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- If the differential is missing:
  - $y(6) = \int_4^6 \ln\left(2 + t^2\right)$ earns the first point and is eligible for the third.
  - $y(6) = \int_4^6 \ln\left(2 + t^2\right) + y(4)$ does not earn the first point but is eligible for the second and third points in the presence of the correct answer.
  - $y(6) = y(4) + \int_4^6 \ln\left(2 + t^2\right)$ earns the first two points and is eligible for the third.

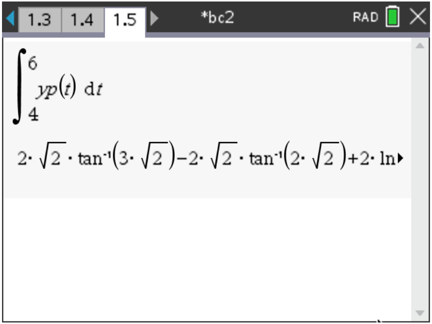
Common Errors

(1) Presentation of only an indefinite integral.

(2) Failure to add the initial position $y(4)$.

(3) Attempts to find $\int \ln(2+t^2)\,dt$ using integration by parts.

(4) CAS results for $\int_4^6 \ln(2+t^2)\,dt$



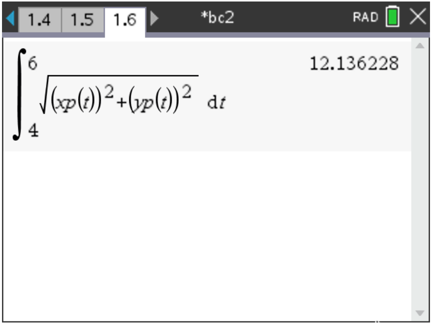
(d) Find the total distance the particle travels along the curve from time $t = 4$ to time $t = 6$.

$\int_4^6 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$	Integrand	**1 point**
$= 12.136228$ The total distance the particle travels along the curve from time $t = 4$ to time $t = 6$ is 12.136.	Answer	**1 point**

Solution

$$\text{Distance} = \int_4^6 \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

$$= 12.136$$



Scoring notes:

- The first point is earned for presenting the correct integrand in a definite integral.

$$\int_{\square}^{\square} \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

- To earn the second point, a response must have earned the first point and must present the value 12.136.

- An unsupported answer of 12.136 does not earn either point.

Common Errors

(1) Presentation errors: missing parentheses.

(2) Presentation of the definite integral $\int_4^6 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dt$.

(3) Presentation of the definite integral: $\int_4^6 \frac{dy}{dx}\, dt$

Additional Questions

(1) Find the x-coordinate of the particle's position at time $t = 6$.

(2) Find the particle's initial position, that is, at time $t = 0$.

(3) Find the particle's distance from the origin at time $t = 6$.

(4) Find the time at which the particle is closest to the origin. What is the position of the particle at that time? What is the distance?

(5) Find the time t_1 at which the particle crosses the x-axis. Find the time t_2 at which the particle crosses the y-axis. Find the total distance the particle travels along the curve from $t = t_1$ to $t = t_2$.

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